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Feasibility of High Yield/High Gain NIF capsules

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Our original ignition "point designs" (circa 1992) for the National Ignition Facility (NIF) were made energetically conservative to provide margin for uncertainties in laser absorption, x-ray conversion efficiency and hohlraum-capsule coupling. Since that time, extensive experiments on Nova and Omega and their related analysis indicate that NIF coupling efficiency may be almost "as good as we could hope for". Given close agreement between experiment and theory/modeling, we can credibly explore target enhancements which couple more of NIF's energy to an ignition capsule. We find that 3-4X increases in absorbed capsule energy appear possible, providing a potentially more robust target and ~10X increase in capsule yield.

Introduction

We can relate the quantity of x-rays absorbed by a NIF [1] indirect drive ignition capsule, E_{cap} , to NIF laser energy, E_{NIF} , via $E_{cap} = \eta_{abs}\eta_{CE}$ $\eta_{HR-cap}E_{NIF}$ where η_{abs} is the fraction of incident laser energy absorbed by the hohlraum, η_{CE} is the conversion efficiency of laser light into x-rays and η_{HR-cap} is the fraction of generated x-rays which are actually absorbed by the capsule. Our original ignition "point designs" [2,3] (circa 1992) were made energetically conservative to provide margin for uncertainties in η_{abs} , η_{CE} , η_{HR-cap} , and E_{NIF} . For NIF's nominal 1.8MJ the 150kJ point design requires $\eta_{abs}\eta_{CE}$ $\eta_{HR-cap} = 8.3\%$. Because the point design's theoretical η_{CE} η_{HR-cap} is 11%, we see that "margin" was built into the program. Since that time an extremely large collection of Nova and Omega experiments and their related analysis indicate that NIF coupling efficiency may be almost "as good as we could hope for". Experiments [4] studying stimulated brillouin and raman backscattering in NIF "plasma emulators" imply that η_{abs} should be >90%. Complementary work examining hohlraum radiation environment shows x-ray production and capsule coupling to be very close to modeling. Quantitatively, we find that η_{CE} η_{HR-cap} will be ~1.04+-0.12 of the coupling predicted by our simulations [5,6].

Given coupling close to simulations, can credibly explore targets which couple more of NIF's energy to an ignition capsule. In this paper we discuss designs in which we greatly increase absorbed capsule energy by a combination of modest improvements. These include: extracting more energy from NIF using a longer, lower power pulse shape matched to a 250eV design; reducing hohlraum wall losses with "cocktails"; allowing the LEH to "close" during the pulse to reduce hole losses; slightly increasing capsule:hohlraum-size; an overall increase in time scale further increasing the radiation fraction absorbed by the capsule. The net result is the prospect of designs which absorb as much as 600kJ to, possibly, 1MJ of x-rays and produce yields in excess of 100MJ.

Improving hohlraum coupling efficiency

To understand the improvements that can be made to ignition hohlraum coupling efficiency, consider as a case-study a target based on a 600kJ variant of a 250eV target with a Beryllium ablator [2] as shown in figure 1. This capsule has an outer radius of 1.77mm and produces 70-120MJ of yield, depending on the detailed drive profile and the amount of DT fuel assumed. We can drive this target with a continuous radiation temperature vs time as shown in figure 2. The hohlraum size and, therefore, the wall area for this target depends on our choice of "case-to-capsule ratio", $R_{\rm CC}=(A_{\rm hohl}/A_{\rm cap})^{0.5}$. Virtually all the NIF point design work, to date, has been done at $R_{\rm CC}=3.65$. This case:capsule ratio would place the capsule in a hohlraum 8.8mm diameter and ~13.3mm long (this is approximately a 5.55X scale-up of a standard Nova hohlraum, or "scale 5.55"). Standard NIF design practice calls for the laser entrance holes to have a diameter of 50% of the hohlraum diameter.

The energy that a capsule absorbs is just one part of the target's overall energy balance. For a given amount of total x-ray production in the hohlraum, = $\eta_{CE}(\eta_{abs}E_{NIF})$ we can write

$$\eta_{CE}(\eta_{abs}E_{NIF}) = E_{WALL} + E_{LEH} + E_{CAP}$$
 (2)

where E_{WALL} is the the x-ray energy absorbed by the high-Z walls of the hohlraum (a diffusive, radiative heat flow) and E_{LEH} is the radiation losses through the laser entrance hole (LEH). Had we used equation 2 to examine a target containing the 600kJ capsule in the early 90's, when we were first exploring NIF possibilities, we would have concluded the capsule/hohlraum combination requires too much energy. At that time we would have assumed pure gold walls, a scale 5.55 hohlraum and 50% laser entrance holes. The energy budget for this target, case A in table 1, shows that it requires 3.3MJ of x-rays. In the early '90's, when there was considerable uncertainty about hohlraum physics, we hoped that hohlraum x-ray conversion efficiency might be as high as 70%. Using that value, we would have concluded that this target would require ~4.7MJ of laser energy; well beyond our expectations for NIF.

