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Scalar mesons in three-flavor Linear Sigma Models

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Abstract

The three flavor linear sigma model is studied in order to understand the role of possible light scalar mesons in the pi-pi, pi-K and pi-eta elastic scattering channels. The K-matrix prescription is used to unitarize tree-level amplitudes and, with a sufficiently general model, we obtain reasonable fits to the experimental data. The effect of unitarization is very important and leads to the emergence of a nonet of light scalars, with masses below 1 GeV. We compare with a scattering treatment using a more general non-linear sigma model approach and also comment upon how our results fit in with the scalar meson puzzle. The latter involves a preliminary investigation of possible mixing between scalar nonets.

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I. INTRODUCTION

In recent years there has been a renewal of interest in the σ meson (the subject of many of the talks at this meeting) and the other scalar mesons below and above 1 GeV - for a list of references to the ideas of various authors see [1] and also [2]. The light scalar mesons are an interesting puzzle because their properties do not fit, in an obvious way, those of a conventional $q\bar{q}$ scalar multiplet.

We begin with the basic problem of extracting information about scalar mesons from experiment. This is related to the problem of describing pseudoscalar meson scattering in an energy regime which is too low for a perturbative QCD treatment, but sufficiently high to make the chiral perturbation theory scheme difficult to implement. Previously the Syracuse group studied $\pi\pi$, πK and $\pi\eta$ scattering [3] in a $\frac{1}{N_c}$-inspired unitarized non-linear chiral Lagrangian framework and found evidence for nine scalar mesons below 1 GeV which have the quantum numbers of an SU(3) nonet. These are the isoscalars σ and $f_0(980)$, the isovector $a_0(980)$ and the isospinor $\kappa(900)$ meson. The σ and in particular the κ meson are rather controversial and emerge as extremely broad states requiring a generalization of a Breit-Wigner parameterization. Many authors have found similar or related results (see [1] and [2]).

In this context it is interesting to study pseudoscalar meson-meson scattering in a different model in order to investigate the model-dependence of the scalar meson parameters. We will work in the classic chiral symmetric Linear Sigma Model, in which the scalar mesons are present from the outset. In an attempt to understand the full scalar spectrum we will also consider the possibility of two linear sigma models with mixing.

II. SU(2) LINEAR SIGMA MODEL AND K-MATRIX UNITARIZATION

Let us focus initially on the $\pi\pi$ scattering amplitude. The result in the original two-flavor Linear Sigma Model [4] is well-known. The tree-level invariant $\pi\pi$ amplitude is the sum of a contact diagram and direct and crossed-channel σ-exchange diagrams:

$$A(s,t,u) = \frac{2}{F_\pi^2}\left(m_b^2(\sigma) - m_\pi^2\right)\left[\frac{m_b^2(\sigma) - m_\pi^2}{m_b^2(\sigma) - s} - 1\right]. \tag{2.1}$$

We have put a subscript b, denoting "bare", on the σ mass since this quantity will be shifted by unitarization as we shall see shortly. This amplitude reproduces the current algebra result, which gives good agreement with experiment near threshold, in the limit where $m_b(\sigma)$ is large. We see that even near threshold the σ resonance plays an important role since there is a delicate cancellation between the constant contact term and the σ pole contribution. The tree-level amplitude clearly blows up at $s = m_b^2(\sigma)$ and so requires regularization.

We use the well-known K-matrix technique [5] whereby the unitarized partial-wave S-matrix element (we only look at elastic scattering channels) is defined by

$$S \equiv \frac{1+iK}{1-iK}. \tag{2.2}$$

Identifying K with the partial wave projection of the tree-level amplitude leads to the following "regularized" partial wave amplitude

$$T_J^I \equiv \frac{T^I_{J\,tree}}{1-iT^I_{J\,tree}}. \tag{2.3}$$

We notice that for $T^I_{J\,tree} << 1$ we have $T_J^I \approx T^I_{J\,tree}$ and also that when $T^I_{J\,tree} \rightarrow \infty$, $\mathrm{Re}(T_J^I) \rightarrow 0$. Applying this unitarization to the partial wave projection of Eq. (2.1) we can get a reasonable fit to the $\pi\pi$ data up to about 1 GeV. Similar results were found for this case in [6]. In order to try and describe the structure of the $\pi\pi$ scattering amplitude beyond 1 GeV and also to investigate the $I = \frac{1}{2}$ and $I = 1$ scattering channels we next extend our analysis to the SU(3) case.

