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Fractal Dynamics of Earthquakes

P. Bak and K. Chen

11.1 INTRODUCTION

Many objects in nature, from mountain landscapes to electrical breakdown and turbulence, have a self-similar fractal spatial structure (Mandelbrot, 1982). This is by no means a trivial observation, since it implies that systems are correlated over large distances. Much effort has been put into computer simulation and characterization of these objects. However the empirical geometrical observation and characterization do not by themselves serve as a physical explanation. It seems obvious that to understand the origin of self-similar structures, we must understand the nature of the dynamical processes that created them: Temporal and spatial properties must necessarily be completely interwoven.

This is particularly true for earthquakes, which have a variety of fractal aspects, as discussed in this volume. The distribution of energy released during earthquakes is given by the Gutenberg–Richter (1956) power law. The distribution of epicenters appears to be fractal with dimension $D \approx 1$–1.3 (Kagan and Knopoff, 1980). The number of after shocks decay as a function of time according to the Omori (1894) power law. There have been several attempts to explain the Gutenberg–Richter law by starting from a fractal distribution of faults or stresses (Kagan and Knopoff, 1987; Huang and Turcotte, 1990; Turcotte, 1989). But this is a hen-and-egg approach: To explain the Gutenberg–Richter law, we assume the existence of another power-law—the fractal distribution.

The Gutenberg–Richter law extends over several orders of magnitude. For instance Johnson and Nava (1985) present data on the New Madrid seismic zone indicating a power law over almost 5 decades. The upper limit is probably due to the fact that measurements were necessarily limited to a period of 167 years, from 1816–1983. Since a human lifetime

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cannot play an essential role for earthquakes, there is no reason to believe that the distribution cannot be extended beyond earthquakes of size $m = 7$ to earthquakes of size 8, 9, and 10, etc., if a geological time period were available for the measurements.

The observation of power laws is of tremendous importance in physics, since it indicates the existence of an underlying scale-invariant mechanism. The Gutenberg–Richter law indicates that the mechanism of small earthquakes is essentially the same as the mechanism for large earthquakes, since otherwise their relative frequency cannot be expected to obey a simple law. Actually the quality of data for earthquakes is excellent compared with other areas of physics, where usually not more than 3 decades are available: Scaling over 8 decades is unheard of. We argue that this is due to the fact that the upper length and time scales for Earth dynamics are much larger than for any system set up by humans.

Recently it has been recognized that many interacting dynamical systems naturally evolve into a self-organized critical state, with avalanches of all sizes—large and small (Bak and others, 1987, 1988; Tang and Bak, 1988; Bak and Chen, 1989). The discovery suggests a rather general dynamical mechanism for the emergence of scaling behavior (including fractal structure) in nature. Shortly after the discovery, it became clear that the simplest and most direct application of this idea might be to earthquakes: The Gutenberg–Richter law, the fractal spatial distribution of epicenters, and other power laws in earthquakes are all manifestations that the crust of the earth operates at a self-organized critical state. Indeed several authors (Bak and others, 1988; Bak and Tang, 1989; Ito and Matsuzaki, 1990; Sornette and Sornette, 1989; Carlson and Langer, 1989) have taken up the idea and presented supporting theoretical evidence, although the Carlson–Langer model fails to reproduce the scaling observed for large earthquakes.

The concept of self-organized criticality is most easily visualized in terms of the prototypical example: a pile of sand. Consider a situation where the pile is built by slowly and uniformly by adding sand, one grain at the time, to a large flat surface with edges where the sand slides off. In the beginning, the sand remains close to the position where it lands. After a while, the pile achieves a slope, and now and then there are small avalanches when the slope somewhere becomes too steep. Avalanches can be thought of as generated by a chain reaction or branching process. Following the initial instability, the falling particle may either stop falling, continue falling, or induce two or more falling particles. Later each falling particle may again stop, continue falling, or induce more falling particles, and so on. The total number of falling particles during this process is a measure of the size of the avalanche. As the process of adding sand continues, the pile becomes steeper and steeper, and larger and larger avalanches appear. Eventually the pile reaches a statistically stationary state where the amount of sand added in average is balanced by the amount of sand falling off the edges, and the growth of the slope stops. The chain reaction for avalanches becomes critical, and avalanches of all sizes occur; this is the self-organized critical state. The frequency of avalanches of different sizes follows a power law distribution similar to the Gutenberg–Richter law (Bak and others, 1987, 1988; Bak and Chen, 1990).

