Shape trends and triaxiality in neutron-rich odd-mass Y and Nb isotopes

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Abstract

New level schemes of odd-Z $^{99, 101}$Y (Z=39) and $^{101, 105}$Nb (Z=41) are established based on the measurement of prompt gamma rays from the fission of ^{252}Cf at Gammasphere. Bands of $\pi 5/2^+$[422], $\pi 5/2^-$[303] and $\pi 3/2^-$[301] are observed and extended to provide spectroscopic information concerning nuclear shapes in this important odd-Z region. In combination with the level structure of the odd-Z Tc (Z=43), Rh (Z=45) and the neighboring even-Z isotopes the level systematics, signature splittings and kinematic and dynamic moments of inertia of the bands in the Y and Nb isotopes are discussed in terms of shape transition and triaxiality. The quadrupole deformations of the N = 60 (and N=62) isotones with Z = 39-45 decrease with increasing Z, and the deformation peaks at N = 60 for the Y (Z=39) isotopic chain and at N = 62 for the Nb (Z=41) isotopic series, following a similar trend in the neighboring even-even neutron-rich nuclei of Z = 38 – 42. The pronounced difference observed in the signature splittings between Y and Tc, Rh isotopes is interpreted as evidence of axially-symmetric deformed shape in the Y isotopes, and, as previously reported, large and near maximum triaxiality in Tc-Rh isotopes. The likely lowering of crossing frequencies of the ground-state bands in Tc and Rh isotones in comparison with those in Y isotones also implies a shape transition from axially-symmetric deformed shapes in Y nuclei to triaxiality in Tc and Rh isotones. Triaxial-rotor-plus-particle model calculations strongly support a pure axially-symmetric shape with large quadrupole deformation in Y isotopes. Triaxial-rotor-plus-particle model calculations yielded γ values ranging from -19^0 to -13^0 for the $5/2^+$[422] ground-state bands of $^{101, 103\ 105}$ Nb and of -5^0 for the two negative-parity bands in ^{101}Nb. The fact that Nb isotopes have intermediate values of signature splitting and band crossing frequencies between those of Y and Tc, Rh isotopes is interpreted as that the Nb isotopes are transitional nuclei with regard to triaxial deformation.

23.20.Lv, 21.10.Re, 25.85.Ca, 27.60.+j

I. Introduction

Studies of shape transitions and shape coexistence in neutron-rich nuclei with A ~ 100 have long been of major importance [1,2]. Intensive investigations have been carried out for even-even nuclei in this region. Large quadrupole deformations [3], the onset of superdeformed ground states and identical bands [2,4], shape evolutions, and shape coexistence [4-8] were observed in the Sr (Z=38) – Zr (Z=40) – Mo (Z=42) region. The sudden onset of superdeformation in the ground states of N=60, 62 Sr isotopes was interpreted in terms of the reinforcement of the proton and neutron shape driving force where the protons and neutrons have shell gaps at the same large deformation [5]. The triaxial degree of freedom has been reported in Mo (Z=42) and Ru (Z=44) isotopes [e.g. 9]. Recently, differences in band crossing frequencies and signature splittings were observed in the $\upsilon h_{11/2}$ bands between Zr and Mo isotopes and ascribed to the triaxial degree of freedom in Mo and the mainly axially-symmetric shape in Zr isotopes [10] indicating a transition from axially-symmetric shape in Zr (Z=40) to triaxiality in Mo (Z=42) isotopes.

Quadrupole deformation is found to decrease with increasing proton number between 38 and 42 [8]. Abrupt shape transitions from spherical to strongly deformed ground states are observed at N=60 in neutron-rich Sr (Z=38) and Zr (Z=40) isotopic chains. Smoother change in shape transition than in the Sr and Zr nuclei are found for isotopes with Z > 40 and are attributed to the appearance of triaxiality for Z > 40 nuclei, and to the partial occupancy of the $\pi g_{9/2}$ orbital at zero deformation for Z > 40 isotopes, while the orbital is not occupied in the spherical limit for Z ≤ 40 isotopes. The deformation varies with changing neutron number and peaks or saturates at N=60-64 for Z=38-42 isotopes. The occupancy of the $g_{9/2}$ proton and $h_{11/2}$ neutron orbitals and their interplay have an important role in the rich diversity of nuclear structure and shapes in this nuclear region.

Spectroscopic information of the odd-Z neighbors is of interest in understanding the shape transitions and the importance of the triaxial degree of freedom in this nuclear region. A shape transition from axially symmetric to triaxial deformation in odd-Z nuclei of this region is of particular interest. However, less has been reported for the odd-Z neighbors so far. High-spin states in $^{99, 101, 103}$ Tc (Z=43, N=56, 58, 60) have been studied and the single-proton states located around the Fermi level were discussed as a function of deformation [11,12,13]. Triaxial deformation was observed in ^{105}Rh (Z=45, N=60) and used to explain the large signature splitting in the ground-state band based on an $g_{9/2}$ odd-proton; while a shape change from triaxial (in the ground-state band) to prolate deformation (in the three- quasiparticle band with very small signature splitting) was discussed [14]. High-spin structures in $^{107, 109, 111, 113}$Rh and $^{105, 107, 109}$Tc were studied and revealed shape coexistence and the role played by triaxiality in these Tc and Rh isotopes [15,16,17,18]. Our GANDS2000 collaboration published detailed results on odd-Z, neutron-rich ^{111,113}Rh (Z=45) [17] isotopes and ^{105,107109}Tc (Z=43) [18] isotopes. Data yielded new level schemes with the highest excitation energies and spins yet established and considerably expanded weakly populated bands to provide rich spectroscopic information [17,18]. Triaxial-rotor-plus-particle model calculations performed with $\varepsilon = 0.32$ and $\gamma = -22.5^0$ on the prolate side of maximum triaxiality yielded the best reproduction of the excitation energies, signature splittings and branching ratios of the positive-parity bands (except for the intruder bands) in Tc isotopes [18]. The model calculations gave the best fits to positive-parity bands in Rh isotopes at near maximum triaxiality with $\gamma = -28^0$ (see Ref. 17). A side band built on an excited $11/2^+$ state with low excitation predominantly feeds the $9/2^+$ state and very weakly the $7/2^+$ state of the $\pi g_{9/2}$ yrast band in Tc and Rh isotopes. This feeding pattern is shown by model calculations to be strong additional evidence of triaxiality in Tc and Rh isotopes [17,18]. Large discrepancies between experiments and triaxial-rotor-plus-particle model calculations for the well-developed K=1/2, $1/2^+$[431] intruder bands possessing large prolate deformation observed in the $^{105, 107}$Tc isotopes stimulates further theoretical interest.