III. SU(3) LINEAR SIGMA MODEL TREATMENT

We consider a general (possibly non-renormalizable) [7] Lagrangian of the form

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\left(\partial^\mu \phi \partial_\mu \phi\right) + \frac{1}{2}\mathrm{Tr}\left(\partial^\mu S \partial_\mu S\right) - V_0 - V_{SB}, \tag{3.1}$$

where $M = S + i\phi$ is a 3×3 matrix field ($S = S^\dagger$ represents a scalar nonet and $\phi = \phi^\dagger$ a pseudoscalar nonet) which transforms linearly under chiral transformations and where V_0 is an arbitrary $SU(3)_L \times SU(3)_R \times U(1)_V$ invariant potential. The symmetry breaker V_{SB} has the minimal form linear in S. Chiral symmetry gives relations among certain parameters of the model [7], for example trilinear coupling constants, masses and decay constants. In particular the "bare" mass of the strange scalar state, denoted κ, is related to the pseudoscalar meson masses and decay constants by:

$$m_b^2(\kappa) = \frac{F_K m_K^2 - F_\pi m_\pi^2}{F_K - F_\pi}. \tag{3.2}$$

If we use a renormalizable potential the model is extremely predictive. Once we fix five input parameters [for example the pseudoscalar parameters m_π, m_K, $m_{\eta'}$, F_π and m_η (or

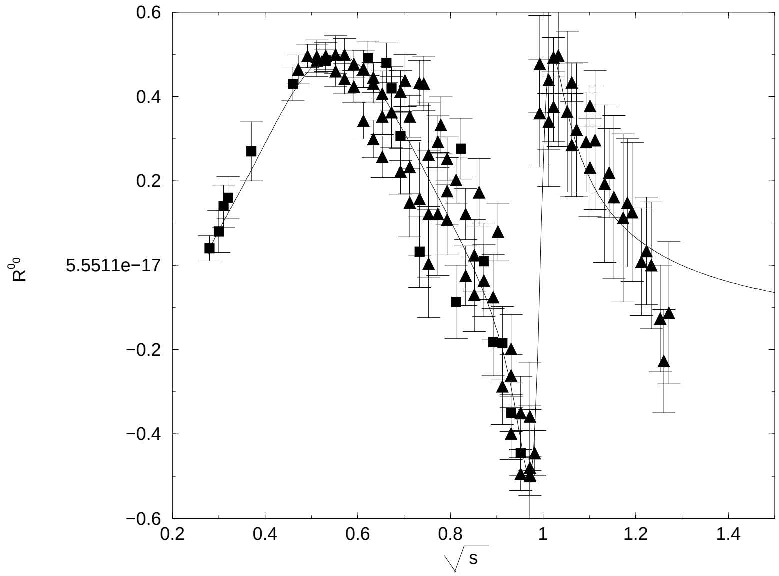

FIG. 1. Comparison of our best fit for the Real part of the I=J=0 $\pi\pi$ scattering amplitude in the non-renormalizable SU(3) linear sigma model with experiment [3].

$F_K)$], it turns out that there is only one free parameter left, which we take to be the bare σ mass, $m_b(\sigma)$.

We calculate the tree-level $\pi\pi$ scattering amplitude and again unitarize the scalar isoscalar partial wave channel according to the K-matrix prescription. It turns out that this one-parameter model does not allow a good fit to the experimental data.

If we consider a more general potential then both of the isoscalar scalar masses [$m_b(\sigma)$ and $m_b(\sigma')$] and also the isoscalar mixing angle are independent parameters. We perform a best fit of the Real part of our unitarized amplitude to the experimental data and show the result in Fig. 1 where it can be seen that we get good agreement with the data up to approximately 1.25 GeV. The best fit values are $m_b(\sigma) = 0.847$ GeV, $m_b(f_0) = 1.3$ GeV and a bare σ-f_0 mixing angle of 48.6^o. We identify the "physical" masses and widths from the poles in the unitarized partial wave amplitude.

We also considered the scalar πK channel where the strange scalar meson κ will of course play an important role. Since $m_b(\kappa)$ is very sensitive [Eq. (3.2)] to the choice of input parameters, in particular to $\frac{F_\pi}{F_K}$, we consider different inputs for F_K. Once again, we calculate the tree-level amplitude from Eq. (3.1) and unitarize using the K-matrix method. A plot of the Real part of the resulting amplitude is given in Fig. 2, where the experimental data is also presented. Finally, we applied the same procedure to study $\pi\eta$ scattering where the $a_0(980)$ appears in the direct scalar channel.