However there are several improvements which can be made to the hohlraum coupling. Wall losses/unit area can be significantly reduced by using hohlraums made of material mixtures. The basic idea is simple; single materials have opacity which is quite high in some parts of the x-ray spectrum but low elsewhere in the spectrum. Radiation will preferrentially flow through these opacity "holes". However, by making the walls from mixtures of complementary materials these opacity holes can be filled in [7, 8]. For example, experiments on Nova showed that ~240eV radiation will flow through a mixture of gold and gadolinium more slowly than through pure gold [7]. The increase in Rosseland opacity inferred from the measurements is close to what was expected from theory.

For ignition pulse-shapes which span a very large range in temperature, very significant decreases in wall losses can be achieved by using mixtures of several materials. For example, figure 3 shows wall loss vs time for three different wall materials exposed to the Tr vs. time of figure 3. These wall estimated wall losses were calculated with the Lasnex [9] code using an average atom [10] atomic physics model. The losses plotted in figure 3 correspond to the area of a scale 5.55 Nova hohlraum made out of the indicated materials. Mixtures can very significantly reduce losses throughout the pulse, including the foot of the pulse. Table 2 lists a variety of cocktail mixtures we have explored and, in the second column, our estimated wall losses for the 600kJ case-study capsule in a hohlraum with $R_{\rm CC}{=}3.65$. The third column shows the ratio of a given mixture's estimated wall loss to that of gold.

Equation 2 also shows that we can also increase E_{CAP} by decreasing the energy lost through the laser entrance hole. In a hohlraum of fixed case:capsule ratio that means that we must decrease the laser entrance hole diameter from its standard value of 50% of the hohlraum diameter. There are at least two techniques for accomplishing this. One is to simply make the holes smaller. The other is to allow the holes to partially close as high-Z blow-off moves inward from the rim of the LEH. Our current work utilizes the latter technique. In our 2D Lasnex simulations of ignition hohlraums we find that the simulated laser entrance holes partially close if we do not coat them with a low-Z layer (as was used in previous work [2], to prevent hole closure). The x-radiation losses through the LEH of our all our integrated simulations of ignition hohlraums are consistently 50-60% of the losses we would expect from $\sigma T_R(t)^4 A_{\rm geometric}$, where $T_R(t)^4$ is the radiation flux that is imploding the capsule, $A_{\rm geometric}$, is the initial area of the LEH and σ is the Stephan-Boltzman constant. This "automatic" decrease in the fractional LEH loss, $E_{\rm LEH}/E_{\rm cap}$, reduces our case-study's x-ray requirement by 350kJ

The potential benefits of reducing the specific wall losses via cocktails and allowing the laser entrance hole to close to 60% of its geometric area are summarized in table 1 as Case B. We see that these two changes reduce the x-ray energy requirement to ~2.35MJ. Additionally, we can achieve further savings of x-ray energy by shrinking the hohlraum size while keeping the capsule fixed; i.e. reduce R_{CC} . Case C in table 2 is for a hohlraum where we decreased R_{CC} to 3.28 (=90% of the conventional 3.65 value). The total x-ray requirements drop to ~2MJ.

We convert hohlraum x-ray energy requirements to laser energy requirements by dividing by the average x-ray conversion efficiency. We mentioned above that in the early 90's we had hoped that the hohlraum x-ray conversion efficiency would be as high as 70%. Since then a broad range of experiments and the associated modeling has shown that hohlraum x-ray conversion efficiency can, in fact, be as high as 85% in Nova hohlraums [11]. In the 1-D and 2-D simulations of the ignition hohlraums described here, we find effective conversion efficiencies of approximately 90%. As described in reference [11], such high conversion efficiencies are a result of the confined nature of the system; plasma blow-off energy that would be lost in open geometry remains in the hohlraum where it can "find" its way into becoming radiation. Using 90% conversion efficiency, the estimated laser requirements for cases B and C of table 1 are 2.6 and 2.2MJ, respectively. This, as we shall see, puts such a target within NIF's design performance envelope.