In the lower odd-Z region with Z = 39-41, where the triaxiality-related shape transition is expected to occur, still less-extended level schemes have been reported because of the comparatively weak populations from fission. Low-lying levels populated by beta decay were reported in [19] and the references therein. The $\pi 5/2^+$[422], $\pi 5/2^-$[303] and $\pi 3/2^-$[301] bands populated in Y and Nb isotopes by fission were proposed. Limited level schemes of $^{99, 101}$Y were reported [8, 20-23], and those of $^{101, 103}$Nb were first given in [8, 24] . Well-developed $\pi 5/2^+$[422] and $\pi 5/2^-$[303] bands can be found in ^{103}Nb in [25]. No decay data have been reported for ^{105}Nb. The $\pi 5/2^+$[422] band with only five levels was proposed in ^{105}Nb based on measurements of prompt gamma transitions from the spontaneous fission of ^{248}Cm [8].

We report in this paper new results on neutron-rich Y (Z=39) and Nb (Z=41) isotopes. New level schemes of ^{99}Y (N=60), ^{101}Y (N=62), ^{101}Nb (N=60) and ^{105}Nb (N=64) are proposed. Signature splittings, kinematic and dynamic moments of inertia as a function of spin and rotational frequencies, respectively, in the Y and Nb isotopes are discussed for the odd-Z, even-N isotopic chains, as well as for the N=60, 62, 64 isotonic chains by combining the data and results of the neighboring even-Z and odd-Z isotopes. Triaxial-rotor-plus-particle calculations were performed and reproduce the level excitations, signature splittings and branching ratios of the observed bands in Y and Nb isotopes. Shape transitions with regard to quadrupole deformation and triaxiality are discussed for neutron-rich nuclei in this Z = 39-45 region.

II. Experiment and data analysis

Measurements of prompt γ rays from a fission source by using multi-gamma detection arrays have been shown to be a powerful tool for population and detection of high-spin states of neutron-rich nuclei [2]. For two runs each taking two weeks in 2000, a ^{252}Cf source of 62 μCi, sandwiched between two 10 mg/cm^2 Fe foils, was placed in an 8-cm-polyethylene ball centered in the Gammasphere, which then consisted of 102 Compton-suppressed Ge detectors. Accumulation of over 5.7 * 10^{11} triple and higher-fold events provided excellent conditions for experimental observations for higher spin states and weakly populated bands.

To clarify ambiguities caused by peak overlapping, a less-compressed Radware cube was used. In comparison with the regular Radware cube with its standard for compression of 8192 channels over ~ 5 MeV one third less compression was performed to build a less-compressed cube.

By following the methods based on the coincident production of the complementary fission partners described in [17], the triple-coincidence data were analyzed to establish the new level schemes of ^{99,}

^{101}Y and $^{101,\ 105}$Nb. Transition energies and relative intensities were determined by using the least-square peak-fitting of Radford's gf3 program.

III. Results and discussions

A. New level schemes, transition energies and relative intensities

Figures 1, 2, 3 and 4 show the new level schemes of ^{99}Y, ^{101}Y, ^{101}Nb and ^{105}Nb, respectively, proposed in the present work. The transitions with asterisks in the level schemes indicate those reported by beta decay studies and related to the level or levels identified in the present fission work, but not directly observed in our experiment.

It can be seen in Fig. 1 that the $\pi5/2^+[422]$ band of ^{99}Y previously reported in [20, 21, 8] is extended from $(19/2^+)$, 1596.0 keV up to $(27/2^+)$, 3179.0 keV. The 3-qp band (For the configuration assignment, see below) is extended from $(15/2^+)$, 2113.9 keV up to $(23/2^+)$, 3389.5 keV. The $\pi5/2^-[303]$ and $\pi3/2^-[301]$ bands with 2-3 levels reported in [19] by means of decay study are observed by prompt gammas from fission for the first time in this work. The 8.6 μs isomer at 2142.1 keV reported in [21] is identified in the present work. The feedings from this isomer to the $13/2^+$, $15/2^+$, $17/2^+$ and $19/2^+$ levels of the ground-state band and to the $13/2^+$ level of the 3-qp band are observed.

In Fig. 2 the $\pi5/2^+[422]$ band of ^{101}Y previously reported in [8, 22, 23] is extended in this work from $(13/2^+)$, 725.0 keV up to $(23/2^+)$, 2396.1 keV. The $\pi3/2^-[301]$ band with four levels observed in decay work [19] is observed for the first time in fission in the present work.

The strongest population of three well-developed bands was seen in ^{101}Nb (see Fig. 3). The $\pi5/2^+[422]$ band reported in [8, 19, 24] is extended in the present work from $(21/2^+)$, 2072.7 keV up to $(27/2^+)$, 3396.8 keV, the $\pi3/2^-[301]$ band from $(11/2^-)$, 1005.9 keV up to $(21/2^-)$, 2624.2 keV, and the $\pi5/2^-[303]$ band from $(21/2^-)$, 2414.7 keV up to $(25/2^-)$, 3206.8 keV.

For ^{105}Nb, the only reported $\pi5/2^+[422]$ band is extended from $(15/2^+)$, 1045.4 keV, reported in [8], up to $(25/2^+)$, 2868.0 keV in the present work.