The basic principle of self-organized criticality is that large interactive dynamic systems naturally organize themselves into a state that is perpetually critical. Dynamic forces inevitably carry the system to the critical state without fine tuning external forces. In contrast to critical, for chaotic few-degrees-of-freedom systems (which can be described by a few variables), the self-organized critical state is robust with respect to any change in local microscopic mechanisms for the system. For example, in terms of the sandpile

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picture, if we try to prevent avalanches by building snow screens, then for a while there will be fewer and smaller avalanches. But eventually the slope adjusts to the new situation, and the critical state is resumed: The critical state is a global *attractor* of the dynamics. This resiliency is important for representing real dynamics in nature. The scaling laws of this critical state are properties of whole systems with many degrees of freedom, and they cannot be deduced by studying local properties. It makes no sense to try to explain large events with a detailed microscopic-engineering approach.

The characterization of the Earth's crust as a system operating at the self-organized critical state is in complete contrast to the view that the crust is a low-dimensional chaotic system. In fact as we demonstrate later, the self-organized critical state is not chaotic at all but operates perpetually at the border of chaos.

We present results of a simple stick slip model of earthquakes, which evolves to a self-organized critical state. Our emphasis is on demonstrating that empirical power laws for earthquakes indicate that the Earth's crust is at the critical state, with no typical time, space, or energy scale. Of course the model is tremendously oversimplified; however in analogy with equilibrium phenomena we do not expect criticality to depend on details of the model (universality).

11.2. MODELS AND SIMULATIONS

In 1956 Gutenberg and Richter observed that the number Q of earthquakes of magnitude greater than m is given by the relation

$$\log_{10} Q = c - bm \tag{1}$$

where b is a universal constant with a value approximately unity, $0.8 < b < 1.2$. The researchers also estimated that the energy E released during an earthquake increases exponentially with m

$$\log_{10} E = c' + dm \tag{2}$$

where d is not known very accurately but generally assumed to be in the range of $1.5 < d < 2.5$. Combining those two relations, we realize that the Gutenberg–Richter law is essentially a power law for the distribution of energy release

$$N(E) = \frac{dQ}{dE} \propto E^{-1-b/d} \equiv E^{-1-\beta} \qquad 0.4 < \beta < 0.6 \tag{3}$$

However despite the universality of the relation, there has been no explanation of this power law behavior. Note that most of the uncertainty lies in relating m to E; there is little doubt that we are indeed dealing with a power law. Kagan (1990) finds the exponent β to lie between 0.5–0.6 from analyzing the Harvard earthquake catalog for large earthquakes.

It is generally assumed that earthquake dynamics are due to a stick-slip mechanism involving the Earth's crust sliding along faults (Burridge and Knopoff, 1967; Otsuka, 1972; Stuart and Mavko, 1979; Sieh, 1978; Mikumo and Miyatake, 1978, 1979; Choi and Huberman, 1984). When a slip occurs at some location, the strain energy at that position is released, and the stress propagates to the near environment. While this picture is rather well-established, no connection between stick-slip models and the actual spatial and temporal correlations has been demonstrated.

The situation that we want to describe is shown in Fig. 11.1a. Two segments of material, representing tectonic plates, are slowly pressed against each other, causing them to slip along their interface. A scaled-down laboratory experiment has actually been performed by Bobrov and Lebedkin (1989), who used aluminum and niobium rods. A fault region was generated as pressure increased, causing a transition from elastic flow (where the rod returns to its original shape once pressure is released) to ductile flow (where compression is irreversible). The researchers indeed observed earthquakes along the fault with a power law distribution independent of the slip material and mechanism (believed to be different for the two materials). In the present context, blocks are tectonic plates grinding against each other along a fault or a fault system. Now and then, parts of the plates slip relative to each other; these slips are ruptures of the crust in earthquakes.

