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Kinematical twist–3 effects in DVCS as a quark spin rotation

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We point out that the kinematical twist–3 contributions to the DVCS amplitude, required to restore electromagnetic gauge invariance of the twist–2 amplitude up to $O(t/q^2)$, can be understood as a spin rotation applied to the twist–2 quark density matrix in the target. This allows for a compact representation of the twist–3 effects, as well as for a simple physical interpretation.

Various authors have recently studied deeply virtual Compton scattering (DVCS) in QCD beyond the leading twist level [1–6]. The main motivation has been the fact that the twist–2 contribution to the DVCS amplitude alone, originally considered in Refs. [7–9], is not transverse (electromagnetically gauge invariant) even when neglecting terms of order t/q^2. It was shown in Refs. [3,4,6] that transversality to this accuracy is restored when one includes in the light–cone expansion certain "kinematical" twist–3 contributions which appear from total derivatives of twist–2 operators. Alternatively, these transversality–restoring terms can be represented as certain Wandzura–Wilczek type contributions to the general twist–3 skewed parton distributions parametrizing the non-forward matrix elements of the relevant light–ray operators beyond the leading twist level [1–6].

In this Brief Report we point out that the kinematical twist–3 contributions to the DVCS amplitude required by transversality can be understood as the result of a certain quark spin rotation applied to the twist–2 part of the quark density matrix in the target. This spin rotation is necessary in order that the quark density matrix satisfy the free–field Dirac equations (up to terms of order t) with respect to the "incoming" and "outgoing" quark, which is both necessary and sufficient for the tree–level DVCS amplitude to be transverse. This observation allows for a very compact representation of the twist–3 effects within the non-local light cone expansion, as well as for a simple physical interpretation. Notations and conventions in this report closely follow Refs. [4,6].

To begin, let us recall how transversality of the tree–level Compton amplitude comes about in a theory of

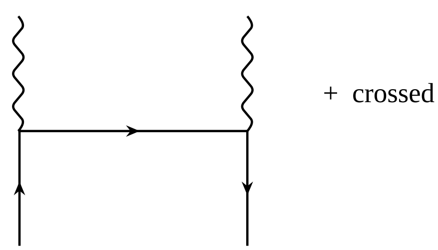

FIG. 1. The virtual Compton amplitude for a free quark.

free quarks (of charge unity, for simplicity). With the vector current given by $J_\mu(x) = \bar\psi(x)\gamma_\mu\psi(x)$, the matrix element of the first contraction of two currents, centered at point X and separated by a distance z, can be written in the form (see Fig.1)

$$\langle p - r/2|\, i\mathrm{T} J_\mu(X - z/2) J_\nu(X + z/2)\, |p + r/2\rangle^{\text{free q.}}$$
$$= \frac{1}{4}\mathrm{tr}\,[\gamma_\mu\, S(z)\,\gamma_\nu\, M(z|X)] \; + \; (\mu \leftrightarrow \nu, z \leftrightarrow -z). \quad (1)$$

Here $S(z) \equiv \hat z/2\pi^2(z^2 - i0)^2$ is the free Dirac propagator in coordinate space ($\hat z \equiv \gamma_\alpha z_\alpha$), and M the density matrix constructed from the incoming and outgoing quark wave functions (plane waves),

$$M_{ij}(z|X) \;=\; u_i \bar u_j\, e^{-i(pz) - i(rX)}, \quad (2)$$

where the spinors satisfy $(\hat p + \hat r/2)\, u = 0$ and $\bar u\,(\hat p - \hat r/2) = 0$. Because of current conservation the expression on the R.H.S. of Eq.(1) should vanish when contracted with the derivatives

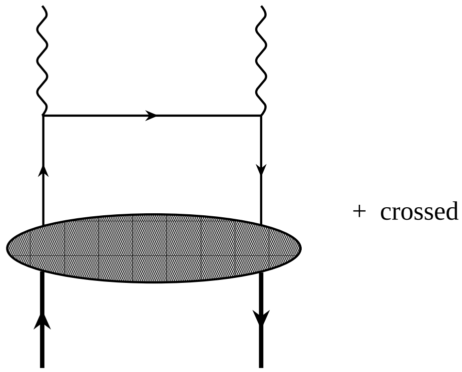

FIG. 2. The twist–2 contribution to the tree–level hadronic DVCS amplitude in QCD. The shaded blob represents the twist–2 part of the quark density matrix in the hadron, (6).

