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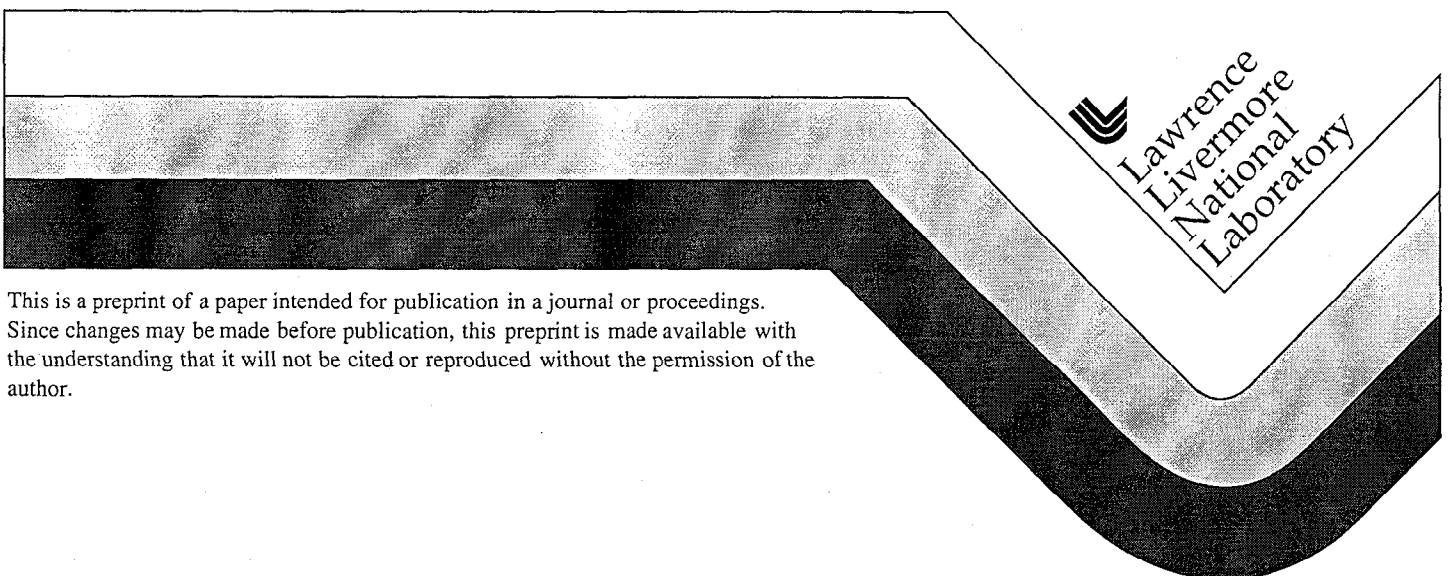
PREPRINT

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# MODAL RESPONSE OF INTERIOR MASS BASED UPON EXTERNAL MEASUREMENTS

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## ABSTRACT

Modal response testing has been used to predict the motion of interior masses of a system in which only external instrumentation is allowed. Testing of this form may occasionally be necessary in validation of a computer model, but also has potential as a tool for validating individual assemblies in a QA process. Examination of the external frequency response and mode shapes can offer insight into interior response. The interpretation of these results is improved through parallel analytical solutions. A simple, three-mass model has been examined experimentally and analytically to demonstrate modal theory. These results show the limitations of the external measurement in predicting internal response due to transmissibility. A procedure for utilizing external testing is described. The question posed through this research is whether or not modal correlation analysis can be adapted for use in systems for which instrumentation of critical components is missing.

## INTRODUCTION

Modern modal correlation techniques link analysis to experiment and greatly increase the overall system understanding through certification of the analytical model. The premise of correlation methods extant, however, is that the system will be well instrumented, at least at critical locations on the system. Thus, it can be envisioned that a class of systems exists in which the well-instrumented test is either impossible or economically unfeasible. For such a system, how can experimentation and analysis be sufficiently mated?

One class of systems satisfying the instrumentation constraint problem is that in which all interior parts are encased in a sealed container. Thus, instrumentation could only be placed upon the exterior surface of the system. Given an adequate correlation procedure, for example, production assemblies could be dynamically tested with a subset of response measurement points and validated

against prior tests or analysis. The correlation would then certify the integrity of the assembly non-destructively.

