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SO(10) GUT Models and Their Present Success in Explaining Mass and Mixing Data*

CARL H. ALBRIGHT

*Department of Physics, Northern Illinois University
 DeKalb, Illinois 60115, USA*

and

*Fermi National Accelerator Laboratory, P.O. Box 500
 Batavia, Illinois 60510, USA†*

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Some features of SO(10) GUT models are reviewed, and a number of such models in the literature are compared. While some have been eliminated by recent neutrino data, others are presently successful in explaining the quark and lepton mass and mixing data. A short description of one very predictive model is given which illustrates some of the features discussed. Future tests of the models are pointed out including one which contrasts sharply with those models based on an $L_e - L_\mu - L_\tau$ type symmetry.

Keywords: SO(10) Gut models; quark and lepton masses and mixings.

1. Introduction

Many mass matrix models in the literature¹ attempt to explain only the recent mass and mixing data in the lepton sector. More ambitious attempts introduce supersymmetric grand unified (SUSY GUT) models to understand both the lepton and quark sectors. In this brief review my attention is restricted to four-dimensional three family SO(10) GUT models with no light sterile neutrinos. One finds that several models are presently quite successful in explaining the data, including the preferred LMA solar neutrino solution. One model is illustrated in some detail, while future critical tests of the presently successful models are described.

2. SO(10) Model Structure

It is well known that the three families of left-handed quarks and leptons and their left-handed charge conjugates fit neatly into three copies of the SO(10) spinor representation, $\mathbf{16}_i$, $i = 1, 2, 3$. In fact, this feature is what has made SO(10) so

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†present address

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attractive as a unification group. Higgs fields appearing in the $\mathbf{45}_H$, $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ are needed to break $\text{SO}(10)$ to the standard model. The two light Higgs doublets which are required to break the electroweak symmetry can be accommodated by a single $\mathbf{10}_H$ of $\text{SO}(10)$, which consists of a $5 + \bar{5}$ of $\text{SU}(5)$ or a $(6, 1, 1) + (1, 2, 2)$ of $\text{SU}(4) \times \text{SU}(2)_L \times \text{SU}(2)_R$. Doublet-triplet splitting of the Higgs fields is required and can be achieved via the Dimopoulos-Wilczek mechanism,² if the $\langle \mathbf{45}_H \rangle$ VEV points in the $B-L$ direction. With only one $\mathbf{10}_H$ effecting the electroweak breaking, $\tan \beta \equiv v_u/v_d \sim 55$.

The above represents the essential ingredients of an $\text{SO}(10)$ model. However, many authors have found it desirable to introduce additional Higgs fields. For example, an additional $\mathbf{16}'_H$, $\overline{\mathbf{16}}'_H$ pair can help to stabilize the double-triplet splitting solution.³ If the VEV $\langle \bar{5}(\mathbf{16}'_H) \rangle \neq 0$, the light Higgs doublet, H_d , can reside in both the vector and spinor representations, i.e.,

$$H_d \sim \bar{5}(\mathbf{10}_H) \cos \gamma + \bar{5}(\mathbf{16}'_H) \sin \gamma, \quad (1)$$

so that Yukawa coupling unification is possible for any value in the range $\tan \beta \sim 1 - 55$. Similar results are also possible with the addition of a $\mathbf{126}_H$, $\overline{\mathbf{126}}_H$ pair in place of the $\mathbf{16}'_H$, $\overline{\mathbf{16}}'_H$ pair of Higgs fields.

It may also be desirable to introduce $\mathbf{16}$, $\overline{\mathbf{16}}$ pairs, $\mathbf{10}$'s, etc. of matter fields provided they get supermassive near the GUT scale. They can then be integrated out in Froggatt-Nielsen type diagrams⁴ which provide higher-order effective interaction contributions to the mass matrix elements.

3. Horizontal Flavor Symmetries

While $\text{SO}(10)$ relates quarks and leptons of one family, it is necessary to invoke some horizontal flavor symmetry in order to avoid the bad $\text{SU}(5)$ relations such as $m_d = m_e$ and $m_s = m_\mu$. This can be done at four different levels of model building with the following prescriptions.

- Level 1: Simply impose a certain texture, such as a modified Fritzsch form for the mass matrices.
- Level 2: Introduce an effective $\lambda \sim 0.22$ expansion for each mass matrix. The prefactors of the expansion parameters typically are not precisely determined, however.
- Level 3: Assign effective operators for each matrix element, possibly with some flavor symmetry imposed.
- Level 4: Introduce a horizontal flavor symmetry which assigns flavor charges to every Higgs and matter superfield. Higgs and Yukawa superpotentials are constructed in terms of renormalizable (and possibly some non-renormalizable) terms which obey that flavor symmetry. Matrix elements then follow from the Froggatt-Nielsen diagrams which can be constructed.

