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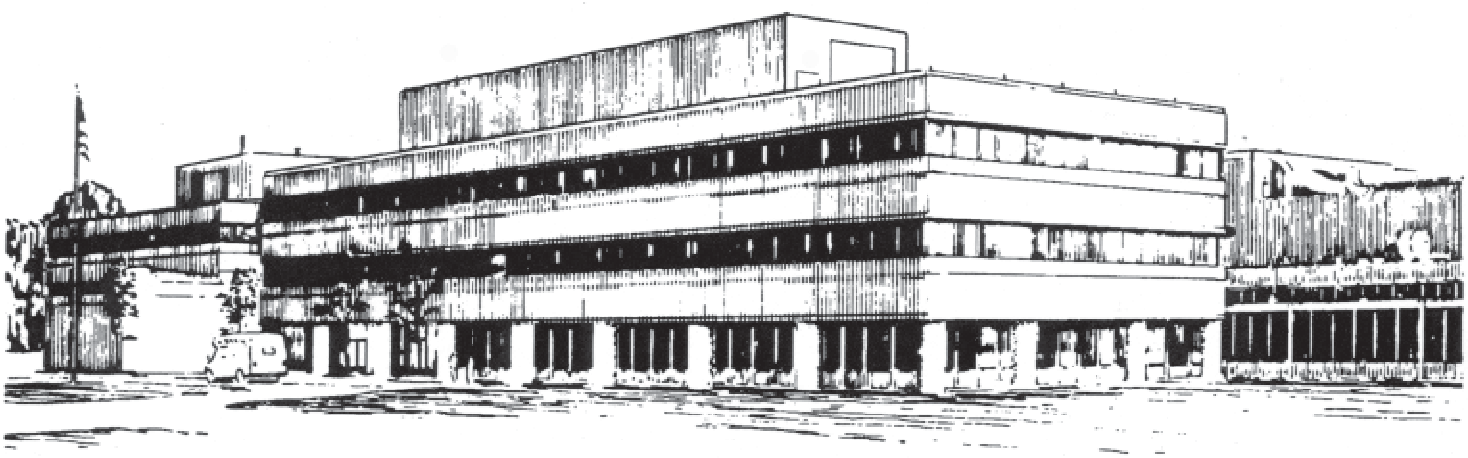

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Plasma Dielectric Tensor for Non-Maxwellian Distributions in the FLR Limit

C.K. Phillips[1], A. Pletzer[1], R.J. Dumont[2] and D.N. Smithe[3]

[1]*Princeton Plasma Physics Laboratory, P.O. Box 451, Princeton, NJ 08540, U.S.A.*
[2]*Association Euratom-CEA sur la Fusion Contrôlée, F-13108 St Paul lez Durance, France*
[3]*Mission Research Corp., Newington, VA 22122, U.S.A.*

Abstract. Previous analytical and numerical studies have noted that the presence of fully non-Maxwellian plasma species can significantly alter the dynamics of electromagnetic waves in magnetized plasmas. In this paper, a general form for the hot plasma dielectric tensor for non-Maxwellian distributions is derived that is valid in the finite Larmor radius approximation. This model provides some insight into understanding the limitations on representing non-Maxwellian plasma species with equivalent Maxwellian components in modeling RF wave propagation and absorption.

INTRODUCTION

Laboratory fusion plasmas as well as many space plasmas can be comprised of both thermal and non-thermal species. In collisionless space plasmas, turbulent heating or shock processes can accelerate particles, resulting in velocity-space distributions that are Lorentzian or power-law-like in nature [1-3]. Neutral beam injection and fusion reactions in laboratory fusion plasmas both introduce energetic ions, which follow a slowing-down type distribution in velocity space. Finally, when electromagnetic waves are applied to heat or else to drive noninductive currents in magnetized plasmas, the wave-induced particle acceleration results in velocity-space distributions that feature energetic “tails” or extended “quasilinear plateaus”. In all of these situations, the question that arises is whether or not these non-thermal plasma species have a noticeable impact on electromagnetic wave dynamics in these plasmas.

Previous analytical and numerical studies [1-6] have shown that wave dynamics can be affected if a sizeable non-thermal ion population is present in the plasma. Most of these studies have focused on modifications to wave absorption or to instability thresholds. A number of these studies [4,5,7] have noted that power absorption on a non-Maxwellian distribution can be approximated by that on an equivalent Maxwellian, chosen so that the thermal speed of the equivalent Maxwellian is equal to the velocity-space averaged perpendicular speed of the non-thermal distribution.

More recently, a 1D all-orders local full wave, METS [8], has been extended to include the effects of non-thermal species on both wave propagation and absorption. Results from this code indicate that the absorption and wave propagation in plasmas with isotropic, non-Maxwellian species can be reasonably simulated with equivalent

Maxwellian in many regimes. However, the spatial profile of power deposition on short wavelength kinetic waves can be narrower and anisotropic effects can lead to larger discrepancies with models based on equivalent Maxwellians [8].