Integrated design analysis

In addition to x-ray and laser energy estimates, as summarized by table 1, our analysis of the hohlraums's x-ray budget also produces x-ray power requirements which we readily convert to laser power requirements using estimated time dependent conversion efficiency. We validate and refine these laser power estimates with 1-D and 2-D Lasnex integrated simulations [2,3,12] which include detailed hohlraum specifications, wall materials, capsule and laser irradiation. Figure 4 shows a laser power which successfully implodes our 600kJ case-study capsule in a scale 5.0 hohlraum ($R_{\rm CC}$ =3.28) made of cocktail materials such as the ones listed in table 2. It has a total energy of 2.25MJ. The yield from our 2-D simulations of this target, which include the affect of time dependent 2D asymmetries and non-Planckian spectra, is 65-70MJ; comparable to the 75-80MJ found for this capsule in 1D simulations using the planckian drive of figure 2. Although these design simulations at $R_{\rm CC}$ =3.28 do show a somewhat greater tendency for an axial jet of fuel to develop at late time than is typically found at the more standard $R_{\rm CC}$ =3.65, the simulated capsules consistently ignite and burns to high yield over a range of tunings .

Besides the 600kJ capsule used for our case-study, we have examined scaled versions of this capsule which absorb between 265 and 1000kJ of x-rays for R_{CC} ranging between 3.65 and 2.98. Our analysis includes validating the estimated laser power with 1-D and 2-D integrated simulations (here the 2D simulations are done with the capsule flux numerically forced to be uniform. This allows us to rapidly assess the energetics of an extensive range of hohlraums without also needing to simultaneously control symmetry.). Figure 5 summarizes the hohlraum coupling efficiency, $\eta_{CE}\eta_{HR\text{-}cap}$ of equation 1, for this survey. At the standard case:capsule ratio, cocktails, slightly reduced LEH's together with longer pulse lengths, combine to produce coupling efficiencies ~20-22% vs. the ~11% of the original point design. If we can successfully reduce the case to capsule ratio without introducing unacceptable asymmety, then couplings ~26-28% are plausible at R_{CC} =3.28 and ~30-33% at R_{CC} =2.98 .

Increasing E_{NIF}

NIF is a glass laser which is capable of producing up to ~4.8MJ (4MJ) of 1μ m (infrared) wavelength laser light when it is completed with 7 (5) slabs of glass in the final booster amplifiers. (The number of slabs that will ultimately be installed is under discussion. The amplifiers being built will accommodate seven). This 1μ m light is converted to the $1/3\mu$ m (blue) light used to irradiate hohlraums in the final optics assembly (FOA) where it is also focussed and aimed onto the target. Two fundamental questions which must be answered in order to assess NIF's capability to produce any given pulse-shape are:

- I- Is there enough 1μm light to create the needed blue pulse-shape?
- II- If so, how much "damage" will the blue light cause in the FOA's?

The answer to the first question depends not only on the intensity dependent conversion efficiency of the KDP crystals (which convert the infrared light to blue light) but also on the operational strategy that we use to produce a given pulse-shape at the target. In the case of the 2.25MJ pulse shape of figure 4, if we elect to generate it by simply running an appropriately shaped, continuous $1\mu m$ pulse-shape through the KDP crystals, then we find that we need 4.5MJ of $1\mu m$ laser light. This is well within the energetics capability of NIF with seven booster slabs

but not with five. However there are operational strategies for significantly reducing the 1µm requirements. These strategies are all based on the "picket fence" approach [13] which replaces a continuous pulse with a train of short, high power pulses which convert to blue light much more efficiently in the KDP crystals during the low power, early time "foot" of the pulse. Now there is a concern, based on simulations, that hohlraums irradiated by widely spaced pickets will have symmetry problems related to cooling of the hohlraum's bulk-plasma between pickets. However it is possible to take advantage of NIF's architecture to produce temporally skewed pickets which convert well in the KDP crystals but provide a continuous pulse after being focussed onto the target [14]. Moreover, by taking advantage of NIF's architecture in which a "quad" (a 2x2 array of four beams) can be treated as a single beam which irradiates the hohlraum, it is possible to interleave four relatively short pulses from each of the four beams to form a continuous pulse. Using techniques such as this, we can envision average 1/3µm conversion efficiencies as high as 70%, as measured at the target. For the pulse shape of figure 4, this "ultra-fast picket" technique could lower the 1µm energy requirement to ~3.5MJ. Indeed, such advanced conversion schemes allow us to contemplate even larger capsules. Table 3 summarizes 1/3µm and 1µm energy requirements for several targets. Using advanced conversion schemes and reduce case:capsule ratios, we can consider driving capsules which absorb as much as 1MJ of x-rays.

