DPST-86-553

OPTIMAL BLENDING QUALITY

By

S. P. HARRIS
Scientific Computations Division

Publication Date: July, 1986

E. I. du Pont de Nemours & Co.
Savannah River Laboratory
Aiken, SC 29808

PREPARED FOR THE U. S. DEPARTMENT OF ENERGY UNDER CONTRACT DE-AC09-76SR00001

This document was prepared in conjunction with work accomplished under Contract No. DE-AC09-76SR00001 with the U.S. Department of Energy.

DISCLAIMER

OPTIMAL BLENDING QUALITY

Problem Statement

The purpose of blending is to form an optimum quality lot from components with known quality attributes. In this application, the components are of the same material but may vary in quality because of process variability.

The product in the lot must satisfy its specifications which are usually tighter than component specifications. Also, the amount of product in the lot must be between 30 to 40 units while the amount in component containers varies between 2.5 to 10 units. When containers are selected for blending, their entire contents must be used. Multiple quality attributes are determined for each container from laboratory analyses.

Summary

We have developed a functional program for product blending. The program is installed at an SRP production site on their VAX computer. A wide range of blending choices is available. The program can be easily changed or expanded. The technology can be applied at other areas where mixing or blending is done.

Our blending strategy optimizes product quality for productions' special needs. Blends which reduce the amount of product sent to scrap recovery can be formed. At other times, a top quality lot may be needed for the customer to demonstrate the production of a quality product.

A top quality lot may be defined as the blend closest to target or in terms of a trade-off between closeness to target and product uniformity. The program provides choices for either definition.

The blending algorithm is based on using Taguchi quadratic loss functions (Kackar,1985) to develop a quality index from multiple quality attributes. The average component quality index and the quality index, determined from the lot attributes, are then used to calculate a performance statistic. The performance statistic is used to rank potential lots in meeting the blending strategy.

Kackar, R.N., (October 1985). Off-Line Quality Control, Parameter Design, and the Taguchi Method. Journal of Quality Technology.

OPTIMAL BLENDING QUALITY

Introduction

The utility of blending is apparent for complex processes where it is difficult or cost prohibitive to eliminate rework. The blending strategies are not intended to be a substitute for this goal.

As the process improves through statistical process control procedures and use of experimental design methods, the need for blending should diminish.

Strategies

Several product blending strategies are considered. Other strategies may also be appropriate.

* Blend closest to target making optimal use of product near specification boundaries. (Blending from tails)

This strategy loosely defines a trade-off between closeness to target and blend variability. The objective is to arrive at a lot close to target (ideal values) using product at opposite sides of the target. For example, suppose the specification for an attribute is to be between 88 and 96 and the target is 92. We wish to blend product close to 88 with product close to 96 in arriving at a lot near the target 92. Even though blending is easy to conceptualize for one attribute, it becomes a complex problem when considering several attributes. A benefit of the strategy is the product remaining in storage should have lower variability and be closer to target, thus making formation of new lots easier.

** Blend closest to target without regard to product variability. (Center Blending)

How close to target can we come for a blend when considering multiple quality attributes (e. g., weight percentages of certain materials, densities)? The strategy is more general than blending from tails. It can provide a benchmark for comparison with other strategies.

*** Blend with near minimum product variability with consideration of the target. (Uniform Blending)

Sometimes it may be desirable to form a blend with very low variability and also be near the target. For example , with a new product, the lot may be examined very closely by the customer. Therefore, we would want to maximize the probability of finding acceptable quality characteristics for resampling. Consideration of the target is necessary since it is possible to have a blend with low variability near the specification boundaries, even though it may not be very likely.

OPTIMAL BLENDING QUALITY

Background Information

Product is stored in 2-liter containers in a vault with maximum storage of 36 containers. Each container will be filled with varying amounts of product (2.5 to 10.0 units).

The objective is to form a blend having 30 to 40 units while meeting certain quality objectives.

The plant lab will produce estimates of four quality attributes from samples taken from each container. The data are stored in the Accountability System on their VAX computer. Each data record will contain a container/product identification number, the amount of product and the laboratory quality attribute values.

Only containers meeting specifications are used in the blending routine. The container and blend specifications are given in Table 1.