$$\frac{\partial}{\partial z_\mu} - \frac{1}{2}\frac{\partial}{\partial X_\mu}, \qquad \frac{\partial}{\partial z_\nu} + \frac{1}{2}\frac{\partial}{\partial X_\nu}. \tag{3}$$

Since the free Dirac propagator satisfies

$$\gamma_\mu \frac{\partial}{\partial z_\mu} S(z) \;=\; \frac{\partial}{\partial z_\nu} S(z)\gamma_\nu \;=\; -i\delta^{(4)}(z), \tag{4}$$

a necessary and sufficient condition for transversality is that the quark density matrix satisfy the "left" and "right" Dirac equations

$$\left.\begin{array}{l} \left(\dfrac{\partial}{\partial z_\nu} + \dfrac{1}{2}\dfrac{\partial}{\partial X_\nu}\right)\gamma_\nu\, M(z|X) \\ \left(\dfrac{\partial}{\partial z_\mu} - \dfrac{1}{2}\dfrac{\partial}{\partial X_\mu}\right) M(z|X)\,\gamma_\mu \end{array}\right\} \;=\; 0. \tag{5}$$

The free–field density matrix (2) clearly satisfies these equation, and thus the R.H.S. of Eq.(1) is transverse.

Turning now to QCD, the twist–2 contribution to the tree–level light–cone expansion of the time–ordered product of two vector currents has a form analogous to Eq.(1), the only difference being that the free quark density matrix is replaced by the twist–2 part of the non-forward matrix element of the quark density $\bar\psi(X - z/2)\ldots\psi(X+z/2)$ between hadronic states (see Fig. 2). The latter is defined as (we suppress the polarization quantum numbers of the hadronic states for brevity)

$$\begin{aligned} & M_{ij}(z|X)^{\text{twist–2}} \\ & \equiv \int_0^1 dv \left\{ (\gamma_\sigma)_{ij}\, \frac{\partial}{\partial z_\sigma} \langle p - r/2| \right. \\ & \quad \left[\bar\psi(X - vz/2)\,\hat z\,\psi(X+vz/2)\right]^{\text{traceless}} |p + r/2\rangle \\ & + (\gamma_5\gamma_\sigma)_{ij}\, \frac{\partial}{\partial z_\sigma}\, \langle p - r/2| \\ & \quad \left. \left[\bar\psi(X - vz/2)\,\hat z\,\gamma_5\psi(X+vz/2)\right]^{\text{traceless}} |p + r/2\rangle \right\} \end{aligned} \tag{6}$$

where the "traceless" string operator satisfies [10]

$$\Box_z \left[\bar\psi(X - z/2)\,\hat z\,\psi(X+z/2)\right]^{\text{traceless}} \;=\; 0, \tag{7}$$

and similarly for the operator with Dirac matrix $\hat z\gamma_5$. Only the chirally even part of the density matrix has been included in Eq.(6); the chirally odd (transversity) part with Dirac structure $\sigma_{\alpha\beta}$ would enter the DVCS amplitude only at power–suppressed level. Noting that the coefficient function for the twist–2 contribution in QCD is the same as that in the free theory — the free quark propagator, $S(z)$ — it is evident that transversality would again require that the quark density matrix satisfy the free Dirac equation. Substituting Eq.(6) in Eqs.(5) one finds

$$\begin{aligned} & \left.\begin{array}{l} \left(\dfrac{\partial}{\partial z_\nu} + \dfrac{1}{2}\dfrac{\partial}{\partial X_\nu}\right)\gamma_\nu\, M(z|X)^{\text{twist–2}} \\ \left(\dfrac{\partial}{\partial z_\mu} - \dfrac{1}{2}\dfrac{\partial}{\partial X_\mu}\right) M(z|X)^{\text{twist–2}}\,\gamma_\mu \end{array}\right\} \\ & = \left[\Box_z + \frac{1}{2}\left(\frac{\partial}{\partial X}\frac{\partial}{\partial z}\right) \pm \frac{1}{2}\left(\frac{\partial}{\partial X}\sigma\frac{\partial}{\partial z}\right)\right] \int_0^1 dv \\ & \quad \times \langle p - r/2| \left[\bar\psi(X - vz/2)\hat z\psi(X+vz/2)\right]^{\text{traceless}} \\ & \quad |p + r/2\rangle \\ & + \ldots, \end{aligned} \tag{8}$$