A simple, linear three-mass model will be examined to explore the availability of modal information at the outermost mass of the system. Experimentation and analysis will demonstrate the applicability of using only external measurement in characterizing system response. A discussion of the extrapolation of these results and of their implications in modal correlation will be given.

## SAMPLE PROBLEM

Equivalent systems of three masses and two compression springs are shown in Figure 1. There is no mathematical distinction between the systems, but it is assumed that the shape of mass 3 prevents measurement of dynamic response on masses 1 and 2 ( $m_1$  &  $m_2$ ) for the system on the left. System response must therefore be inferred from motion of mass 3 ( $m_3$ ). This three-mass system has been tested and analytically modeled using the information in Table 1. Analysis used a lumped-mass model with the SAP2000 structural analysis program. As shown in Table 1, two values were used experimentally for  $m_2$ .

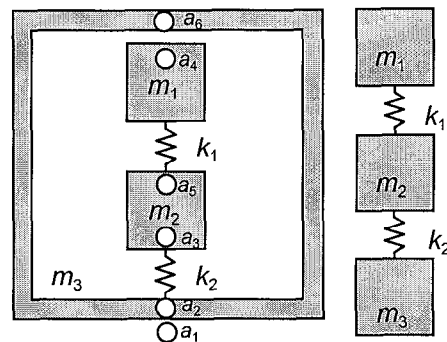


FIGURE 1. Sample three-mass systems demonstrating potential enclosure constraint.

$m_1$	1.84 lb	$k_1$	109,500 lb/in
$m_2$	3.33 lb	$k_2$	66,600 lb/in
	1.55 lb		
$m_3$	3.56 lb		

TABLE 1. Experimental test data.

For testing, the assembly is hard-mounted to a B&K shaker Model 4817/4802, with 1.9g (rms) burst-random excitation. The base plate and shaker head mass are included in  $m_3$  shown above. Accelerometers are mounted with MBond 200 as shown in Figure 1. The accelerometers used are tri-axial Endevco 63B-100 at  $a_3$  and  $a_5$ , and Endevco 63A-500 at  $a_1$ ,  $a_2$ ,  $a_4$ , and  $a_6$ . Response of all channels was used to verify that motion was one-dimensional and that the mass lumps were rigid. The modal peaks functions are shown in Figures 2-3. These figures show the response of all accelerometers and the response from only the external accelerometers ( $a_1$ ,  $a_2$  &  $a_6$ ). As shown, the exterior measurements do show the same form of response as the complete system, but the magnitude of the response is substantially reduced.

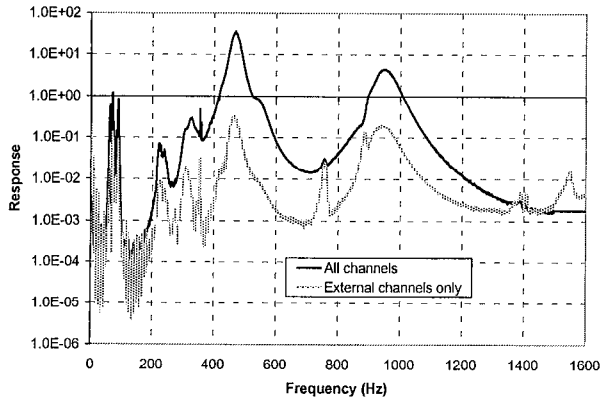


FIGURE 2. Experimental response for  $m_2=3.33$  lb.

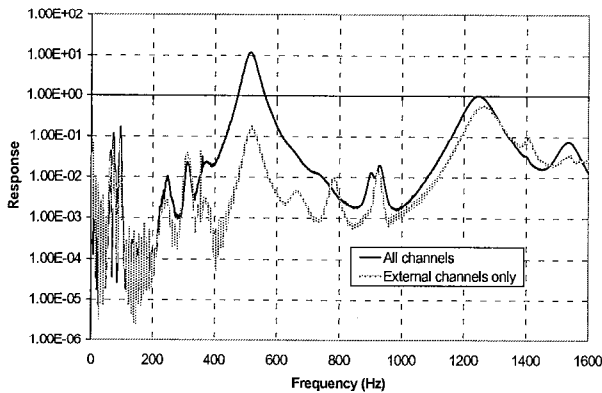


FIGURE 3. Experimental response for  $m_2 = 1.55$  lb.