4. Some General Observations

- The SO(10) models found in the literature differ by their choice of Higgs structure, horizontal flavor symmetry and the flavor charge assignments, if any.
- The desirable Georgi-Jarlskog relations,⁵ $m_s = m_\mu/3$, $m_d = 3m_e$, can be readily obtained if a $\langle \mathbf{45}_H \rangle$ Higgs VEV points in the $B - L$ direction, or if a $\langle \bar{\mathbf{5}}(\mathbf{126}_H) \rangle$ VEV is involved.
- The presence of a $\langle \bar{\mathbf{5}}(\mathbf{16}_H) \rangle$ VEV and a flavor symmetry will typically lead to lopsided⁶ down quark and charged lepton mass matrices, D and L . This is useful to explain the small V_{cb} and large $U_{\mu 3}$ mixing matrix elements.⁷ A consequence of this lopsided nature is an enhanced flavor-violating $\tau \rightarrow \mu\gamma$ decay rate that is within one or two orders of magnitude of the present experimental limit. Hence future improved experiments will be able to confirm or rule out this mechanism.
- Most early models were easily able to accommodate the SMA solar neutrino solution, while some could accommodate the LOW or QVO solution as well. However, to obtain the LMA solution in the SO(10) GUT model framework with the seesaw mechanism, some fine tuning is generally required. Typically, models which require special features of the Dirac and right-handed Majorana mass matrices, N and M_R , to get maximal atmospheric mixing have trouble getting the LMA solar solution. That is easier to achieve if the M_R matrix can be independently adjusted to yield the LMA solution, while N and L conspire to give maximal atmospheric mixing.

5. Some Selected SO(10) Models

A number of SO(10) SUSY GUT models can be found in the literature.¹ To illustrate the success of some well-known models, I have confined my attention to four-dimensional models with three quark and lepton families for which the seesaw mechanism applies with three right-handed (conjugate left-handed) singlet neutrino fields. I have also assumed that the presently-preferred LMA solution⁸ will be confirmed by KamLAND.⁹ On this basis already some of the models, as constructed, have been ruled out by more recent mass and mixing data, while others still survive. It is instructive, however, to compare the various features of all the models considered.

Table 1 lists the models with their level of construction, flavor symmetry, texture, applicable range of $\tan\beta$, whether or not they fit the CKM mixing matrix and their preferred solar neutrino solution. Some textures correspond to lopsided mass matrices, while others have only symmetric or both symmetric and antisymmetric entries. The latter favor large values of $\tan\beta$ to give the desired Yukawa coupling unification at the GUT scale, while the lopsided models tend to require low or moderate values of $\tan\beta$ in order that the matrices be lopsided enough. Thus the determination of $\tan\beta$, as well as the observation of the $\tau \rightarrow \mu\gamma$ mentioned earlier, will serve to rule out one choice or the other.

Some features of the models warrant specific remarks. In the Blazek-Raby-Tobe

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Table 1. Features of some selected SO(10) models.

Model	Ref.	Level	Flavor Sym.	Texture	$\tan \beta$	CKM	Solar	Viable
AB	¹⁰	4	$U(1) \times Z_2 \times Z_2$	Lopsided	~ 5	Yes	LMA	Yes
BPW	¹¹	3	effective operators	Sym/Asym	low	Yes	LMA	Yes
BR	¹²	4	$SU(3)$	Lopsided	1-10	Yes	SMA	No
BRT	¹³	4	$U(2) \times U(1)^n$	Sym/Asym	~ 55	No	LMA	No
BW	¹⁴	1	postulated	Sym	?	Yes	LMA	?
CM	¹⁵	4	$U(2) \times (Z_2)^3$	Sym	10	Yes	LOW	No
CW	¹⁶	4	$\Delta(48) \times U(1)$	Sym/Asym	~ 2	Yes	LMA	No
KM	¹⁷	2	$SU(3) \times U(1)$	Lopsided	small	?	LMA	?
M	¹⁸	2	$U(1)_A \times Z_2$	Lopsided	small	Yes	LMA	No
RV-S	¹⁹	2	$SU(3)$ and Abelian	Sym/Asym	?	Yes	LMA	Yes