The second fundamental question about a given pulse shape is how much "damage" will it cause in the FOA? A basic problem is that surface imperfections will slightly absorb blue light causing local heating. Too much heating produces local damage. The figure of merit for this process, known as the "damage integral", increases with fluence (j/cm²) but decreases with pulse-length as $1/t^{0.5}$, since heat can diffuse away from the absorbing imperfections. NIF's specification for damage integral is $8j/cm^2$, 3ns gaussian equivalent. This means that a 3ns gaussian pulse of $8j/cm^2$ passing through the FOA would be acceptable. Likewise, the $t^{0.5}$ scaling means a 12ns pulse of $16j/cm^2$ could also be acceptable. For an arbitrary pulse shape [15]

Damage integral =
$$1.1 \times \int_{0}^{t} \frac{I(s)}{\sqrt{t-s}} ds$$

where I is the blue light intensity in units of GW/cm², t and s are in ns. The final column in table 3 lists the damage integral values for several higher absorbed energy designs. All are within NIF's 8j/cm² 3ns gaussian equivalent damage specification.

Discussion

The 600kj capsule driven at 250eV that we used as a case-study is part of a larger study exploring the limits of capsule coupling energy. This work indicates that NIF may be able to drive some surprisingly energetic targets. Figure 6 is an "engineering plot" which summarizes our findings at 250eV. It relates capsule absorbed energy to laser performance. The solid lines represent the three case:capsule ratios we studied; $R_{\rm CC}$ =3.65, 3.28 and 2.98. The broken line at the upper right shows where we would run out of 1 μ m energy using an advanced pulse shaping technique such as the ultra-fast pickets described above. At $R_{\rm CC}$ =3.65, we may be able to implode a capsule which absorbs 600kJ before exceeding NIF's 8J/cm² blue light fluence specification. If we can successfully implode capsules in hohlraums with reduced case:capsule ratio, then absorbed energies approaching 800kJ to 1MJ are possible. The star on the plot indicates one 220eV target we investigated. It is based on a 1000kJ absorbed energy capsule which produced 380MJ of energy [16].

Although we allow the laser entrance hole to close to $\sim\!80\%$ of its initial diameter, it is important to realize that these hohlraums are bigger than the typical point design hohlraum [2,3] (a scale 3.45 hohlraum). Consequently, the laser entrance holes are bigger, even after the partial closure. This, coupled with a lower peak power ($\sim\!300\text{-}350\text{TW}$ vs. 450TW) leads to the prospect of relatively low intensities in the LEH; $\sim\!2\text{-}4\times10^{14}\text{w/cm}^2$ may be possible.

Finally, figure 7 is a plot of yield vs. capsule absorbed energy which demonstrates some of the benefits of increased absorption. We see that a factor of two to four increase in absorption over

the original 150-200kJ moves us much further from the "cliff" where the penalty for small errors in understanding can be very large. These increases in absorbed energy can also very significantly increase the capsule yield.

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	case A	case B	case C
E _{WALL} (MJ)	1.8	1.2	0.95
E _{LEH} (MJ)	0.9	0.55	0.4
E _{CAP} (MJ)	0.6	0.6	0.6
total x-rays (MJ)	3.3	2.35	1.95
C.E.	0.7	0.9	0.9
Laser energy (MJ)	4.7	2.6	2.2

Table 1- Energy budget for three different assumptions of hohlraum wall material, laser entrance hole and conversion efficiency. Capsule absorbed energy remains fixed.

material	wall loss (kJ)	wall loss/Au
Au	1850	1.00
Au:Gd	1540	0.83
U:At:W:Gd:La	1200	0.65
U:Bi:W:Gd:La	1200	0.65
U:Bi:Ta:Dy:Nd	1170	0.63
Th:Bi:Ta:Sm:Cs	1250	0.68
U:Pb:Ta:Dy:Nd	1170	0.63
U:Ta:Dy:Nd	1240	0.67
U:Au:Ta:Dy:Nd	1190	0.64
U:Au:Ta:Dy:Nd	1220	0.66
U:Nb.14:Au:Ta:Dy	1230	0.66
-		

Table 2- A variety of mixtures of materials can reduce x-ray wall losses to ~2/3 that of pure Au.

target	1/3µm energy	1µm energy	1µm energy	damage integral
E _{CAP} /Yield/case:capsule	(MJ)	CW pulse (MJ)	fast pickets (MJ)	(j/cm2 3ns equiv)
600kJ/70MJ/RCC=3.28	2.25	4.5	3.5	7.2
600kJ/120MJ/RCC=3.65	2.55	5	3.9	7.8
850kJ/150MJ/RCC=3.28	2.65	5.1	4	7.8
1000kJ/380MJ/RCC=3.28	3	6	4.5	8

Table 3- 1μm energy need to drive various targets, assuming two different operational strategies, and 1/3μm damage integral for each target's pulse-shape.