Table 1

PRODUCT SPECIFICATIONS

Attribute	Container	Blend	Blend-Optimization	Target
1	88 - 100	90 - 94	91.5 - 92.5	92
2	0 - 5	0 - 5	0 - P75	0
3	2.4 -	3.0 -	P25 -	5-
4	125 - 177	125 -177	140 -160	150

(P25 & P75 are the 25th and 75th attribute percentiles, respectively)

Container and blend specifications are set by the customer. The blend-optimization specifications were determined by us and the plant Quality Control group with the aim of providing a cushion if the blend is resampled by the customer. Notice that they are tighter than the other specifications.

Some quality attributes are more important than others to the production group. Thus, they are prioritized as 40%, 25%, 25%, and 10% for attributes 1, 2, 3, and 4, respectively. The priority reflects the relative importance of the attribute. For example, attribute 1 is the most important. It is considered to be four times as important as attribute 4. Attributes 2 and 3 are considered equally important.

OPTIMAL BLENDING QUALITY

Blend Quality Attributes

Quality attributes are estimated numerically for a potential blend from the container attributes measured by the Lab. A weighted average of the container attributes is used. The weights are the amount of product in the containers.

For example, consider attribute 1. Suppose containers 1, 2, 3, and 4 are used to form a potential blend. Also, suppose the attributes and units are as follows:

	Container				
	1	2	3	4	Blend
Attribute 1	90.8	94.7	93.2	91.7	92.5
Units	10	7	9	5	31

The blend value is estimated as

$$[90.8x10 + 94.7x7 + 93.2x9 + 91.7x5]/31 = 92.5.$$

Quality Measures

The plant lab will initially measure four quality attributes per container but eventually up to ten may be measures.

Usually, it is difficult to even rank the product according to overall quality when given multivariate data on product quality and attribute priorities. Exceptions would be when the product is on target or near specification boundaries over all attributes.

To facilitate comparison among containers and for input into the optimization, we have applied Taguchi quadratic loss functions (QLF). The QLF are typically used in Quality Assurance / Quality Control applications in estimating the deviation of a product's functional characteristics from its desired target value.

Consider the QLF

$$L(y) = K\ (y-m)^2 \qquad (1)$$

where K is some constant, m is the target, and y is the attribute value.

The QLF (1) lends itself to mathematical manipulation. It measures closeness to target across the attributes. The points, i.e. assigned loss , are additive across containers, thus enabling the calculation of variability for a blend. Attribute priority and the amount of product in each container are also used in the calculations. The following considerations indicate why the QLF is an acceptable approximation in a wide variety of situations. Suppose the loss L(y) depends only on the difference y-m. Let L(y)=f(y-m), where f is a non-negative function that can be differentiated at least twice and f(0)=0. If f is expanded in a Taylor series through terms of second order, we obtain the approximation

$$L(y)=f(y-m)=K0 + K1(y-m) + K2(y-m)^2 . \quad (2)$$

Terms of higher order are usually very small and can be ignored. The fact that f(0)=0 implies that K0=0 and the fact that f is non-negative implies that K1=0 and K2 > 0. Thus the form of equation (2) reduces to equation (1). If there is more than one measurement, y, the average loss is

$$\begin{aligned} L &= E\ L(y) \\ &= K\, E(y-m)^2 \\ &= K\, sig^2 \end{aligned}$$

where E is the expected value.

There are three types of QLF's.

* N-Type : The *nominal*(closest to target)-The best.

** S-Type : The smallest-The best.

*** B-Type : The bigger - The better.

<u>N-Type</u>

(a) Suppose the plus and minus specification limits (m + - D) are equal to D, then the loss is defined as

$$L(y)=P(y - m)^2 / D^2$$

where P= Attribute Priority
y= Attribute Value
m= Target value of y.

(b) Suppose the plus and minus limits are not equal, i.e., the upper spec=m + D1 and the lower spec=m - D2. Then the loss is defined as

$$L(y)=P(y - m)^2 / D1^2 \quad \text{if } y < m$$

or

$$L(y)=P(y - m)^2 / D2^2 \quad \text{if } y > m.$$

S-Type

Suppose the target m = 0 and the upper specification is D. Then the loss is defined as

$$L(y)=P\, y^2 / D^2 \quad y > 0.$$

The S-type QLF is a special case of the N-Type function.