Tables I, II, III and IV give the transition energies and relative intensities determined in $^{99, 101}$Y and $^{101, 105}$Nb, respectively. Also shown in the tables are those transitions obtained earlier by decay studies or by fission measurements reported by other authors [8, 19-24]. We report our energy determinations to two decimal places, so as not to lose accuracy in sum and difference checking and in signature splitting determinations. However, we caution that systematic errors are of the order of 0.1 keV, for comparison with energy determinations by others. We also show the statistical standard deviations reported by the Radware gf3 least-squares fitting program but caution especially the very small values of 0.01 keV given for many of the stronger transitions. Even with the special Radware cube program with lower compression of channels that we used and with the fixed peak-width formula, there may be too few degrees of freedom to get meaningful statistical standard deviations for some of the peaks. We do not attempt to show systematic uncertainty of the relative photon intensities, but they probably range from around 5% for stronger peaks to 20% for weaker ones, and to even larger values for peak-overlapping cases. We have not attempted to correct for internal conversion, as the multipolarities are often not known.

Table I. Energies and relative intensities of the transitions observed in ^{99}Y

E_γ (keV)	Statistical standard devi. σ (keV)	Relative Intensities	E_γ[21] (keV)	E_γ^{β}[19] (keV)	Initial level (keV)	Half life[19] (ns)
			27.7		2142.1	8.6 μs
121.7	0.1			120.58	657.6	
125.26	0.01	100	125.1	125.12	125.3	0.047
158.73	0.01	65.5	158.7	158.63	284.0	
				160.73	818.3	
				169.56	657.6	
198.57	0.01	51.6	198.5	198.49	482.5	
214.08	0.01	9.6	214.0		1868.9	
223.89	0.01	28.9	223.9		706.4	
244.97	0.02	4.4	245.2		2113.9	
269.55	0.01	14.3	269.6		976.0	
273.24	0.09	2.9	272.9		2142.1	8.6 μs
275.54	0.07	2.3			2389.3	
283.30	0.01	8.8	283.5		1259.3	
283.8	0.1	27.2	283.8	283.7	284.0	
304.7	0.1	1.2			2694.0	
330.82	0.08			330.30	818.3	
334.1	0.1	0.8			3028.0	
336.84	0.02	4.1	336.3		1596.0	
337.39	0.03	2.1			1933.3	
340.4	0.1			340.81	624.4	
357.23	0.01	21.9	357.2	357.2	482.5	
361.5	0.2				3389.5	
362.24	0.14			362.11	487.5	
385.61	0.04				2717.9	
398.91	0.04	1.8			2332.3	
422.41	0.01	17.4	422.3		706.4	
459.18	0.03	2.1	459.2		2113.9	

461.1	0.1				3179.0	
				487.31	487.5	
493.52	0.01	16.8	493.5		976.0	
				499.26	624.4	
520.36	0.03	2.0			2389.3	
				531.75	657.6	
534.3	0.1			533.9	818.3	
535.9	0.1			536.12	535.9	
546.1	0.1	0.4	546.2		2142.1	8.6 μs
552.89	0.01	13.1	553.2		1259.3	
580.21	0.09	1.4			2694.0	
619.88	0.02	11.4	619.7		1596.0	
				624.32	624.4	
638.7	0.1	1.2			3028.0	
673.98	0.02	4.6			1933.3	
				692	818.3	
695.5	0.1	1.0			3389.5	
736.47	0.03	3.6			2332.3	
784.47	0.05	1.1			2717.9	
				817	818.3	
846.7	0.1	0.9			3179.0	
882.83	0.07	1.5	882.5		2142.1	8.6μs
1166.14	0.08	1.2	1166.0		2142.1	8.6μs
1370.92	0.04	3.5	1371		1654.8	
1435.63	0.04	0.9	1435.5		2142.1	8.6μs
1529.43	0.03	8.6	1529.5		1654.8	

Table II. Energies and relative intensities of the transitions observed in ^{101}Y

E_γ (keV)	Statistical standard deviations σ (keV)	Relative Intensities	E_γ[8] (keV)	E_γ^{β}[19] (keV)	Initial level (keV)
79.9	0.12			79.7	590.4
124.31	0.02			123.97	714.7
128.43	0.04	100	128.2	128.34	128.4
158.92	0.02			158.43	873.7
163.29	0.01	52.2	163.4	163.35	291.7
202.73	0.05	28.8	202.6	202.63	494.4

204.1	0.1			203.9	714.7
230.59	0.01	15.3	230.7	230.7	725.0
276.43	0.02	9.6			1001.4
283.32	0.03			282.7	873.7
289.78	0.01	6.9			1291.2
291.64	0.02	5.6	291.6	291.72	291.7
348.13	0.02	2.6			1639.3
354.97	0.02	1.2			1994.3
365.93	0.03	5.4	366	365.97	494.4
401.8	0.1				2396.1
423	0.1	3.3		422.84	714.7
433.37	0.01	5.3	433.3		725.0
				462.14	590.4
506.92	0.04	3.1			1001.4
510.7	0.1			510.73	510.6
566.29	0.04	3.4			1291.2
				586.13	714.7
				590.4	590.4
637.82	0.11	2.3			1639.3
702.9	0.1	1.9			1994.3
				714.3	714.7
				744.1	873.7
756.8	0.2				2396.1

Table III. Energies and relative intensities of the transitions observed in ^{101}Nb

E_γ (keV)	Statistical standard deviations σ (keV)	Relative Intensities	E_γ[24] (keV)	E_γ^{β}[19] (keV)	Initial level (keV)	Half life[19] (ns)
119.49	0.01	100	119.3	119.3	119.5	0.084
136.02	0.01	64	135.9	135.9	255.5	0.035
140.82	0.01	14.1	141.5	140.6	346.5	0.024
165.54	0.03	29.2	165.7	165.8	374.2	0.019
186.04	0.02	9.5	186.2	186	532.6	0.013
197.13	0.01	20.7	197	197.7	729.2	
198.59	0.01	17.6	198.6		572.8	
205.72	0.01	21.3	205.6	205.6	205.7	
208.69	0.02	36.4	208.5	208.5	208.7	
214.04	0.01	5.9	214.1		746.6	
				226.8	346.5	