In this paper, the hot plasma dielectric susceptibility for a fully non-Maxwellian but still gyrotropic particle distribution function is derived that is valid in the finite Larmor radius limit (FLR). This model provides some insight into understanding the limitations on representing non-thermal species as equivalent Maxwellians in modeling RF wave absorption and propagation. It also provides the basis for generalizing the plasma dielectric operators in FLR-based 2D full wave simulation codes.

DERIVATIONS

The hot plasma susceptibility for a given species, "s", described by the gyrotropic particle velocity distribution $f_0(v_\perp,v_{//})$, in a homogeneous, uniformly magnetized plasma can be written in the following form:

$$\vec{\vec{\chi}}_s = \frac{\omega_{ps}^2}{\omega}\int_0^\infty 2\pi v_\perp dv_\perp \int_{-\infty}^\infty dv_{//}\ \hat{z}\hat{z}\frac{v_{//}^2}{\omega}\left(\frac{1}{v_{//}}\frac{\partial f_0}{\partial v_{//}} - \frac{1}{v_\perp}\frac{\partial f_0}{\partial v_\perp}\right)_s + \frac{\omega_{ps}^2}{\omega}\int_0^\infty 2\pi v_\perp dv_\perp \int_{-\infty}^\infty dv_{//} \sum_{n=-\infty}^{n=\infty}\left[\frac{v_\perp U}{\omega - k_{//}v_{//} - n\Omega}\vec{\vec{T}}_n\right]_s \quad , \tag{1}$$

where $\vec{\vec{T}}_n$ is given in Equation (10-48) of reference [9] by Stix. In the FLR limit, the expressions in Equation (1) may be simplified by replacing the Bessel functions in $\vec{\vec{T}}_n$ by their series expansions, integrating by parts the terms involving $\int_0^\infty 2\pi v_\perp dv_\perp (v_\perp^m)\partial f_0/\partial v_\perp(.......)$, and retaining terms through order $\lambda \sim k_\perp^2 w_\perp^2/2\Omega_s^2$, where Ω_s is the cyclotron frequency and $w_\perp^2$ is the velocity space average of the perpendicular velocity. The resulting form of the plasma susceptibility is a generalization of the Maxwellian-based FLR susceptibility given by Stix in Equations (59-63) in Chapter 10 in reference [9]. It may be written in similar but generalized form as:

$$\chi_{xx} = \frac{\omega_p^2}{\omega}\left\{\frac{1}{2}\left[\tilde{A}_{1,0} + \tilde{A}_{-1,0}\right] - \frac{\lambda}{2}\left[\tilde{A}_{1,1} + \tilde{A}_{-1,1}\right] + \frac{\lambda}{2}\left[\tilde{A}_{2,1} + \tilde{A}_{-2,1}\right]\right\} \tag{2}$$

$$\chi_{xy} = i\frac{\omega_p^2}{\omega}\left\{\frac{1}{2}\left[\tilde{A}_{1,0} - \tilde{A}_{-1,0}\right] - \lambda\left[\tilde{A}_{1,1} - \tilde{A}_{-1,1}\right] + \frac{\lambda}{2}\left[\tilde{A}_{2,1} - \tilde{A}_{-2,1}\right]\right\} \tag{3}$$

$$\chi_{xz} = \frac{\omega_p^2}{\omega}\left(\frac{1}{2}\frac{k_\perp}{\Omega}\right)\left\{\left[\tilde{B}_{1,0} + \tilde{B}_{-1,0}\right] - \lambda\left[\tilde{B}_{1,1} + \tilde{B}_{-1,1}\right] + \frac{\lambda}{2}\left[\tilde{B}_{2,1} + \tilde{B}_{-2,1}\right]\right\} \tag{4}$$

$$\chi_{yy} = \frac{\omega_p^2}{\omega}\left\{2\lambda\tilde{A}_{0,1} + \frac{1}{2}\left[\tilde{A}_{1,0} + \tilde{A}_{-1,0}\right] - \frac{3\lambda}{2}\left[\tilde{A}_{1,1} + \tilde{A}_{-1,1}\right] + \frac{\lambda}{2}\left[\tilde{A}_{2,1} + \tilde{A}_{-2,1}\right]\right\} \tag{5}$$

$$\begin{aligned}\chi_{yz} = i\frac{\omega_p^2}{\omega}\left(\frac{k_\perp}{\Omega}\right)\{&\tilde{B}_{0,0} - \lambda\tilde{B}_{0,1} - \frac{1}{2}\left[\tilde{B}_{1,0} + \tilde{B}_{-1,0}\right] \\ &-\lambda\left[\tilde{B}_{1,1} + \tilde{B}_{-1,1}\right] - \frac{\lambda}{4}\left[\tilde{B}_{2,1} + \tilde{B}_{-2,1}\right]\}\end{aligned} \tag{6}$$