B-Type

Suppose the target value m = + infinity (ideal), the lower specification = D, and the attribute values y > 0.

Let z= 1/y in the S-Type function. Then z>0, the target m=0, the upper specification= 1/D and

$$L(y)=P\, D^2 / y^2 .$$

Using the specifications in Table 1, the loss functions for attributes 1, 2, 3, and 4 are the following :

Attribute	Con Spec	Target	Con QLF	
1	88-100	92	$L1(y)=0.025\ (y - 92)^2$	$y<92$
			$L1(y)=0.00625(y - 92)^2$	$y>92$
2	0 - 5	0	$L2(y)=0.01\ y^2$	
3	2.4-	5-	$L3(y)=0.03698(y - 5)^2$	$2.4<y<5$
			$L3(y)=0$	$y>5$
4	125-177	150	$L4(y)=0.000160(y - 150)^2$	$y<150$
			$L4(y)=0.000137(y - 150)^2$	$y>150$

Similarly, blend optimization QLF's are formed from the specifications in Table 1. Plots of container quality points are shown in Exhibit 1 (attached). Notice that maximum points is obtained at boundaries of the specifications. There the points are equal to the attribute priority. If a container is on the specification boundary across all four attributes, then the maximum value of 1.0 is obtained. The minimum points of 0.0 is obtained when the product is on target across all four attributes.

Optimization Criteria

Potential blends have two associated statistics.

* The lot points which is sum across the points of the four calculated quality attributes for the blend.

** Average container points which is the weighted average of the container points. The weights are equal to the amount of product in each container.

Both of these values are used in a performance statistic associated with each potential blend. The statistic is used in ranking blends with respect to the optimization criteria (e. g., blending from tails).

OPTIMAL BLENDING QUALITY

The program enumerates all possible combinations of 4, 5, 6, and 7 containers. From these combinations, an optimum is selected for each criteria. The maximum number of combinations when the storage area is full (36 containers) is given below.

Number of containers	Number of combinations
4	58,905
5	376,992
6	1,947,792
7	8,347,680
Total	10,731,369

The enumeration is done efficiently in the program. When the maximum blend product amount (40.0) is reached for a certain combination of containers, additional containers are not considered. For example, if containers 1, 3, 10, 14 total max units of 40.0, the program does not consider other combinations which include these four containers.

It is possible to develop ad hoc procedures for obtaining near optimal blends from a reduced number of combinations. However, a run time of 20 minutes for 36 containers on the Vax was considered, by the computer group, to be acceptable. The need for reducing CPU time may arise when the number of attributes is increased.

Strategy 1

- Identify blends closest to target using containers furthest from target.
(Blending from tails)

Each potential blend has an associated performance statistic, S1 , which is a function of blend points (BP) and average (weighted) can points (ACP)

$$S1=BP + 0.5/ACP.$$

The smaller S1 is, the more desirable the blend with respect to strategy 1. We want the blend to be close to target, making BP near zero. In addition, if the containers are far from target, ACP will be large. Thus, the ratio 0.5/ACP will be small. As such, S1 precisely defines a trade-off between blend closeness to target and container variability. The 0.5 factor in S1 was chosen on the basis of our judgement in observing the performance of the criteria on simulated data. It is a stringency factor in defining the trade-off between closeness to target and can variability.

Strategy 2
- Identify blends closest to target using all combinations of containers. (Center blending)

No restriction is placed on container variability. Therefore,

$$S2=BP.$$

Using S2 as a basis of sorting potential blends, we will arrive closer to target, if possible, than using strategy 1. Blends from strategy 2 can be used for comparison with other strategies.

Strategy 3
- Identify blends with minimum can variability with consideration of the target. (Uniform Blending)

$$S3= 0.60 \times ACP + 0.40 \times BP$$

Again a trade-off is offered between closeness to target and container variability. The can QLF's were used in computing BP so that ACP and BP are additive. However, we assign greater weight , 0.60 , to container variability than to blend closeness to target. The weights were selected on the basis of our judgement.