Figure 11.1b shows a one-dimensional model of a single fault. For simplicity one plate is assumed to be rigid and the other to be an elastic medium represented by an array of blocks at positions x_1, x_2, x_3 . . . connected by springs. The blocks interact with the rigid plate by means of static and dynamic friction forces. We assume that the array is open at one end and extends infinitely in the other direction. Whenever the spring force on a particular block exceeds the critical static friction force, it slides until interaction forces have been reduced below the critical dynamical friction. In the aluminum rod experiment, the process may be dislocation motion caused by an atomic bond shifting. During this process, potential energy is first converted into kinetic energy, then dissipated (radiated) when the blocks are decelerated by the frictional forces.

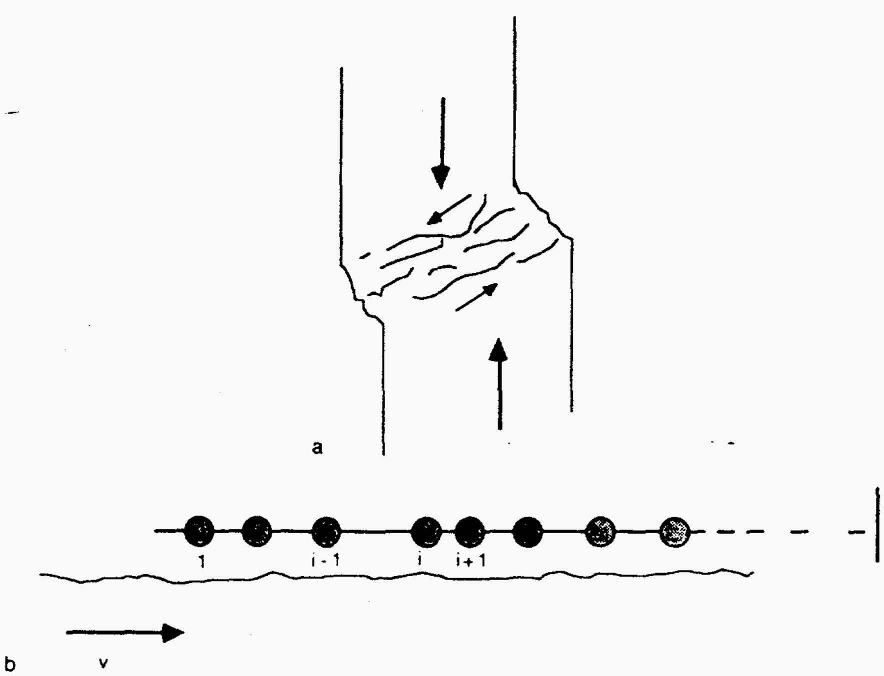

FIGURE 11.1. (a) Blocks slipping along fault system when subjected to stress. (b) One-dimensional illustration of our model. The block-spring chain is pushed along a rough surface with a low velocity v.

Of course since the blocks are at rest between slips, the total force on each block is zero; thus spring forces exactly balance friction forces. When a block slides, the friction force on the block is reduced and so is the spring force on that block. There must be exact conservation of friction forces (or equivalently spring forces) at the individual sliding event since the blocks are at rest both before and after the event. For simplicity we assume that the force is redistributed evenly among nearest neighbors. Note that while forces are conserved, the density of blocks is not: There can be wide fluctuations of the local density of blocks. This distinguishes the model from leaf-spring models (Burridge and Knopoff, 1967) where the average distance between blocks is fixed by leaf springs hooked to a rigid rod. We believe that this model is not physical and introduces a characteristic length into the model. This length leads to deviations from the Gutenberg–Richter law (dominating characteristic large events) not found in nature (Carlson and Langer, 1989). We must explicitly include the perpendicular to the fault system.

The model is driven by slowly pushing the rigid surface relative to the other surface. The time scale set by the pushing is a geological one, and it can be viewed as infinitely large compared with a realistic observation time, so that there is no typical time scale. This is essential for generating power laws and fractal scaling for spatial and temporal correlation functions.