model,¹³ a sterile neutrino is present while the apex of the CKM triangle is in the second quadrant which is now at odds with the recent quark mixing determination. The Chou-Wu model¹⁶ requires a sterile neutrino to get the solar LMA solution. The Chen-Mahanthappa model¹⁵ prefers the LOW solar solution, for the LMA solution can not be obtained without violating the upper CHOOZ bound²⁰ on U_{e3} . The Maekawa model¹⁸ also violates the CHOOZ bound on U_{e3} . For the Buchmuller-Wyler model¹⁴ and the Kitano-Mimura model¹⁷, it is not clear from their solutions whether the LMA mixing is in the presently allowed range. Of the three remaining apparently successful models, the Babu-Pati-Wilczek model¹¹ requires a non-seesaw contribution to the left-handed Majorana matrix, M_L , in order to fit both the atmospheric and solar LMA solutions. The Ross-Velasco-Sevilla model¹⁹ is rather recent and has not been completely specified. To illustrate some of the features of SO(10) models cited earlier, some detailed features of the very predictive Albright-Barr model¹⁰ are presented in the next Section.

Of the models listed in Table 1, some are already essentially ruled out by the more accurate recent quark and lepton mixing data, but as we have seen, several are still viable. In making this judgment I have assumed there are no light sterile neutrinos and that the LMA solution is the correct one. Of course, some models which are on the verge of being ruled out may be revived by their authors with further adjustments.

6. Example of the LMA Solution in One Predictive SO(10) Model

The model developed in Ref. 10 is based on a $U(1) \times Z_2 \times Z_2$ flavor symmetry that stabilizes the Dimopoulos-Wilczek solution to the doublet-triplet splitting problem by the introduction of a second pair of $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ Higgs fields.³ The Higgs and Yukawa superpotentials can be written down after flavor charges for that symmetry are assigned to all the Higgs and matter fields. The mass matrices then follow from Froggatt-Nielsen diagrams involving the vertex terms appearing in the superpotentials.

The Dirac mass matrices for the up and down quarks, neutrinos and charged leptons are found to be

$$\begin{aligned}
 U &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U, & D &= \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} M_D, \\
 N &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U, & L &= \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma + \epsilon & 1 \end{pmatrix} M_D,
 \end{aligned} \tag{2}$$

Several texture zeros appear in elements for which the flavor symmetry forbids the appearance of any Froggatt-Nielsen diagrams. The antisymmetric ϵ terms arise from diagrams involving the adjoint $\langle \mathbf{45}_H \rangle$ Higgs VEV pointing in the $B - L$ direction. The lopsided nature of the large σ terms in D and L arises from the appearances of diagrams involving the $\langle \overline{\mathbf{5}}(\mathbf{16}_H) \rangle$ Higgs VEV as suggested earlier.

The eight input parameters are defined at the GUT scale and are set equal to

$$\begin{aligned}
 M_U &\simeq 113 \text{ GeV}, & M_D &\simeq 1 \text{ GeV}, \\
 \sigma &= 1.78, & \epsilon &= 0.145, \\
 \delta &= 0.0086, & \delta' &= 0.0079, \\
 \phi &= 54^\circ, & \eta &= 8 \times 10^{-6}.
 \end{aligned} \tag{3}$$

With these values, the structures of the D and L matrices lead to the Georgi-Jarlskog relations at the GUT scale with Yukawa coupling unification for $\tan \beta \sim 5$. All nine quark and charged lepton masses plus the three CKM angles and CP phase are well-fitted with these input parameters after evolution from the GUT scale:

$$\begin{aligned}
 m_t(m_t) &= 165 \text{ GeV}, & m_\tau &= 1.777 \text{ GeV} \\
 m_u(1 \text{ GeV}) &= 4.5 \text{ MeV}, & m_\mu &= 105.7 \text{ MeV} \\
 V_{us} &= 0.220, & m_e &= 0.511 \text{ MeV} \\
 V_{cb} &= 0.0395, & \delta_{CP} &= 64^\circ
 \end{aligned} \tag{4}$$

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determine the input parameters which lead to the following predictions:

$$\begin{aligned}
 m_b(m_b) &= 4.25 \text{ GeV}, & m_c(m_c) &= 1.23 \text{ GeV} \\
 m_s(1 \text{ GeV}) &= 148 \text{ MeV}, & m_d(1 \text{ MeV}) &= 7.9 \text{ MeV} \\
 |V_{ub}/V_{cb}| &= 0.080, & \sin 2\beta &= 0.64.
 \end{aligned}
 \tag{5}$$

The Hermitian matrices $U^\dagger U$, $D^\dagger D$, and $N^\dagger N$ are diagonalized by small LH rotations, while $L^\dagger L$ is diagonalized by a large LH rotation. This accounts for the fact that $V_{cb} = (U_U^\dagger U_D)_{cb}$ is small, while $U_{\mu 3} = (U_L^\dagger U_\nu)_{\mu 3}$ is large and responsible for the maximal atmospheric neutrino mixing for any reasonable M_R .