Example of Calculated Values

Suppose we are considering containers 2, 4, 12, and 16 as a potential blend. These were selected from the 36 container example in the following section. The lab quality attributes and calculated values are summarized in Table 2.

Table 2

		Attribute				Quality Points
Id	Units	1	2	3	4	
2	9	92.04	0.80	4.13	160	0.048
4	8	92.84	0.50	4.18	150	0.032
12	9	92.21	1.70	4.75	160	0.045
16	10	91.24	2.00	5.17	145	0.058
Blend	36	92.04	1.29	4.59	153.6	0.077

OPTIMAL BLENDING QUALITY

To illustrate the calculation of container points, consider container 2. Its' quality points (0.048) are calculated as in the following table.

Attribute	Points
1	$L1(92.04) = 0.00625(92.04 - 92)^2 = 0.00001$
2	$L2(0.80) = 0.01(0.80)^2 = 0.0064$
3	$L3(4.13) = 0.037(4.13 - 5)^2 = 0.028$
4	$L4(160) = 0.000137(160 - 150)^2 = 0.0137$
	Sum = 0.048

Average container point is calculated as :

$$\{0.048x9 + 0.032x8 + 0.045x9 + 0.058x10\} / 36 = 0.0465$$

and is a index for quality.

The statistics for ranking this potential blend with other blends within each optimization criteria are

$$S1 = 0.077 + 0.5/ 0.0465 = 10.830$$

$$S2 = 0.077$$

$$S3 = 0.60 \times 0.0465 + 0.40 \times 0.02465 = 0.038.$$

Blending Example

Consider the 36 container example in Table 3. Optimal blends for each strategy are shown below.

Strategy	Container Number
S1	3 17 19 32 33
S2	13 16 17 26
S3	4 12 13 28

The optimal blend attributes estimates are

Attribute	Strategy S1	S2	S3
1	92.04	91.97	92.42
2	1.51	1.01	0.89
3	3.94	4.73	4.43
4	147.50	146.47	150.29
Units	40	34	35
Blend Pts	0.3240	0.0721	0.3822
Container Pts	0.4211	0.0925	0.0429.

Consider the graphs in Exhibit 2 (attached) for container points. The distribution of the data are shown along the horizontal axis for each attribute (*'s). Also shown are the container attributes for optimal blends from S1 (circles) and S3 (squares). Notice the greater variability associated with S1 vis-a-vis S3 for, say, attribute 1. Recall, S1 is blending from tails and S3 is uniform blending.

Future Directions

Future directions for this application could include:

- Blending containers not meeting specifications
- Simulation for the long run performance of the optimization criteria
- Probabilistic assessment of blend specifications
- Procedures for reducing CPU time.