Let us monitor the friction force z_i (which equals minus the spring force) on the ith block. Initially a random distribution of subcritical forces is chosen. The z_is grow at a small rate p until somewhere z reaches the critical value, and a slip event takes place. Without loss of generality, the critical friction force is chosen to be an integer Z_{cr}, and the reduction of friction force is taken to be two units, so

$$z_i \rightarrow z_i - 2 \qquad z_{i\pm1} \rightarrow z_{i\pm1} + 1 \qquad \text{when } z_i > Z_{cr} \tag{4}$$

Bak and Tang (1989) present a model driven by letting $z_i \rightarrow z_i + 1$ at random positions. This has the advantage that all operations are integers; i.e., the model is a random cellular automaton. In contrast the present model is completely deterministic, with all randomness entering through the initial condition. The model appears to be more physical, since no external random forces are needed. Nevertheless the results, including critical exponents, fractal dimensions, etc., remain the same.

The process initiated by the event in Eq. (4) transfers force to neighbors, allowing for a chain reaction. This chain reaction is the earthquake. As the process continues, the forces z_i generally increase, causing larger and larger earthquakes. Eventually the system is pumped up to a minimally stable state where all forces are near the critical value; that is $\mathrm{Int}(z_i) = Z_{cr} - 1$ for all i at this state. The next instability is propagated throughout the system until the excess force is released at the boundary and the system is back to a minimally stable state. Thus statistically stationary state has been reached. We assume that the Earth's crust has had sufficient time to reach a stationary state, so we are generally concerned with this state only.

The dynamics of this one-dimensional model is rather trivial, and the preceding discussion should be viewed as a pedagogical exercise only. To achieve nontrivial critical behavior, it is sufficient to generalize the model to include next-nearest neighbor interactions (Kadanoff and others, 1989). Here we generalize the model to two and three dimensions, keeping in mind that the Earth's crust, and particularly the fault region, is a higher dimensional system. The generalization is rather trivial: Blocks are situated on a d-dimensional lattice, and each block is connected with its $2d$ nearest neighbors. Now z

that the model is not in the same universality class as earthquakes. What is important is that there are power laws indicating that the Earth's crust is at a critical state, with no typical time, space, and energy scale.

11.3. DISCUSSION AND CONCLUSIONS

As discussed in the introduction, the critical state is robust with respect to randomness, etc. It is trivial to see that a random distribution of critical forces has no effect, since the model can be transformed into the uniform one by simply shifting the variable z (a gauge transformation). We have also studied models where a fraction of the springs were randomly removed (Bak and others, 1988) and a critical state with the same critical exponents was reached. If the properties of the system are changed during the simulation (due to some external event), the system returns to the self-organized critical state after a transient period. The critical state is a global *attractor* of the dynamics. This resiliency is important for self-organized criticality to apply to a wide range of natural phenomena.

Systems with few degrees of freedom (like the Feigenbaum map, coupled oscillators, circle maps, etc.; for reviews, see Hao, 1984) may also exhibit critical points with power law correlations. Since these have no spatial degrees of freedom, then can not possibly have fractal power law spatial correlations. However critically requires fine tuning some parameter, and the critical point, separating regular from chaotic states, has no robustness at all. Thus any small perturbation throws the system off the critical point by destroying long-time memory effects. Attempts to explain the complicated behavior of earthquakes as low-dimensional (few degrees of freedom) chaos must be considered fundamentally misguided, since chaos implies exponentially decaying correlations, not power laws. The belief that there may be a connection between low-dimensional chaos and fractals is without mathematical foundation. Our model cannot be reduced to a few degrees of freedom at the critical state. Sooner or later, information from far away affects the dynamics of any given point.

Figure 11.3 compares the number of blocks slipping versus time in a simulation where z increases by $p = 0.00002$ per unit time for a system of the size 50×50. Note the irregular evolution of the individual events. The outcome of a single earthquake is quite unpredictable, since it depends on minor details far removed from the initial point of instability.

Forecasting individual earthquakes in such a system is quite impossible, since accurate global information is needed. How do we characterize this unpredictability? Usually the unpredictability of dynamic systems is characterized by the Lyapunov exponent, which defines the amplification of small differences in the initial condition as the system evolves. A positive Lyapunov exponent indicates chaos. We have simulated systems at the critical stationary state that initially differ by a small force f_i, where f_i is a random number from $-q$–q' and q is a small number of the order 10^{-5}. Figure 11.4 shows the average difference of z per site as a function of time (the Hamming distance). The straight line indicates power law behavior. Hence the Lyapunov exponent is zero, and the system is at the border of chaos. Nevertheless the fact that the power is positive indicates the uncertainty of the state of the system grows, albeit much less dramatically than for chaotic systems. The situation for predicting earthquakes is less desperate than for fully chaotic systems, although there is the added complexity of having to deal with many degrees of freedom. We denote such systems as weakly chaotic. Since many dynamic systems are expected to be self-organized critical, we expect weak chaos to be quite ubiquitous in nature.