The type of $\nu_e \leftrightarrow \nu_\mu, \nu_\tau$ solar neutrino mixing is determined by the texture of M_R , since the solar and atmospheric mixings are essentially decoupled in this model. Further study reveals the LMA solution requires a nearly hierarchical texture¹⁰ which can also be understood with Froggatt-Nielsen diagrams. The texture suggested is

$$M_R = \begin{pmatrix} b^2 \eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R,
 \tag{6}$$

with the parameters ϵ and η specified in Eq. (3). Here Λ_R then sets the scale of the heavy right-handed Majorana neutrino masses and determines Δm_{32}^2 for the atmospheric neutrino mixing by the seesaw mechanism.

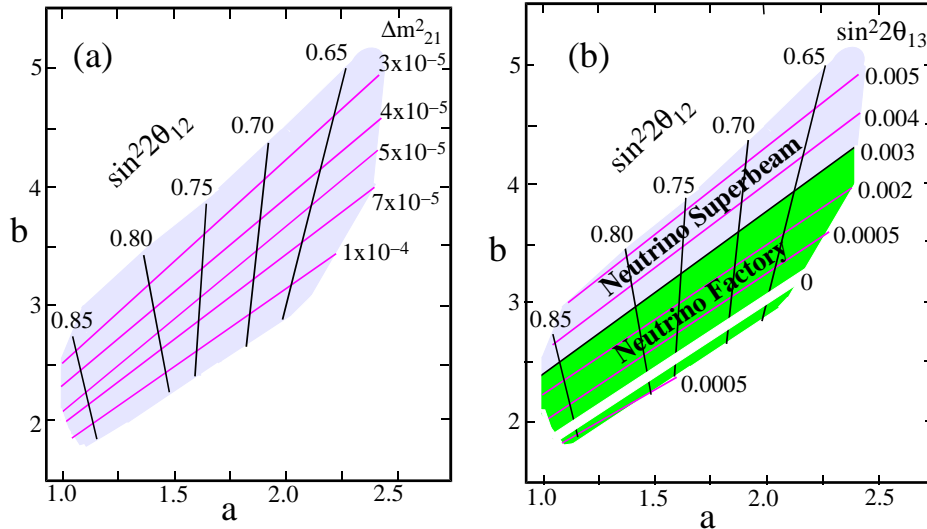


Fig. 1. The viable region of GUT parameter space consistent with the present bounds on the LMA MSW solution. Contours of constant $\sin^2 2\theta_{12}$ are shown together with (a) contours of constant Δm_{21}^2 and (b) contours of $\sin^2 2\theta_{13}$.

The allowed parameter space in the $a-b$ plane²¹ shown in Fig. 1 corresponds to the pre-SNO allowed LMA solar neutrino region²² in the $\Delta m_{21}^2 - \sin^2 2\theta_{12}$ mixing plane. With the recent SNO and Super-Kamiokande results,⁸ it should be understood that part of the allowed parameter region corresponding to higher values of a , i.e., lower values of $\sin^2 2\theta_{12}$, has been eliminated. In Fig. 1(a) contours of constant $\sin^2 2\theta_{12}$ and Δm_{21}^2 are shown, while contours of constant $\sin^2 2\theta_{12}$ and $\sin^2 2\theta_{13}$ are given in Fig. 1(b). Once Δm_{21}^2 and $\sin^2 2\theta_{12}$ are known, the model parameters a and b are determined by Fig. 1(a) from which the reactor neutrino mixing $\sin^2 2\theta_{13}$ can be found from Fig. 1(b). We observe that the reactor angle, θ_{13} , as determined in this model is generally much smaller than that determined from the present CHOOZ bound,²⁰ i.e., $|U_{e3}| \simeq \sin \theta_{13} < 0.16$ or $\sin^2 2\theta_{13} < 0.10$. As indicated, a Neutrino Factory will be required to determine θ_{13} for a large part of the presently allowed region.