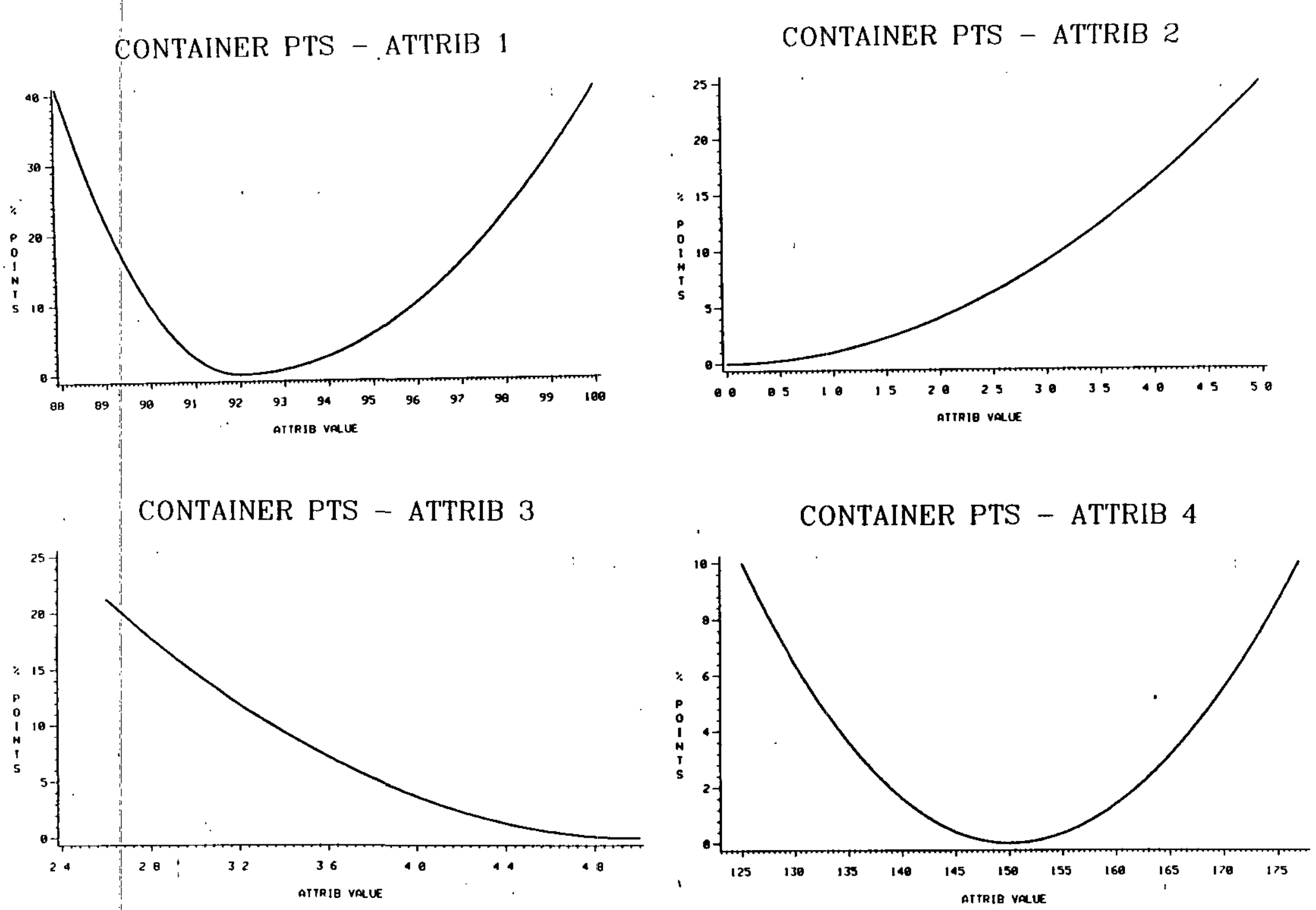

EXHIBIT 1

Quadratic Loss Functions

Table 3

36 Container Example with the Amount of Product (Units) and Quality Attributes.

ID	Units	Attribute 1	Attribute 2	Attribute 3	Attribute 4
1	10	88.39	5.00	2.40	147
2	9	92.04	0.80	4.13	160
3	8	89.46	0.47	3.09	140
4	8	92.84	0.50	4.18	150
5	6	96.49	5.00	2.86	160
6	6	96.68	1.80	4.36	160
7	10	97.01	2.40	3.33	150
8	9	96.10	1.50	4.28	140
9	8	94.87	3.20	3.98	160
10	10	96.48	1.20	5.27	145
11	10	90.05	0.60	2.40	147
12	9	92.21	1.70	4.75	160
13	8	90.92	0.50	4.32	140
14	8	91.07	3.00	4.84	150
15	6	92.59	3.00	4.47	160
16	10	91.24	2.00	5.17	145
17	6	96.83	1.20	5.97	160
18	10	97.61	2.20	6.05	150
19	9	98.00	3.80	6.06	140
20	8	94.02	4.11	5.69	160
21	10	91.71	5.00	5.14	147
22	9	92.19	4.68	4.27	160
23	8	90.46	1.70	4.46	140
24	8	92.01	1.70	3.57	150
25	6	93.20	1.60	4.67	160
26	10	90.63	0.30	3.86	145
27	6	91.87	1.10	3.92	160
28	10	93.48	0.80	4.43	150
29	9	94.28	1.00	5.08	140
30	8	95.16	0.60	4.74	160
31	10	94.94	0.70	4.84	147
32	9	88.67	1.50	2.60	160
33	8	88.16	0.20	2.40	140
34	6	95.98	2.00	5.00	150
35	8	96.01	1.10	5.01	160
36	10	96.79	1.20	4.03	145