236.49	0.01	7.2	236.5		809.3
246.58	0.01	7.2	246.6		1339.6
255.36	0.06	6.8	255.2	255.2	255.5
259.38	0.01	5.3	259.4		1005.9
263.78	0.02	4.5	263.9		1073.1
275.24	0.05	3.3			1281.1
276.69	0.01	30.3	276.8		532.2
299.74	0.04	2.4			2072.7
300.56	0.02	3.6			1373.6
318.43	0.04	1.9			1599.5
319.33	0.11	1.1			1692.9
325.44	0.11	1.6			1925.0
326.76	0.21	2.9	327.7	327.1	532.6
335.0	0.1				2624.2
346.4	0.1	3.4			346.5
353.2	0.2				2046.2
359.1	0.1				2906.5
363.82	0.01	12.8	364.1		1093.0
364.14	0.01	20.8	364.4		572.8
364.2	0.2				2289.1
374.12	0.02	19.5			374.2
400.01	0.02	7.6	400.3		746.6
412.84	0.11	5.8	412.7		532.2
433.32	0.02	3.7	433		1772.9
435.14	0.03	18.8	435.1		809.3
473.38	0.02	7.1	473.5		1005.9
473.71	0.01	19	474.1		729.2
474.73	0.06	1.1			2547.4
490.3	0.1				3396.8
500.34	0.01	17.2	500.4		1073.1
534.63	0.04	5.3			1281.1
560.81	0.01	4.4	561.4		1093.0
564.44	0.01	10.9	564.4		1373.6
593.53	0.04	3.6			1599.5
610.44	0.01	9.4	610.7		1339.6
619.85	0.02	9.6	620.1		1692.9
643.94	0.06	3.1			1925.0
672.54	0.03	4.5			2046.2
679.84	0.02	2.4	679.6		1772.9
689.5	0.1	0.9			2289.1
699.3	0.1	1.4			2624.2
721.77	0.04	3.4	722		2414.7
733.1	0.02	3.2	733		2072.7
763.98	0.12	1.1			2810.2
774.44	0.06	1.1			2547.4
792.11	0.07	0.9			3206.8
833.88	0.09	1.8			2906.5
849.33	0.16	0.6			3396.8

Table IV. Energies and relative intensities of the transitions observed in ^{105}Nb

E_γ (keV)	Statistical standard deviations σ (keV)	Relative Intensities	E_γ[8] (keV)	E_γ^{β}[19] (keV)	Initial level (keV)
127.99	0.01	100	127.9		128.0
162.43	0.01	66.3	162.3		290.4
220.64	0.01	36.4	220.8		511.1
223.36	0.01	17.9	223.6		734.5
272.83	0.01	5.5			1318.3
290.45	0.04	11.4	290.2		290.4
310.93	0.01	11.7	311		1045.4
313.44	0.11	1.1			2033.8
342.7	0.1				2868.0
383.13	0.03	13.8	383.1		511.1
402.23	0.04	3.1			1720.4
444.12	0.01	14.1	444.4		734.5
491.72	0.13				2525.4
534.39	0.01	8.5	534.6		1045.4
583.73	0.02	6.4			1318.3
674.93	0.05	2.2			1720.4
715.39	0.12	1.5			2033.8
805.03	0.15	0.9			2525.4
834.2	0.1	1			2868.0

B. Discussions and interpretations

1. $^{99, 101}$Y

An assignment of $\pi 5/2^+[422]$ $\pi g_{9/2}$ was made to the ground-state band in $^{99, 101}$Y based on particle-rotor model calculations for the low spin parts of the bands [22, 23]. Assignments of $\pi 5/2^-[303]$ and

$\pi 3/2^-[301]$ $\pi f_{5/2}$ were made to the two negative-parity side-bands identified in ^{99}Y, respectively, and the latter assigned to the only side-band observed in ^{101}Y. As reported in [21], the 8.6 μs isomer at 2142.1 keV in ^{99}Y is found to be depopulated through transitions with similar intensities into the $13/2^+$, $15/2^+$, $17/2^+$ and $19/2^+$ (but not the $11/2^+$) levels of the ground-state band (see Fig. 1). Based on the above observations and the arguments of K-forbiddeness and the related formula $K^\pi=17/2^+$ was proposed for this isomer state. A 3-qp configuration π5/2[422]υ3/2[411]υ9/2[404] was assigned to this level (see ref. 21). The isomer state is also found to feed, via 27.7- and 273.2- keV transitions, respectively, the 2113.9- keV level and the 1868.9- keV level of the high-lying band starting with the 1654.8- keV level. Based on the measurements of total internal conversion coefficients of this 27.7- keV low-energy transition, on the 1.4 ns life-time determined for the 1654.8- keV level, and the high-energy decay-out transitions of 1529.4 and 1370.9 keV feeding the $7/2^+$ and $9/2^+$ level of the ground-state band, respectively, $K^\pi=11/2^+${ π5/2[422]υ3/2[411]υ9/2[404]} was assigned to the 1654.8- keV level [21]. In the present work the non-observation of the 1654.8- keV transition between the 1654.8- keV level and the ground state ($5/2^+$) supports this assignment. The positive-parity rotational band built on this 1654.8- keV, ($11/2^+$) level reaching 3389.5 keV, ($23/2^+$) identified in the present work is thus a 3 qp band.

The ground-state bands of $^{99,\,101}Y$, extended in the present work, are very similar to each other and exihibit properties of well-deformed prolate rotors. The signature splittings of the ground-state bands of $^{99,\,101}Y$ are shown in Fig. 5. It can be seen in this figure that the pronounced similarities of the signature splitting of the ground-state bands remain up to the high-spin region. Shown in Fig. 6a and 6b are the kinematic and dynamic moments of inertia, respectively, as a function of rotational frequency for the ground-state bands in $^{99,\,101}Y$. Despite the fact that the bands do not reach sufficiently high spins to show band crossing , the variations of the $J^{(1)}$ and $J^{(2)}$ curves in Fig. 6 follow similar trends over a wide frequency region, with a slightly larger alignment for ^{99}Y than that for ^{101}Y seen in the upper part of the band. The characteristics of a well-deformed prolate rotor and the similarities of the ground-state band behavior in the Y isotopes can be interpreted by the deformed shell gaps at Z = 38 and N=60, 62. It is reasonable to consider $^{99,\,101}$Y as a single quasi-proton coupled to the deformed cores ^{98}Sr and ^{100}Sr, respectively, where identical yrast bands are seen [4].