Figure 1- Be ignition capsule designed to operate at $250 \, \text{eV}$. The 50mm thick doped layer next to the DT ice is Be with 2% (atomic fraction) Na and 0.4% Br on the inside of the layer, linearly decreasing to 0.5% Na and 0.1% Br on the outside of the layer. The yield is $\sim 75 \, \text{MJ}$ with the amount of DT ice shown and the pulse shape of figure 3. By increasing the ice thickness and adjusting the pulse shape, yields up to 120 MJ are achieved in simulations.

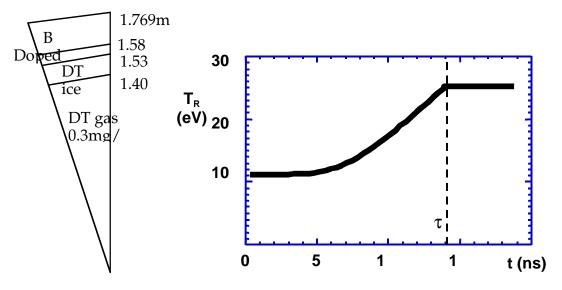


Figure 2- The 250eV Be capsule capsule can be driven with a continuous pulse shape parameterized by a "plateau time", τ . The functional form is $T_F^4 = T_0^4 + (T_F^4 - T_0^4)(t/\tau)^n$ for $t < \tau$. n = 5 typically allows ignition over the widest range of τ .

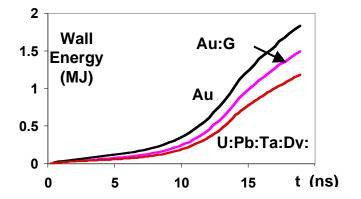
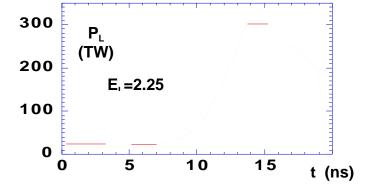


Figure 3- Solid lines: x-ray energy (MJ) absorbed by walls of various materials vs. time when exposed to the temperature vs. time of figure 3. Area of all three corresponds to that of a scale 5.55 hohlraum. Dotted lines: albedo vs. time for gold and for a multi-component cocktail.

Figure 4- Laser power vs. time which drives a 600kJ, 250eV capsule inside a scale 5.0 hohlraum made with cocktail walls.



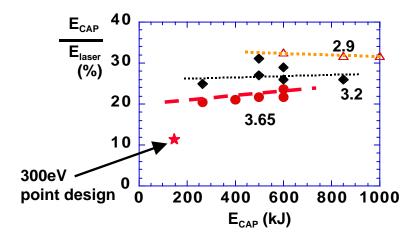
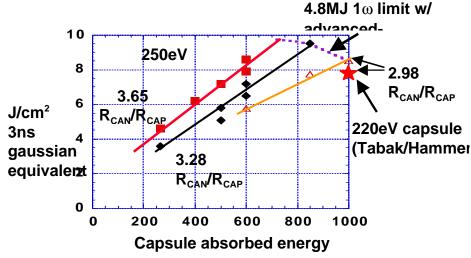


Figure 5-Hohlraum coupling efficiency (_CE _HR-cap) vs capsule absorbed energy for various scales of the 250eV Be capsule. The coupling efficiency ranges between 20 and 33%, depending on the case:capsule ratio.

Figure 6- This plot relates energy absorbed by the 250eV capsule to NIF performance parameters. Solid lines indicate three different case:capsule ratios. Broken line indicates the limit set by NIF's available 1_m energy, assuming seven booster amplifier slabs and advanced conversion schemes. NIF's design specification for 1/3_m light fluence is "8 J/cm2 3ns gaussian equivalent".



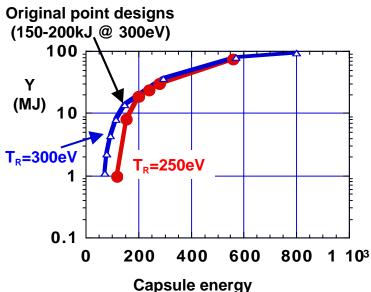


Figure 7- Yield vs absorbed capsule energy from 1-D simulations of a capsule driven at 250eV peak radiation temperature and a capsule driven at 300eV. Significantly increasing the capsule absorbed energy will move us away from the ignition "cliff", thereby providing a more robust target.

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