where the ellipsis stands for the corresponding term with the operator $\bar\psi\hat z\gamma_5\psi$, which we have not written out for brevity. The first term in the bracket vanishes because the string operator satisfies Eq.(7). The second term, involving the contracted derivatives, is proportional to $t \equiv r^2$ or m^2 (the target mass squared) and can be dropped if we are interested in the DVCS amplitude up to terms of order t/q^2 or m^2/q^2. The third term, however, involving the structure

$$\left(\frac{\partial}{\partial X}\sigma\frac{\partial}{\partial z}\right) \;\equiv\; \frac{\partial}{\partial X_\alpha}\sigma_{\alpha\beta}\frac{\partial}{\partial z_\beta}, \tag{9}$$

where $\sigma_{\alpha\beta} \equiv (1/2)[\gamma_\alpha, \gamma_\beta]$, is not proportional to either t or m^2, and thus cannot be neglected in DVCS kinematics. Thus, the twist–2 density matrix (6) does not satisfy the free Dirac equation even when neglecting terms proportional to t or m^2. This is the reason why the twist–2 contribution to the DVCS amplitude is not transverse up to terms of order t/q^2 or m^2/q^2, as has been observed within various different approaches in Refs. [1–6].

One may now ask how the twist–2 part of the quark density matrix (6) could be modified such that it satisfies the free–field Dirac equations up to terms of order

t or m^2. Since the problematic term in Eq.(8) involves the Dirac matrix $\sigma_{\alpha\beta}$, it seems natural to look for a transformation in the form of a quark spin rotation. Since we require that the transformation matrix should reduce to unity for forward matrix elements, the only possibility (up to a variable factor in the exponent) is a transformation of the form

$$\Sigma\,(z/2) \equiv \exp\left[-\frac{1}{4}\left(z\sigma\frac{\partial}{\partial X}\right)\right], \tag{10}$$

$$\left(z\sigma\frac{\partial}{\partial X}\right) \equiv z_\alpha \sigma_{\alpha\beta}\frac{\partial}{\partial X_\beta}, \tag{11}$$

which satisfies

$$\Sigma\,(z/2)^{-1} \;=\; \Sigma\,(-z/2). \tag{12}$$

It is easy to see that a density matrix with the desired properties is obtained by the following transformation:

$$\begin{aligned} M_{ij}(z|X)^{\mathrm{rot}} \;\equiv\; & \int_0^1 dv\,\Big\{ \\ & [\Sigma\,(\bar{v}z/2)\,\gamma_\sigma\,\Sigma\,(\bar{v}z/2)]_{ij}\,\frac{\partial}{\partial z_\sigma}\,\langle p-r/2| \\ & \left[\bar\psi(X-vz/2)\,\hat{z}\,\psi(X+vz/2)\right]^{\mathrm{traceless}}|p+r/2\rangle \\ & + [\Sigma\,(\bar{v}z/2)\,\gamma_5\gamma_\sigma\,\Sigma\,(\bar{v}z/2)]_{ij}\,\frac{\partial}{\partial z_\sigma}\,\langle p-r/2| \\ & \left[\bar\psi(X-vz/2)\,\hat{z}\gamma_5\,\psi(X+vz/2)\right]^{\mathrm{traceless}}|p+r/2\rangle\Big\}, \end{aligned} \tag{13}$$

where $\bar{v} \equiv 1-v$. Indeed, by a straightforward calculation, making use of the explicit form of the matrix exponential

$$\begin{aligned} \Sigma\,(\bar{v}z/2) = & \cosh\left[\frac{\bar{v}}{4}\left(z\frac{\partial}{\partial X}\right)\right] \\ & - \frac{\sinh\left[\frac{\bar{v}}{4}\left(z\frac{\partial}{\partial X}\right)\right]}{\left(z\frac{\partial}{\partial X}\right)}\left(z\sigma\frac{\partial}{\partial X}\right) \\ & + \text{terms}\;\left(\frac{\partial}{\partial X}\frac{\partial}{\partial X}\right), \end{aligned} \tag{14}$$

one can convince oneself that the "rotated" twist–2 density matrix (13) satisfies the free–field left– and right Dirac equations (5) up to terms involving contracted derivatives of the string operator of the form