In Fig. 6 one can find that the deformation of ^{99}Y is larger than that of ^{101}Y, which, in view of the information concerning shape transitions of the neighboring isotopes, may imply a peak position at N = 60 of the deformation for the Z = 39 isotopic series. Model calculations made by Skalski et al. [1] predicted the largest deformation cluster at the middle of the shell, i.e. around Z = 38, 40 and N = 60, 62 and 64. It has been shown by experiments that for the even-even neighbors the deformation is saturated at N = 60-62 for Z = 38 (Sr), and is a maximum at N = 64 for Z = 42 (Mo); and for Z = 40 (Zr) the situation is intermediate [8]. Since $^{99,\,101}$Y can be considered as a quasi-proton coupled to ^{98}Sr and ^{100}Sr, respectively, the saturation or peak positions of the deformations observed in these even-even neighbors support the above conclusion that for the Z = 39 (Y) isotopes the quadrupole deformation peaks at N = 60.

2. $^{101,\,105}$Nb

The ground-state band and the two side-bands of ^{101}Nb are extended up to higher excitations in the present work (see Fig. 3), with the two side-bands reaching much higher excitations and spins than those in the Y isotopes. However, the population of ^{105}Nb is much weaker, with only one band observed (see Fig. 4). The ground-state bands in $^{101,\ 105}$Nb were assigned the $\pi5/2^+[422]$ configuration, while $\pi5/2^-[303]$ and $\pi3/2^-[301]$ were assigned to the two side-bands in ^{101}Nb.

The interpretation of the ground-state bands in Nb isotopes as having the same configurations as those of $^{99,\ 101}$Y can be justified as follows: Figures 7a and 7b show, respectively, the variations of kinematic and dynamic moments of inertia as functions of rotational frequency of the ground-state bands for the N = 60 odd-Z isotones. Shown in Fig. 8a and 8b are those for N = 62 odd-Z isotones. It can be seen in Fig. 7 and 8 that in the region of Z = 39-45 the deformation is decreasing with increasing Z, following the same trend observed in even-Z nuclei in the region of Z = 38-42 (see e.g. [8]). In the Nilsson diagram for the Z ~ 40 region, one can see a crossing between the upsloping $3/2^-[301]$ and downsloping $5/2^+[422]$ orbital above the Z = 38 deformed shell gap. So for Z = 41 (Nb) with lower deformation than Z = 39 (Y) the odd proton can occupy the $5/2^+[422]$ orbital with the $3/2^-[301]$ lying below the Fermi level; and for Z = 39 (Y) with larger deformation the odd proton can also occupy the $5/2^+[422]$ orbital, while the $3/2^-[301]$ lies above the Fermi level.

Figures 9a and 9b indicate the kinematic and dynamic moments of inertia, respectively, of the ground-state bands of ^{101,103,105}Nb (Z = 41). From this figure and based on the same arguments given in the preceding section one may conclude that for the odd-Z of the Z = 41 isotopic chain the deformation is saturated at N = 62, also following, and supported by, the consistent trend observed in the neighboring even-even isotopes.

3. Triaxiality trends in the odd-A isotopes of Y(39) – Nb(41)– Tc(43)

As reviewed in the introduction, a shape transition from axially-symmetric shape to a triaxial degree of freedom was suggested between the neighboring even-Z Zr (Z = 40) and Mo (Z = 42) isotopes [10]. It was pointed out that triaxial deformation plays an important role in the Mo isotopes while the Zr isotopes still have an axially-symmetric shape. For the odd-Z nuclei in this region, triaxial deformations have been observed in heavier nuclei with Z = 43 (Tc), 45 (Rh), and N = 60-66: Large signature splitting, excitations and branching ratios in the odd-Z Tc (Z = 43, N = 62-66) and Rh (Z = 45, N = 66, 68) have been interpreted by triaxial-rotor-plus-particle model calculations as triaxial shapes of $\gamma = -22.5^0$ and -28^0, respectively [17, 18], and triaxial deformations were also reported in ^{103}Tc (Z = 43, N = 60) [13] and in $^{105, 107, 109}$Rh (Z = 45, N = 60, 62, 64) [14, 15]. The spectroscopic information of the Y (Z = 39) and Nb (Z = 41) isotopes thus allows searches for the expected triaxiality shape transition in the Z = 39(Y) – 41(Nb) – 43(Tc) nuclei in this region.

Figure 10 shows the level systematics of the positive-parity ground-state bands in N = 60, 62 and 64 isotonic chains with odd-Z ranging from 39 through 45. In this figure a distinct difference in level pattern can be seen between the Y and the Tc and Rh isotones. An almost equally-spaced level pattern is seen in Y, while level bunching in Tc and Rh isotones is observed. The difference can be more clearly seen in Figs. 11 and 12, which show signature-splitting functions S(I) of the positive-parity ground-state bands of N = 60 and N = 62 isotones, respectively, in a wide odd-Z range of Z =

39 – 45. Here we use the signature-splitting function S(I) used in our refs. 17 and 18 given by Gelberg et al. The signature splitting S(I) is defined as

$$S(I) = \frac{E(I) - E(I-1)}{E(I) - E(I-2)} \cdot \frac{I(I+1) - (I-2)(I-1)}{I(I+1) - (I-1)I} - 1 \qquad (1)$$

For both the N = 60 and N = 62 isotonic chains the pronounced difference in S(I) between the Y and Tc and Rh isotones is clearly seen. Very small signature splitting is observed in Y, in pronounced contrast to the quite large splittings in the Tc and Rh isotones. In fact S(I) increases significantly from ~ ±(0.02-0.04) in $^{99, 101}$Y to ~ ±(0.40-0.60) in $^{103, 105, 107, 109}$Tc and ~ ±(0.20-0.40) in $^{105, 107, 109, 111, 113}$Rh. Since large triaxial deformation has been reported in the Tc and Rh isotopes and the signature splitting function S(I) is very sensitive to triaxiality, the above observations indicate that, while triaxial deformation plays a significant role in the Tc and Rh isotopes, mainly axially-symmetric deformed shapes remain in ^{99}Y, implying a shape transition from axially-symmetric deformed shape in Z = 39 isotopes to triaxial deformations in Z = 43, 45 isotopes (see the following section for discussions based on calculations). It should be noted that the difference in the signature splitting between Y (Z = 39) and Tc (Z = 43), Rh (Z = 45) discussed above is in fact even more pronounced than that reported for the $\upsilon h_{11/2}$ band of the even-odd neighbors ^{101}Zr and ^{103}Mo [10], which was attributed to a triaxial degree of freedom in ^{103}Mo and axially-symmetric shape in ^{101}Zr by performing particle-rotor model calculations [10]. It can also be seen from Fig. 11 and 12 that the Nb isotones have intermediate values of signature splitting functions S(I) ~ ±(0.10-0.20) between those of Y and Tc, Rh isotones, which may imply a transitional character for the Nb isotones with regard to triaxial deformation.