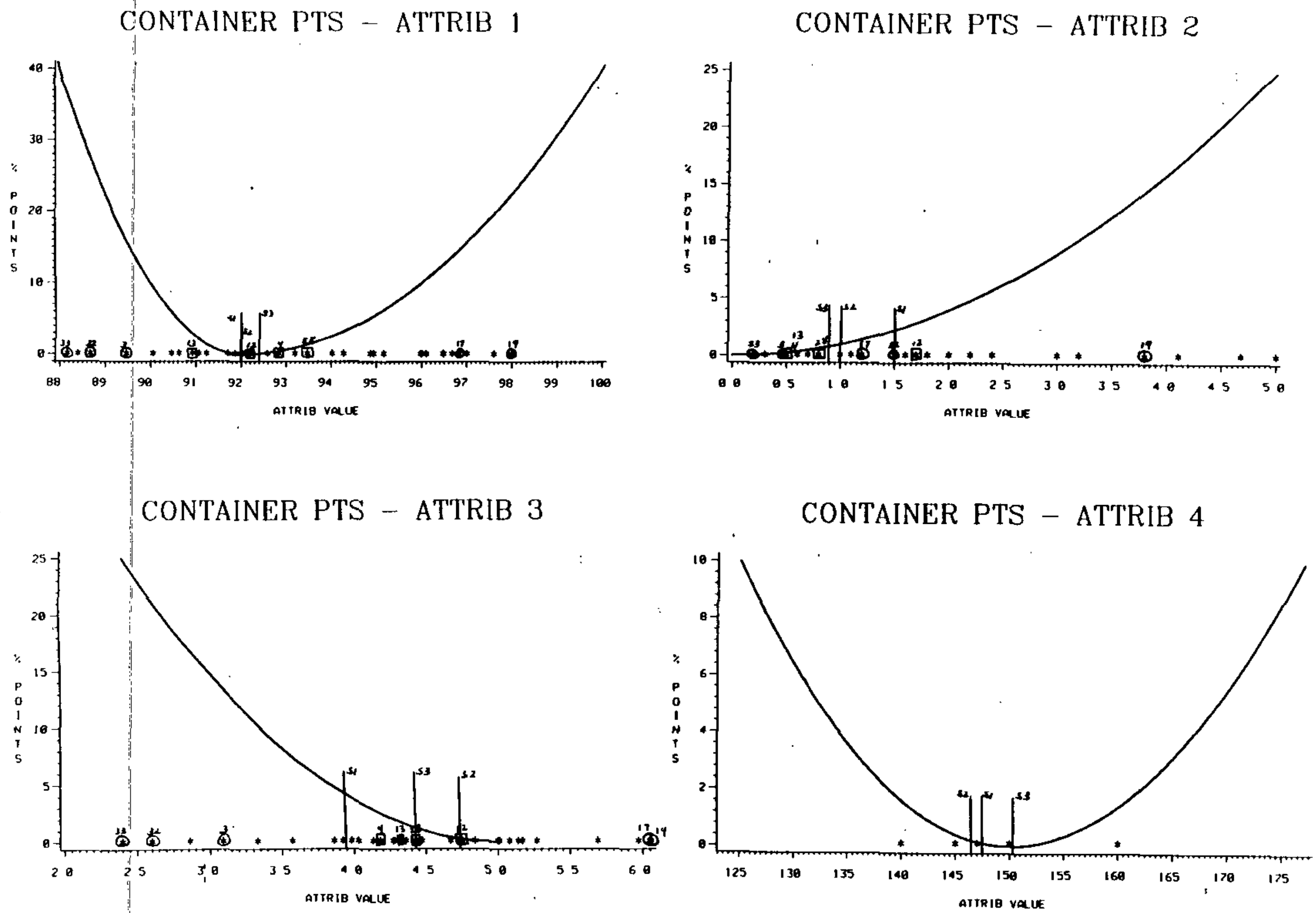

EXHIBIT 2

36 Can Example