$$\left(\frac{\partial}{\partial z}\frac{\partial}{\partial X}\right), \qquad \left(\frac{\partial}{\partial X}\frac{\partial}{\partial X}\right) \tag{15}$$

which have matrix elements of order t. Diagrammatically, the spin rotation of the twist–2 density matrix may be represented as in Fig.3, as a kind of "evolution kernel" acting between the twist–2 matrix element and the free quark propagator in the coefficient function. Note that since in the matrix element the total derivative operator just turns into the momentum transfer, $\partial/\partial X_\alpha = ir_\alpha$, the "rotated" density matrix does not involve any information beyond the basic twist–2 matrix elements, as parametrized by the twist–2 skewed distributions.

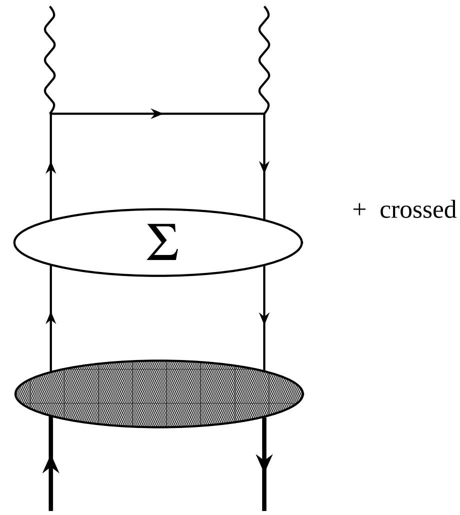

FIG. 3. Diagrammatic representation of the quark spin rotation, Eqs.(13) and (16), as an intermediate step between the twist–2 quark density matrix in the hadron and the free–quark virtual Compton amplitude.

The contribution to the light–cone expansion generated by the "rotated" density matrix (13) may be written in the form

$$\begin{aligned} & \frac{1}{4}\mathrm{tr}\left[\gamma_\mu\,S(z)\,\gamma_\nu\,M(z|X)^{\mathrm{rot}}\right] \;+\; (\mu\leftrightarrow\nu, z\leftrightarrow -z). \\ & = \frac{1}{2\pi^2 z^4}\int_0^1 dv\,\Big\{\frac{1}{4}\mathrm{tr}\,[\Sigma\,(\bar{v}z/2)\,\gamma_\mu\,\hat{z}\,\gamma_\nu\,\Sigma\,(\bar{v}z/2)\gamma_\sigma] \\ & \quad \times\frac{\partial}{\partial z_\sigma}\,\langle p-r/2| \\ & \quad \left[\bar\psi(X-vz/2)\,\hat{z}\,\psi(X+vz/2)\right]^{\mathrm{traceless}}|p+r/2\rangle \\ & + \frac{1}{4}\mathrm{tr}\,[\Sigma\,(\bar{v}z/2)\,\gamma_\mu\,\hat{z}\,\gamma_\nu\,\Sigma\,(\bar{v}z/2)\,\gamma_5\gamma_\sigma] \\ & \quad \times\frac{\partial}{\partial z_\sigma}\,\langle p-r/2| \\ & \quad \left[\bar\psi(X-vz/2)\,\hat{z}\gamma_5\,\psi(X+vz/2)\right]^{\mathrm{traceless}}|p+r/2\rangle\Big\} \\ & + (\mu\leftrightarrow\nu, z\leftrightarrow -z), \end{aligned} \tag{16}$$

where we have substituted the explicit form of the free coordinate–space quark Green function. In this expression the spin rotation appears as acting on the coefficient functions rather than the matrix elements; this is of course a matter of interpretation. One can easily verify that Eq.(16) is identical to the expression in Eq.(3.51) of Ref. [6], which was obtained by making repeated use of an string operator identity following from the QCD equations of motion and neglecting quark–gluon operators. Thus, we see that the "kinematical" twist–3 contributions to the non-forward light–cone expansion (and consequently to the DVCS amplitude) can be understood as the effect of a quark spin rotation applied to the twist–2 part of the quark density matrix.

We have chosen here to present the spin rotation at the level of non-forward matrix elements of the quark density, as relevant to DVCS. It is clear, however, that this representation of "kinematical" twist–3 contributions in the light–cone expansion is valid at the operator level, and thus can be applied also in the context of other exclusive reactions where the relevant matrix elements are of distribution amplitude type.

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