It is worthwhile noting the band built at the excited $11/2^+$ state with excitation energy as high as 1654.8 keV in ^{99}Y which is well-developed in the present work(see Fig. 1). This band de-excites mainly via a 1529.4-keV transition to the $7/2^+$ state, and a weak 1370.9-keV transition to the $9/2^+$ of the ground-state band, just opposite to what is observed in the side-bands in the Tc and Rh isotopes [see ref. 17 and 18], where the side-band built on an excited $11/2^+$ level with low excitation energy (~570 – 770 keV) predominantly feeds the $9/2^+$ state and very weakly the $7/2^+$ state of the yrast bands in Tc and Rh isotopes. However, this band is not an analogous collective sideband to the ground band as in Rh nuclei, because its half life of 1 ns for the 11/2 bandhead represents a retardation of three orders of magnitude if M1 or two orders of magnitude if E2. The 3-quasiparticle assignment of Meyer et al [21]. is the more likely interpretation.

The fission of ^{252}Cf does not populate sufficiently high excitations and spin states to allow observations of the complete band crossings in these Y and Nb isotopes. However, the variations of the kinematic and dynamic moments of inertia versus rotational frequencies for ground-state bands of the N = 60 isotones with odd-Z shown in Fig. 7 and those of N = 62 isotones in Fig. 8 indicate a clear tendency for the crossing frequencies of Rh (Z = 45) and Tc (Z = 43) to be lower than that of Y (Z = 39) isotones. This behavior is similar to the case of $\upsilon h_{11/2}$ band in the neighboring even-even Zr and Mo isotopes (N = 60, 62), where a shift of band crossing to lower rotational frequency observed in the $\upsilon h_{11/2}$ bands of Mo compared to Zr isotopes was accounted for by triaxiality in Mo and axially-symmetric shape in Zr isotopes in CSM calculations [10]. Despite the fact that the bands of Y isotopes do not extend to high enough spins to allow a similar CSM calculation to address the difference in the band crossing in terms of triaxiality, the similar shift of band crossing observed in Y compared to Tc and Rh isotopes supports the interpretation of axially-symmetric shape in Y isotopes. It can also be seen in Fig. 7 and 8 that the crossing frequencies of Nb isotones is likely between the values of Y and Tc, Rh isotones, which may also suggest a transitional character for Nb isotones with regard to triaxial deformation.

IV. Rigid triaxial-rotor-plus-quasiparticle model calculations

In previous publications [refs.17,18] we have described the rigid triaxial-rotor–plus-quasiparticle model. Therefore, in the present paper we will mention only a few basic features of this model. A more detailed description can be found in the seminal paper by Larsson et al. [26]. By "rigid" we mean that the shape, which is defined by the deformation parameters of the core [27], is the same for all states. The deformation parameters are fixed, and this model does not contain either β or γ vibrations.

We use the hydrodynamic irrotational flow formula for the ratios of the moments of inertia along the principal axes, which depend only on the triaxial deformation parameter γ. We normalize them with a scaling factor, which is a free parameter. This parameter is expressed through an effective value of $E(2^+)$ of the core.

The single particle Hamiltonian contains an anisotropic oscillator potential which depends on the deformation parameters ε_2 and γ. We use the Lund convention for γ, which is confined to the 0 to -60^0 interval.

In the present work this model is used to calculate the eigenvalues and eigenfunctions of states with positive and negative parity (if available) in ^{99}Y, ^{101}Y, ^{101}Nb, ^{103}Nb and ^{105}Nb. Theoretical excitation energies, signature splittings and some branching ratios are compared with the experimental results. The fitted parameters are summarized in Table V, and Fig. 13 indicates the spherical single-particle levels used in the calculations.

Table V Model calculation parameters for Y and Nb isotopes

Nucleus and band	Quadrupole deformation ε_2	Triaxiality γ (deg.)	Coriolis attenua. factor ξ	Inertial parameter $E(2^+)$ (MeV)
^{99}Y $5/2^+$[422] band	0.41	0	1.0	0.14
^{101}Y $5/2^+$[422] band	0.39	0	0.95	0.16
^{101}Nb $5/2^+$[422] band	0.35	-19	0.83	0.2
$5/2^+$[422] band	0.36	-14	0.8	0.16
$3/2^-$ [301] band	0.25	-5	1.0	0.13
$5/2^-$ [303] band	0.25	-5	1.0	0.13
^{103}Nb $5/2^+$[422] band	o.37	-15	0.8	0.155
^{105}Nb $5/2^+$[422] band	0.36	-13	0.8	0.16

Before proceeding further, we must make a remark on the validity of the model at relatively high spins. In previous studies, especially in those of ^{111,113}Rh, a band crossing (backbending) was clearly visible at spins around 23/2. This was interpreted as the crossing of the one-quasiparticle ground band with a three-quasiparticle band based on a $\pi g_{9/2}\ \upsilon h_{11/2}^2$ configuration. On the contrary, we do not see any clear indication of backbending in the Y and Nb nuclei mentioned above. Therefore, one is tempted to consider the observed cascades, with spins up to I=27/2, as one-quasiparticle rotational bands. It is more plausible to assume that also in this case we deal with band mixing. The mixing matrix elements are probably stronger than in the nuclei studied in [17], so that the mixing is distributed over several states. This may be the reason for seeing only a smooth transition. In practical terms, this means that the calculated wave functions and excitation energies can be taken at face value only below I=23/2.