As an interesting special case, we note that with $a = 1$, $b = 2$ and $\Lambda_R = 2.72 \times 10^{14}$ GeV, the seesaw mechanism leads to the simple light neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0 & -\epsilon & 0 \\ -\epsilon & 0 & 2\epsilon \\ 0 & 2\epsilon & 1 \end{pmatrix} M_U^2 / \Lambda_R. \quad (7)$$

From M_ν , L and the input parameters we then find

$$\begin{aligned} M_1 &= 3.2 \times 10^8, \quad M_2 = 3.6 \times 10^8, \quad M_3 = 2.8 \times 10^{14} \text{ GeV}, \\ m_1 &= 4.9 \text{ meV}, \quad m_2 = 8.7 \text{ meV}, \quad m_3 = 51 \text{ meV}, \\ \Delta m_{32}^2 &= 2.5 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\text{atm}} = 0.994, \\ \Delta m_{21}^2 &= 5.1 \times 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta_{\text{sol}} = 0.88, \quad \tan^2 \theta_{13} = 0.49, \\ U_{e3} &= -0.014, \quad \sin^2 2\theta_{\text{reac}} = 0.0008 \end{aligned} \quad (8)$$

Note that Λ_R has not only set the scale for the atmospheric neutrino mixing Δm_{32}^2 but also for the solar neutrino mixing Δm_{21}^2 . For example, a value of $\Delta m_{32}^2 = 2.8 \times 10^{-3} \text{ eV}^2$ results in $\Delta m_{21}^2 = 5.7 \times 10^{-5} \text{ eV}^2$, while the mixing angles remain unchanged. Whereas one might have expected an inverted hierarchy with M_1 and M_2 so close together and much smaller than M_3 , the resultant form of M_ν leads to a normal but rather mild hierarchy for the light left-handed neutrino masses.

7. Future Tests of SO(10) Models

Several critical tests will be made in the future with long baseline experiments involving Superbeams, and possibly Neutrino Factories. These tests involve the nature of the light neutrino mass hierarchy, i.e., normal vs. inverted; the value of the reactor neutrino mixing angle θ_{13} or the element $|U_{e3}| \simeq \sin \theta_{13}$; and the determination of the leptonic Dirac CP-violating phase δ . For the three models

Table 2. Predictions of the presently successful models considered.

Model	Ref.	Hierarchy	$ U_{e3} $	$\sin^2 2\theta_{13}$	CP Violation
AB	¹⁰	Normal	0 - 0.035	0 - 0.005	Small
BPW	¹¹	Normal	?	?	?
RV-S	¹⁹	Normal	~ 0.07	~ 0.02	?

considered which clearly appear to be still viable, the predictions are listed in Table 2.

It is apparent that the presently successful SO(10) GUT models favor a normal hierarchy. This is in stark contrast with the models with a conserved lepton number quantity,²³ such as $L_e - L_\mu - L_\tau$, which favor an inverted hierarchy.^a On the other hand, the predicted value for $|U_{e3}|$ is apparently quite model dependent, with some models predicting values very close to the CHOOZ bound, while others require a Neutrino Factory to pin down the correct value. Unfortunately, the leptonic CP violating phase δ , which is of great interest if the LMA solution is the correct one, is not well determined in most models.

8. Summary

A number of SO(10) SUSY GUT models have been proposed in the literature with a small but interesting sample considered here. Some have been, or are on the verge of being, eliminated, while others still survive and are able to explain all the known quark and lepton mass and mixing data. Long baseline experiments which can determine whether the neutrino mass hierarchy is normal or inverted appear to have a direct bearing on the survival of SO(10) vs. nearly-conserved $L_e - L_\mu - L_\tau$ type models. This particular test appears to be one of the most promising for narrowing down the list of successful model candidates.

The observed value of $\sin^2 2\theta_{13}$ appears to be less discriminatory between models of the SO(10) or the conserved lepton type. Some models of both types predict that θ_{13} lies just below the CHOOZ bound and will be observable with off-axis beams and/or Superbeams. Others favor such low values of θ_{13} that a Neutrino Factory will be required to determine its value.

The issue of proton decay via dim-5 operators is potentially a serious one for GUT models, if proton decay is not detected shortly.²⁵ On the other hand, by formulating an SO(10) model in five dimensions, one can eliminate the dim-5 operator contributions entirely.²⁶ The dim-6 operators will still be present and possibly somewhat enhanced, but they typically lead to lifetimes for proton decay which are presently two to three orders of magnitude larger than the present lower bounds.

^aFor a recent variant of such models and additional references, see Ref. 24.

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