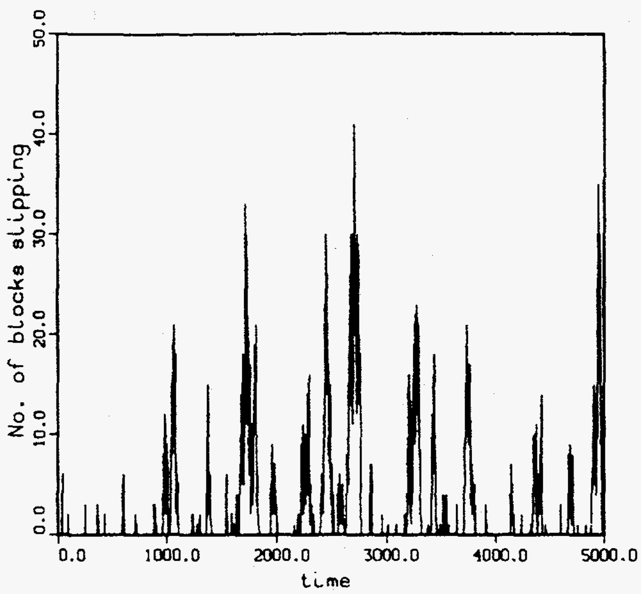

FIGURE 11.3. Evolution of activity, including several earthquakes, for 30 × 30 system driven at a rate of p = 0.0001. The plot shows the number of sliding blocks versus time.

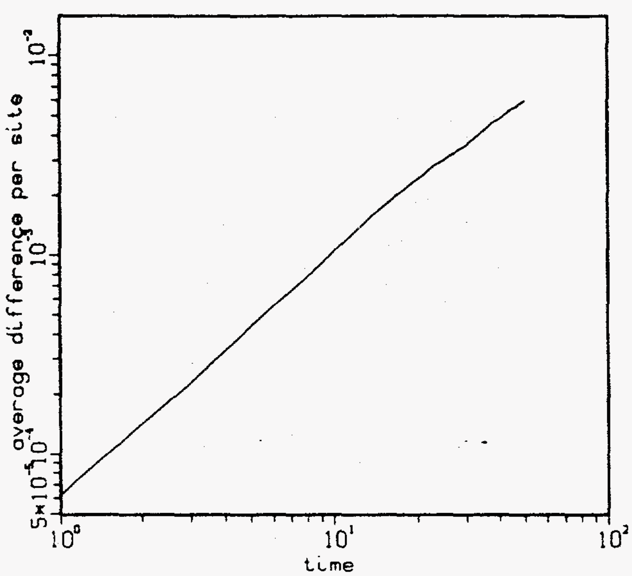

FIGURE 11.4. Power law growth of a small random difference in the initial condition (weak chaos). The plot shows the evolution of the Hamming distance, which is the sum of absolute values of the difference between a system in the critical state and the same system with a small initial, random perturbation versus time.

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Once the existence of the self-organized critical state has been established, it is not so difficult to derive other exponents, such as the fractal dimension, characterizing different correlation functions (Tang and Bak, 1988). In particular Ito and Matsuzaki (1990) have generalized our model by adding a random disturbance to sites just subjected to an earthquake. They obtained a spatial clustering of epicenters with a fractal dimension of 1.1. They were also able to obtain a power law distribution of aftershocks (Omori's law). Sornette and Sornette (1989) have shown the existence of $1/f$ noise in the time gap between large earthquakes. A number of other works applying the principle of self-organized critically to earthquakes have been performed. In collaboration with S. Obukhov, Chen and others (1990) have proposed a crack propagation model of earthquakes, which includes realistic features of a long-range redistribution of elastic forces following local ruptures. The model evolves to a self-organized critical state with exponent β close to the observed one. We also notice that Brown and others (1990) have studied a spring block model of earthquakes similar to the one discussed in that paper. Their study confirms the general picture just presented.

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