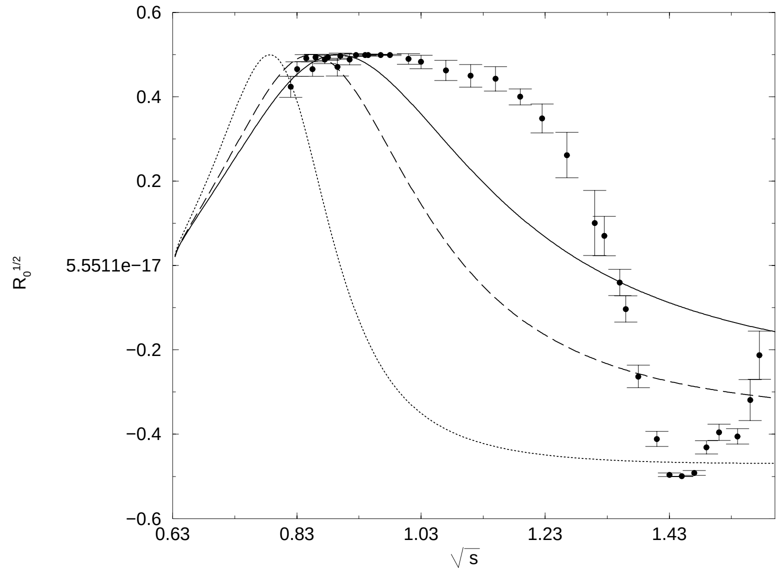

FIG. 2. Comparison of our prediction for the Real part of the I=$\frac{1}{2}$, J=0 πK scattering amplitude in the non-renormalizable SU(3) Linear Sigma Model with experiment. The curves correspond to $m_{\rm BARE}(\kappa)$ = 1.3 GeV (solid), 1.1 GeV (dashed) and 0.9 GeV (dotted).

IV. DISCUSSION

We studied $\pi\pi$, πK and $\pi\eta$ scattering in a K-matrix unitarized version of the SU(3) Linear Sigma Model. One crucial feature is that the masses of the scalar resonances are shifted by unitarization from their "bare" tree-level values to "physical" values which are presented in Table I.

	σ	f_0	κ	a_0
Present Model mass (MeV), width (MeV)	$457, 632$	$993, 51$	$800, 260-610$	$890-1010, 110-240$
Comparison mass (MeV), width (MeV)	$560, 370$	$980 \pm 10, 40-100$	$900, 275$	$985, 50-100$

TABLE I. Predicted "physical" masses and widths of the scalar mesons of the Linear Sigma Model. Suitable comparison values are also given as discussed in text.

For comparison, we have also listed in Table I the $f_0(980)$ and $a_0(980)$ parameters given in the 2001 Review of Particle Properties [10]. For the σ and κ parameters we present for comparison the results of our previous analyses [3]. We see that the masses are consistent to within about 50-100 MeV, while the widths are more model-dependent.

The overall conclusion is that the scalar mesons of the Linear Sigma Model emerge with physical masses below the 1 GeV region. In particular even though we studied a range of bare κ masses between 0.9 and 1.3 GeV, unitarization of the πK scattering amplitude always forced a physical mass of about 800 MeV.

V. FINAL NOTE

In Fig. 2 we see that although the region just above the πK threshold can be explained in this model, the structure observed experimentally around 1.4 GeV is not accounted for. This is associated with the well-established $K_0^*(1430)$ resonance and thus suggests that this state is not described in the simple version of the Linear Sigma Model studied so far. Similarly, the heavier isovector $a_0(1450)$ state does not fit in this picture. If there is a nonet of light (masses $<$ 1 GeV) scalar mesons the question remains not only of understanding their quark substructure (a long-standing puzzle in meson spectroscopy), but also that of the experimentally observed heavier scalar states such as the $K_0^*(1430)$ and $a_0(1450)$.