We will first consider the lowest-Z nucleus here, ^{99}Y (Z=39). Figure 14 shows a comparison of the calculated signature splitting with the experiment for the ground-state band of ^{99}Y. The calculations were performed for axial symmetry ($\gamma=0^0$) and for small triaxiality ($\gamma=-12.5^0$). The axially symmetric calculation is obviously satisfactory for low spins up to I=19/2. At higher spin, the experimental signature splitting function S(I) [17] is of the same sign, but significantly smaller than in the calculation for axial symmetry. Similar deviations of experiment from theory at higher spins were also noticed in our previous papers on Rh and Tc [17,18]. In the latter cases, the deviation was associated with the sharp backbending and attributed to the alignment of a pair of $h_{11/2}$ neutrons. As noticed above, in this case the $h_{11/2}$ neutrons are also expected to be major participants in carrying angular momentum along the rotation axis. Since signature splitting in an axially symmetric nucleus can be considered as a consequence of Coriolis coupling, the lowered value of S(I) at the higher spins could also be attributed to a decrease of the summed Coriolis matrix elements as spin increases. Another characteristic is the quite small value of S(I), which already points at axial symmetry.

The theoretical excitation energies in ^{99}Y and ^{101}Y are compared with experiment in Fig. 15. We consider the fit at lower spins satisfactory. The calculation assumes a constant moment of inertia, so that this overall fit does not reproduce the experimental increase of the moment of inertia at higher spin,

In Table V, we fitted a quadrupole deformation ε_2=0.41 ,which roughly correponds to β_2=0.43. If we look at the deformation parameters of the neighbouring even-even nuclei, we find β_2=0.408(6) and β_2=0.423 for ^{98}Sr and ^{100}Sr, respectively [28]. This means that the fitted value of β_2 is plausible. ^{101}Y has been fitted with similar parameters and a similar result is obtained (see Table V and Fig. 15) .

Let us now examine the Z=41 Nb nuclei. Here we have treated bands of both parities. Figure 16 gives an overview of the comparisons with experiment for excitations of the three bands in ^{101}Nb. In order to describe correctly the excitations and signature splitting (see below) in ^{101}Nb, we had to use a triaxiality parameter $\gamma=-19^0$ for the $5/2^+$ positive parity band, and $\gamma=-5^0$ for the two negative-parity bands.

The signature splitting at low spins in ^{101}Nb is quite reasonably reproduced with $\gamma=-19^0$ (see Fig. 17), but at high spins the theoretical values are again too high. For the negative-parity bands we fitted $\gamma=-5^0$. As shown in Fig. 18, the experimental signature splitting for the negative-parity band based on 3/2-[301] is quite small and well reproduced with $\gamma=-5^0$. The previously mentioned overestimation can be noticed again at higher spins. The generally small signature splitting in the negative-parity bands is understood in terms of the availability of comparable coupling to the K=1/2 bands in the near-lying orbitals with j values of 1/2, 3/2 and 5/2. The admixture to the j=3/2 will give a signature splitting contribution of opposite sign to the other two orbitals. We would, of course, not claim a rigid shape with $\gamma=-5^0$, but the model calculations may simulate other degrees of freedom, e.g. γ

vibrations around a prolate, axially symmetric minimum.

In order to fit the ground-state band excitation energies in ^{103}Nb and ^{105}Nb, we took γ=-15^{0}and γ=-13^0, respectively (see Fig. 19). The corresponding signature splittings can be seen in Figs. 20 and 21. Contrary to the negative-parity bands in ^{101}Nb, we can consider the positive-parity bands in ^{101}Nb, ^{103}Nb and ^{105}Nb as really displaying triaxial deformation.

The comparison between theoretical and experimental branching ratios shows a rather poor agreement, although the general trends are reproduced. As an example, we show several branching ratios in ^{99}Y and ^{101}Nb in Table VI.

Table VI Experimental and theoretical branching ratios for positive-parity bands in ^{99}Y and ^{101}Nb

	^{99}Y		^{101}Nb	
Intensity ratio	Exp.	Ther.	Exp.	Ther.
I (9/2 -->5/2) / I (9/2-->7/2)	0.33	0.23	0.11	0.25
I (11/2 -->7/2) / I (11/2-->9/2)	0.42	0.56	0.19	0.63
I (13/2 -->9/2) / I (13/2-->11/2)	0.6	1.07	0.92	1.75
I (15/2 -->11/2) / I (15/2-->13/2)	1.17	1.61	0.34	1.53
I (13/2 -->9/2) / I (13/2-->11/2)	0.25	-5	1.0	0.13

It is interesting to examine the systematics of the ε_2 and γ deformation parameters for neutron-rich nuclei of Z=39 to Z=45 shown in Fig. 22. We can see that when going from Z=39 to Z=45, with neutron numbers roughly between 60 and 70, the ε_2 deformation parameter decreases from values slightly above 0.40 to 0.27. This trend can be understood, at least at a qualitative level. Due to the large value of N, the "quasimagic" Z=40 does not manifest itself. Therefore, when Z decreases from 45 to 39, the proton number reaches the middle of the Z=28-50 shell, the deformed shell gap at Z=38, and this causes an increase in deformation. This increase in collectivity can be related to the increase of $N(\pi)N(\upsilon)$, i.e. of the product of the valence proton and neutron numbers (particles or holes) [29]. In the present case, this product obviously increases when we go from Rh to Y.

At the same time, the nuclear shape changes from axial symmetry in the case of Y to nearly maximum triaxiality in Rh. Such an anticorrelation of the quadrupole deformation and of triaxiality is generally known. It has been examined in quantitative terms in [30].

V. Summary

New level schemes of $^{99,\,101}$Y and $^{101,\,105}$Nb proposed in the present work provide spectroscopic information about the shape trends and triaxiality in neutron-rich nuclei with odd-Z = 39, 41 in the A ~ 100 region. Following a systematic trend, quadrupole deformation peaks at N=60 and 62, respectively, in these Y and Nb isotopes. By combining the data of Tc and Rh isotopes, it is found that ε_2 decreases from values slightly above 0.40 to 0.27 when going from Z=39 to Z=45. Very small signature splitting is observed in yrast bands of $_{39}$Y isotopes, in pronounced contrast to the large ones in $_{43}$Tc and $_{45}$Rh isotopes, the latter being reported to have triaxial deformations in the previous papers [17,18]. There is a trend of band crossing occurring somewhat later in the ground-state bands of Y compared to the Tc and Rh isotopes. This is similar to the observations in even-even $_{40}$Zr and $_{42}$Mo isotopes. However, intermediate values of signature splitting and band crossing frequency between $_{39}$Y and $_{43}$Tc , $_{45}$Rh are observed in $_{41}$Nb isotopes.