$$\begin{aligned}\chi_{zz} = &\frac{2\omega_p^2}{\omega k_{//} w_\perp^2}\int_{-\infty}^{\infty} dv_{//}\int_0^{\infty} 2\pi v_\perp dv_\perp \; v_{//}\left[\frac{-1}{v_\perp}\frac{\partial f_0}{\partial v_\perp}\frac{w_\perp^2}{2}\right] \\ &+ \frac{\omega_p^2}{\omega}\left[\frac{2\omega}{k_{//} w_\perp^2}\right]\{(1-\lambda)\tilde{B}_{0,0} \\ &+ \int_{-\infty}^{\infty} dv_{//}\int_0^{\infty} 2\pi v_\perp dv_\perp \; \frac{v_{//}}{\omega - k_{//}v_{//}}\left[\frac{1}{2}\frac{w_\perp^2}{v_\perp}\frac{\partial f_0}{\partial v_\perp} + f_0\right]\left(\frac{1-k_{//}v_{//}}{\omega}\right)\} \\ &+ \frac{1}{2}\frac{\omega_p^2}{\omega}\lambda\left\{\frac{2(\omega-\Omega)}{k_{//}w_\perp^2}\tilde{B}_{1,0} + \frac{2(\omega+\Omega)}{k_{//}w_\perp^2}\tilde{B}_{-1,0}\right\},\end{aligned} \tag{7}$$

where, for j=0,1:

$$\tilde{A}_{n,j} = \int_{-\infty}^{\infty} dv_{//}\frac{1}{\omega - k_{//}v_{//} - n\Omega}\int_0^{\infty} 2\pi v_\perp dv_\perp Hj(v_{//}, v_\perp) \tag{8}$$

$$\tilde{B}_{n,j} = \int_{-\infty}^{\infty} dv_{//}\frac{v_{//}}{\omega - k_{//}v_{//} - n\Omega}\int_0^{\infty} 2\pi v_\perp dv_\perp Hj(v_{//}, v_\perp) \tag{9}$$

$$H_0(v_{//}, v_\perp) = \frac{1}{2}\frac{k_{//}w_\perp^2}{\omega}\frac{\partial f_0}{\partial v_{//}} - (1 - \frac{k_{//}v_{//}}{\omega})f_0(v_{//}, v_\perp) \tag{10}$$

$$H_1(v_{//}, v_\perp) = \frac{1}{2}\frac{k_{//}w_\perp^2}{\omega}\frac{\partial f_0}{\partial v_{//}}\frac{v_\perp^4}{w_\perp^4} - (1 - \frac{k_{//}v_{//}}{\omega})f_0(v_{//}, v_\perp)\frac{v_\perp^2}{w_\perp^2} \quad . \tag{11}$$

In the limit that $f_0(v_\perp, v_{//}) = f_{max}(v_\perp)\, h(v_{//})$, then the generalized susceptibility reduces to that given in reference [9].

DISCUSSION

The local hot plasma susceptibility for a given species, "s", described by the gyrotropic particle velocity distribution $f_0(v_\perp, v_{//})$, that is valid in the FLR approximation has been derived by expanding the full hot plasma susceptibility to first order in $\lambda \sim k_\perp^2 w_\perp^2 / 2\Omega_s^2$. In the limit that $k_{//} \Rightarrow 0$, the FLR-based susceptibility for a general distribution differs from that of a Maxwellian distribution only in terms of $O(\lambda)$. The xx, xy, and yy elements will be the same as that of an equivalent Maxwellian in this limit, provided that the thermal speed of the Maxwellian is chosen to equal $w_\perp^2$. Hence, the propagation of waves, such as fast waves or ion Bernstein waves, which depends on these elements, can be simulated exactly using the equivalent Maxwellian. The χ_{xz} and χ_{yz} elements will be approximately equal in this limit, provided the ratio:

$$\frac{I_v}{\langle v_{//} \rangle} = \frac{\int_0^\infty 2\pi v_\perp dv_\perp \int_{-\infty}^\infty dv_{//} \; v_{//} f_0(v_{//}, v_\perp) \frac{v_\perp^2}{w_\perp^2}}{\int_0^\infty 2\pi v_\perp dv_\perp \int_{-\infty}^\infty dv_{//} \; v_{//} f_0(v_{//}, v_\perp)} \tag{12}$$

is close to unity. More generally, the FLR-based susceptibility elements for a general, gyrotropic distribution can be computed using equivalent Maxwellians, if the $(\text{velocity})^n$ moments for the general distribution are similar to those of the equivalent Maxwellian. Finally, the expressions for the FLR-based susceptibility given in Equations (2)-(11) may be utilized to generalize the dielectric operator in 2D FLR-based full wave codes. Such a generalization is required in order to self-consistently integrate such codes with Fokker-Planck packages.

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