Previously [11] we found in a non-linear chiral Lagrangian approach that various properties of the light and heavier scalar mesons could be neatly understood by considering them to be mixtures of two nonets. This leads us to look at the possibility of mixing in the context of the Linear Sigma Model also. We observe that since there are infinitely many different quark substructures which lead to meson fields $M(x)$ with identical transformation properties under $SU(3)_L \times SU(3)_R$, C and P, it is not possible, at the level of the meson Lagrangian, to distinguish between these quark substructures (except perhaps through their $U(1)_A$ transformation properties). In fact there are various interesting proposals in the literature (for more detail and for explicit realizations of such objects see [1] and references therein) to explain the light scalar mesons, for example as meson-meson "molecules" or as multiquark ($qq\bar{q}\bar{q}$) states.

Let us consider a toy mixing model given by the following Lagrangian density,

$$
\begin{aligned}
\mathcal{L} = & \frac{1}{2}\mathrm{Tr}\left(\partial_\mu M \partial^\mu M^\dagger\right) + \frac{1}{2}\mathrm{Tr}\left(\partial_\mu M' \partial^\mu M'^\dagger\right) \\
& + c_2 \mathrm{Tr}\left(MM^\dagger\right) - c_4 \mathrm{Tr}\left(MM^\dagger MM^\dagger\right) - d_2 \mathrm{Tr}\left(M'M'^\dagger\right) - e\mathrm{Tr}\left(MM'^\dagger - M'M^\dagger\right).
\end{aligned}
\tag{5.1}
$$

Here c_2, c_4 and d_2 are positive real constants. The M matrix field, which we consider to be $q\bar{q}$, is chosen to have a wrong sign mass term so that there will be spontaneous breakdown of chiral symmetry. A pseudoscalar octet will thus be massless (for simplicity we do not include a quark mass effective term V_{SB}) . A second matrix field M', which may represent an unconventional quark substructure such as $qq\bar{q}\bar{q}$, is introduced with trivial dynamics except for its mixing with M. The mixing is controlled by the parameter e and the e-term is the only one which violates $U(1)_A$ symmetry.

The mass spectrum resulting from Eq. (5.2) has two scalar octets and two pseudoscalar octets, each with an associated $SU(3)$ singlet. Each octet has eight degenerate members since the quark mass terms have been switched off. In order to see whether the mixing can give a spectrum in reasonable agreement with experimental observations let us focus on the I=1, positively charged particles as an example. It turns out that the mass eigenvalues for the pseudoscalar states are:

$$m^2\left(\pi_p\right) = 0, \qquad m^2_{\mathrm{BARE}}\left(\pi'_p\right) = \frac{2e^2}{d_2} + 2d_2. \tag{5.2}$$

We put the subscript "BARE" on $m^2\left(\pi'_p\right)$ to indicate that it may receive non-negligible corrections from K-matrix unitarization.

The mixing angle for the I=1 scalar states is given by

$$\tan 2\omega = \frac{4e}{2d_2 - 4c_2 - \frac{6e^2}{d_2}} \tag{5.3}$$

and the corresponding mass eigenvalues are

$$m^2_{\mathrm{BARE}}\left(a_p, a'_p\right) = 2c_2 + d_2 + \frac{3e^2}{d_2} \mp 2e\csc 2\omega, \tag{5.4}$$

where the upper (lower) sign stands for a_p, $\left(a'_p\right)$. It is interesting to examine the masses of the degenerate octets in a little more detail. For orientation we begin with the case when the mixing parameter e vanishes. The usual "$q\bar{q}$" pseudoscalars π_p are zero mass Goldstone bosons in this limit while a_p (the scalar partner of π_p) and a'_p (the "$qq\bar{q}\bar{q}$" scalar) are degenerate. If $2c_2 > d_2$ then a_p and a'_p are both heavier than π'_p. Then, as the mixing is turned on a four quark condensate develops and the mass ordering becomes

$$m_{\mathrm{BARE}}(a_p) > m_{\mathrm{BARE}}(\pi'_p) > m_{\mathrm{BARE}}(a'_p) > m_{\mathrm{BARE}}(\pi_p) = 0. \tag{5.5}$$

The mixing angle ω remains small, because the denominator of Eq. (5.3) is always negative and increases in magnitude as e^2 increases, which means that a'_p is predominantly $qq\bar{q}\bar{q}$. The ordering in Eq. (5.5) is generally consistent with an identification of these states with the experimentally observed states $a_0(1450)$, $\pi(1300)$, $a_0(980)$ and π respectively. It also shows that the lowest-lying scalar may not come out as predominantly $q\bar{q}$, and that in fact the scalar state closer to $q\bar{q}$-type can be heavier, and so possibly in the mass range expected from the quark model.

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