Triaxial-rotor-plus-particle model calculations favor a pure axially-symmetric shape with large quadrupole deformations for the ground-state bands in Y isotopes. The best fits for the $5/2^+[422]$ ground-state bands in Nb isotopes imply smaller triaxiality with γ from -13^0 to -19^0 , while nearly axially-symmetric shape with $\gamma = -5^0$ is obtained for the negative-parity bands in the Nb isotopes.

All the above observations and interpretations imply that while large and near maximum of triaxial deformations are identified in $_{43}$Tc and $_{45}$Rh isotopes an axially-symmetric and strongly deformed shape is seen in $_{39}$Y isotopes. An anticorrelation of quadrupole deformation and of triaxiality is seen in nuclei with Z ranging from 39 to 45. The $_{41}$Nb isotopes, having intermediate values of signature splitting and band crossing frequency between Y and Tc, Rh isotopes, are transitional regarding the triaxial degree of freedom. One may conclude that in the A~100 neutron-rich nuclei the triaxial shape is prevalent for the bands based on a one-quasiparticle $g_{9/2}$ proton state in the region with Z>41. More detailed information can be useful for further understanding of the transitional Nb nuclei.

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Figure captions

1. New level scheme of ^{99}Y proposed in the present work.

2. New level scheme of ^{101}Y proposed in the present work.

3. New level scheme of ^{101}Nb proposed in the present work.

4. New level scheme of ^{105}Nb proposed in the present work.

5. Signature splitting S(I) of the ground-state bands of $^{99,\,101}$Y. ■ ^{99}Y, ● ^{101}Y.

6. Kinematic (a) and dynamic (b) moments of inertia of the ground-state bands of $^{99,\,101}$Y.

7. Kinematic (a) and dynamic (b) moments of inertia of the ground-state bands of N = 60 isotones with odd Z= 39 – 45. The data of ^{103}Tc are taken from [13], and ^{105}Rh from [14].

8. Kinematic (a) and dynamic (b) moments of inertia of the ground-state bands of N = 62 isotones with odd Z = 39 - 45. The data of ^{103}Nb are taken from [25], ^{105}Tc from [18], and ^{107}Rh from [16]. ▲ ^{101}Y, ♦ ^{103}Nb, ● ^{105}Tc, ■ ^{107}Tc.

9. Kinematic (a) and dynamic (b) moments of inertia of the ground-state bands of Z = 41 isotopic chain with N = 60, 62 and 64. The data of ^{103}Nb are taken from [25].

10. Level systematics of ground-state bands of N = 60, 62 and 64 isotones with odd-Z ranging from 39 through 45. The data of ^{103}Nb are taken from [25], ^{103}Tc from [13], $^{105,\,107}$Tc from [18], ^{105}Rh from [14], and $^{107,\,109}$Rh from [16].

11. Variations of signature splitting S(I) with spins of the ground-state bands in N = 60 isotones with odd-Z = 39 – 45. The data of ^{103}Tc and ^{105}Rh are taken from [13] and [14], respectively. ♦ ^{99}Y, ▲ ^{101}Nb, ● ^{103}Tc, ■ ^{105}Rh.

12. Variations of signature splitting S(I) with spins of the ground-state bands in N = 62 isotones with odd Z = 39 - 45. The data of ^{103}Nb, ^{105}Tc and ^{107}Rh are taken from [25], [18] and [16], respectively. ♦ ^{101}Y, ▲ ^{103}Nb, ● ^{105}Tc, ■ ^{107}Rh.

13. Spherical single-particle levels used in the triaxial-rotor-plus-particle model calculations in the present work.

14. Triaxial-rotor-plus-particle model calculations for the signature splitting of the ground-state band of ^{99}Y. The calculations are performed with $\varepsilon_2 = 0.41$, $\gamma = 0^0$, $\xi = 1.0$, $E(2^+) = 0.14$ MeV (■) and $\varepsilon_2 = 0.41$, $\gamma = -12.5^0$, $\xi = 1.0$, $E(2^+) = 0.15$ MeV (●), respectively. Experimental values are indicated by ♦.

15. Comparison of experiment and theory for excitation energies of the ground bands in ^{99}Y and ^{101}Y. As in Fig. 14 the model calculations assume axial symmetry but are made with the same code and shell-model parameters as the triaxial rotor calculations.

16. Experimental and theoretical energy comparisons for three bands in ^{101}Nb, namely, positive-parity ground $5/2^+[422]$band and negative-parity excited $3/2^-[301]$ and $5/2^-[303]$ bands. The model calculations use a triaxiality γ parameter of -19^0 for the $5/2^+$ground band and γ parameters of -5^0 for the negative-parity bands.

17. Signature splitting plot for ^{101}Nb $5/2^+$ ground band. The fit for $\gamma = -19^0$ is judged better than that for -14^0 at low spins and *vice versa* for the higher spins. See discussion in text.

18. Signature-splitting plot for the ^{101}Nb $3/2^-[301]$ band with the best theoretical fit at lower spins coming at γ of -5^0.

19. Ground-band energy comparisons between experiment and theory for ^{103,105}Nb. Data of ^{103}Nb are taken from [25].

20. Signature-splitting comparison plots for ^{103}Nb ground band. Data of ^{103}Nb are taken from [25]. Theoretical calculations used $\varepsilon_2 = 0.37$ and $\gamma = -15^0$. The fit is unusually good over the full range of spins, although theory overestimates splitting somewhat at higher spins.

21. Same as Fig. 20 except for ^{105}Nb and $\gamma = -13^0$, $\varepsilon_2 = 0.36$.

22. Systematics of quadrupole and triaxial deformations observed in the neutron-rich Z=39, 41, 43, 45 isotopes.

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