Results of the SAP2000 analysis are shown in Figures 4-6. The pseudo-spectral response at  $m_1$ ,  $m_2$ , and  $m_3$  are shown for multiple mass values of  $m_2$ . These results compare very well with the experimental analysis in both frequency

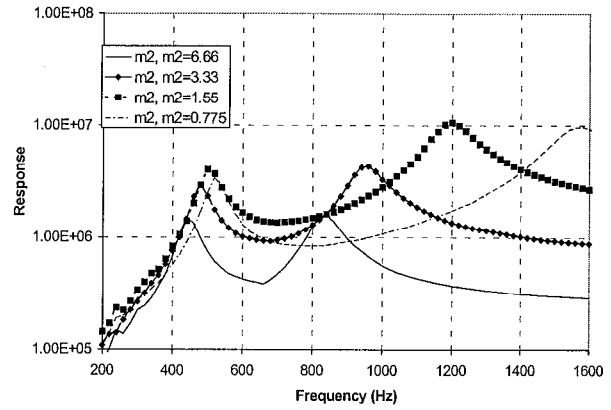


FIGURE 4. Analytical response of interior mass 1 for various values of mass 2.

content and shape of the response curves. For the interior masses both natural frequencies are observable, but at lump 3 the second natural frequency becomes less observable as mass 2 increases. The corresponding mode shapes (from analysis) are shown in Figure 7. As shown, the second mode is the motion of  $m_2$ , with  $m_1$  and  $m_3$  acting as anti-nodes. The low stiffness of  $k_2$  compared to the mass at  $m_2$  causes poor transmissibility of mode 2 motion at  $m_3$ . For a good summary of transmissibility refer to Lin & Ewins [1]. Although the test case is simple, the implications on more complex systems are important. In systems with greater internal complexity, there may exist layers of sufficient mass or inadequate stiffness to mask motion inside that layer. A non-rigid exterior surface may provide better frequency response functions but may obscure interior motion as well.

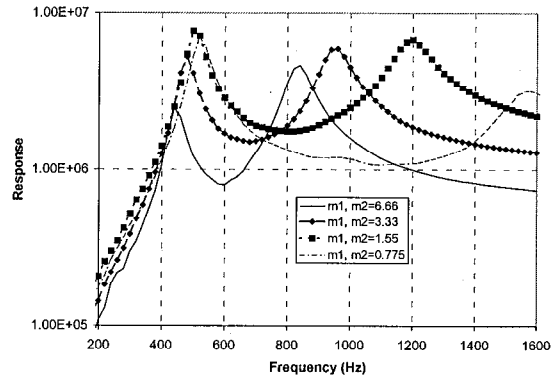


FIGURE 5. Analytical response of interior mass 2 for various values of mass 2.

Transmissibility of interior motion presents the most challenging aspect of using external measurements to characterize the system. In an actual assembly, one could hope that excitation in a different axis or orientation would provide additional information, taking advantage of any system asymmetries. Depending on analysis needs, however, sub-testing of the internal part may be required to ensure accurate modeling. When transmissibility is at fault, the prohibitive boundary mass or spring does provide a good boundary condition for the subsystem analysis.

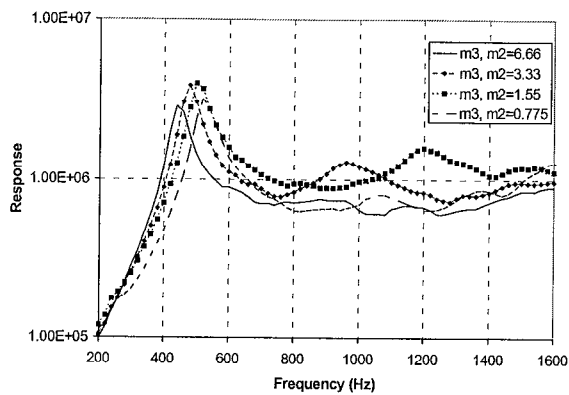


FIGURE 6. Analytical response of external mass 3 for various values of mass 2.

### DEVELOPING A VALID MODEL

The stated goal of this work is to predict internal response using external measurements and analysis. The procedure for accomplishing this would be as follows. First, fully instrument the system as appropriate (considering limitations of equipment and redundancies of measurements). Second, perform the experiment using appropriate suspension system and excitation. The data taken during the experiment would be used for the third step of the process. This involves curve fitting the data to extract natural frequencies, damping values, and mode shapes. Some systems that might be tested may be nearly symmetrical or symmetrical in certain DOF, giving multiple responses in a narrow frequency band. Most commercial software will allow fitting of multiple frequencies in one peak. Upon examining the mode shapes, any internal motion presents itself fundamentally as a rigid body motion of the external surface. For example, in the three mass system presented above, there will be three "rigid" motions of  $m_1$ , the first is the rigid body mode of the free-free system, the next is associated with the first mode and the last with the second or interior response. If lumped masses were assumed for each component, a system of  $n$  masses should show  $n$  rigid body responses per degree of freedom.

An analytical model should be developed concurrently with experimental analysis. For example, the model may be used in suggesting excitation or response measurement points. Problems with transmissibility due to mass to stiffness ratios can be predicted, indicating if subsystem tests will be necessary. In developing the model, mass and inertia properties should be precise, with stiffness and damping approximated as possible. Comparisons of natural frequency and transfer functions may lead to adjustments in stiffness and damping of the analytic model. Validation of the analytical model is made through mode shape comparison. Exterior instrumentation, when compared to the analytical mode shape, will reveal a great deal of information regarding interior motion. In this way, the analytical model prejudices (and defines) interpretation of the experimentation. Analytical frequency response functions are plotted for the representative measurement locations (as in the TAM). For matching motion in the mode

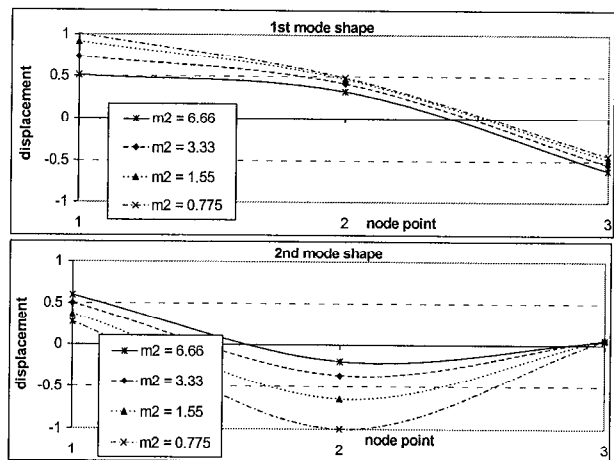


FIGURE 7. Analytical mode shapes for various values of mass 2.

shapes, the frequency and receptance of the mode can then be compared between analytical and experimental systems. In summary, when mode shape and frequency agree well between exterior measurements and analysis, the analytical model is viewed as valid.

Multiple levels of the above process may occur if sub-system testing is used to obtain information lost through transmissibility. Following tests on simple sub-systems, complexity may be added, but care must be taken in performing the test and in reporting the results. This is where extensive interaction between the experimentalist and the analyst is required. This interaction will help refine the model and identify incorrect assumptions. The currently missing final step is correlation of analytical and experimental software. Published methods of correlating the analytical and experimental results rely on a sufficiently populated Test-Analysis Model (TAM) [2-5]. For example, one of the first steps in establishing the TAM is the reduction of the analytical mass matrix. If only exterior measurement points are allowed, the mass of the entire system must be reduced to the external points, which is intuitively incorrect. Thus, it is unclear how correlation can be best applied to the described system.

### VALIDATING PHYSICAL SYSTEMS

The most practical application of external measurements is in the manufacturing cycle. As a quality assurance tool, the production unit could be instrumented at pre-determined locations and excited to measure system response. This response could then be compared to baseline values obtained through experimentation or validated computer models. This test assures that internal components are correctly assembled.

To test production units, the system must first be fully tested in either traditional ways or as described above. A validated computer model is useful in determining the optimal location of the external measurement points of the production unit. To speed production, a minimum number of excitation and measurement points are desired. Frequency and damping are compared, or a correlation routine could be used to compare each production unit with predefined accept/reject

criteria. Techniques such as CoMAC and CORTHOOG [5], if applicable, could also be useful in identifying the cause of rejection, such as identifying a missing part in a partial assembly. This would provide a powerful QA tool.

#### **SUMMARY**

An attempt has been made to use frequency response measured on the exterior surface of a system to identify internal response. This technique is insufficient when transmissibility prevents interior response from reaching the surface measurement points. However, in many cases sufficient information will reach the surface instruments to validate analytical models or to prove the system integrity. Development or appropriate application of available correlation tools would improve the effectiveness of the process.

#### **ACKNOWLEDGEMENTS**

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