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# Thickness Effects on the Plastic Collapse of Perforated Plates with Triangular Penetration Patterns

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## ABSTRACT

This paper investigates the effects of plate thickness on the accuracy of limit load solutions obtained using an elastic-perfectly plastic [EPP] equivalent solid [EQS] procedure for flat perforated plates with a triangular array of penetrations. The EQS approach for limit loads is based on an EQS collapse surface that is valid for generalized plane strain. This assumption is applicable for very thick plates but is known to be less reasonable for very thin plates where plane stress may be a better assumption. The limits of applicability of the generalized plane strain assumption are investigated by obtaining limit load solutions for perforated plates of various thicknesses that are subjected to in-plane and bending loads. Plastic limit load solutions obtained using three-dimensional EPP finite element analysis [FEA] of models which include each penetration explicitly are compared with solutions obtained using the EQS approximation. The penetration pattern chosen for this study has a ligament efficiency (ligament width-to-pitch ratio, h/P) of 0.32.

For plates thicker than the pitch, the limit load calculated using the EQS method for both in-plane and bending loads is shown to be very accurate (within 4%) of the limit load calculated for the explicit model. On the other hand, for thin plates ( $t/P < 2$ ), the EQS limit load is 5% greater than the explicit limit load for bending and 8% greater than the explicit limit load for in-plane loads. For thinner plates, the collapse surface is tied to the local geometry deformation and, hence, an equivalent solid plate representation of plastic collapse is a function of deformation mode and thickness.

## NOMENCLATURE

| | |
|---|---|
| $P$ | Pitch of pattern, mm |
| $t$ | Plate thickness, mm |
| $h$ | Minimum ligament width, (P-d), mm |
| $\mu$ | Ligament efficiency, h/P |
| $d$ | Penetration diameter, mm |
| $S_y$ | Yield stress of material, MPa |
| $E$ | Young's modulus, MPa |
| $p$ | Transverse pressure, MPa |
| $F_x$ | In-plane traction, N/m |
| $E^*, \nu^*$ | Equivalent solid plate material properties, MPa, unitless |
| $E_z^*$ | Equivalent transverse solid plate material property, MPa |
| $G_z^*$ | Equivalent shear solid plate material property, MPa |

## INTRODUCTION

This paper assesses the effects of plate thickness for the calculation of limit loads of flat perforated plates utilizing equivalent solid analysis methods. Reinhardt [1,2] developed a fourth-order yield function to represent yielding of thick perforated plates with triangular penetration patterns. Gordon et al. [3] carried this a step further by developing a fourth order collapse surface that represents the limit load response of thick perforated plates. Using this collapse surface, Jones et al. [4] developed an elastic-perfectly plastic flow model to provide an equivalent solid [EQS] elastic-perfectly plastic [EPP] plasticity formulation that is useful in calculating limit loads for thick perforated plates with triangular penetration patterns.

Gordon et al. [3] developed the fourth order collapse surface based on the assumption that the penetration pattern is in a state of generalized plane strain – i.e. the plate is relatively thick. If the penetrations are normal to the mid-surface of the plate, then the top and bottom surfaces of the plate are considered to remain flat and parallel to the mid-surface while the average force normal to each surface remains zero. Referring to Figure 1, generalized plane strain is implemented by coupling all of the nodes on the top surface and setting the net force in the Z direction to zero. These assumptions place a constraint on the response of the penetration pattern, which is expected to have some effect on the limit load as the plate becomes thin relative to the pitch of the pattern. This paper investigates the effects of plate thickness on the limit load calculated by the EQS-EPP procedure of Jones, et al. for both bending and in-plane stretching modes of loading.

## PROBLEM DESCRIPTION

Consider a rectangular plate of finite length (52.58 P), thickness t, and infinite width as shown in Figure 1. Limit load solutions are determined for a number of penetration thickness-to-pitch (t/P) ratios for both a transverse pressure load (p) and in-plane traction load ( $F_X$ ). The plates have their short sides clamped - i.e. all nodes on the Z = 52.58 P surface are clamped. The axes of the penetrations are normal to the mid-surface of the plates. A solid region surrounds the perforated region so that transverse shear stresses at the perforated-to-solid interface will play a significant role in the plastic collapse of the thick plates. With transverse pressure p applied, the plate is subjected to bending modes of deformation. With traction $F_X$ applied, the plate is subjected to in-plane stretching modes of deformation. The loads p and $F_X$ are applied separately to allow the study of limit loads due to bending and stretching.

It is expected that the results of the plates which are subjected to bending will differ from the results of those plates which are subjected to in-plane tension loads. For thicker plates, it is anticipated that the transverse shear stresses at the interface of the solid and the perforated material regions will affect the limit load. For thinner plates, it is anticipated that the limit load will be governed by the formation of a plastic hinge in the plate – i.e. a bending failure. The fourth order collapse surface used by Gordon et al. [3] uses a superposition of in-plane and out-of-plane responses. In order to isolate the effects of thickness on each of these responses, limit load solutions are obtained for the direct tension case (p = 0) as well as the bending ( $F_X$ = 0) case. In this way, it will be possible to determine if the thickness effects are due to the inability of the assumed yield surface to represent plane stress yielding or if there is a deficiency in the yield surface to represent bending loads.

## METHOD OF SOLUTION

O'Donnell [5] experimentally demonstrated the effects of plate thickness on bending for plates where the plasticity effects are negligible. The results of O'Donnell's work, in addition to the work by Slot and O'Donnell [6], have demonstrated that elastic EQS methods based on the generalized plane strain assumption are reasonable for an elastic plate whose thickness is greater than twice the pitch.

In this paper, limit load solutions for semi-infinite plates of varying thickness are determined numerically to investigate the effects of thickness on the accuracy of the EQS method. Here, ABAQUS [7] is used with an elastic-perfectly plastic stress-strain curve to obtain limit load solutions for models where the holes are represented explicitly and for models where the holes are treated using the EQS method. The EQS limit loads were obtained using the work developed by Jones et al. [4]. The effect of thickness can be determined by comparing the resulting limit loads calculated by the different models.

In these analyses, the limit load is defined as the load for which a small increase in load produces a very large increase in deflection – i.e. the slope of the load-deflection curve approaches zero. For a small-strain, small-deformation finite element formulation, this load will comply with the theoretical definition of a lower bound limit load because the nodes are in equilibrium and the stresses are below the yield stress.

## FEA MODELS

Figure 2 shows the FEA mesh that was used for the explicit model. An enlarged view of the mesh within the penetration pattern is shown in the insert of Figure 2.

Limit loads are calculated for bending and in-plane loads for penetration thickness-to-pitch [t/P] ratios of 21.33, 2, 1, 0.75, and 0.50. Symmetry boundary conditions are applied along the Y = 0 and $\sqrt{3}$ P/2 surfaces and on the X = 0 plane. There are 41 rows of penetrations in the model. A solid region surrounds the perforated region so that the transverse shear stresses at the perforated-to-solid interface will play a significant role in the plastic collapse of the plates.

The plate is modeled as a slice from the semi-infinite plate, as shown in Figure 2, such that it extends one pitch width in the Y-direction and is fixed at a solid boundary many pitch lengths from the center in the X direction. Thus the same finite element model is used to represent both a perforated plate subjected to transverse bending by setting $F_X = 0.0$ or subjected to in-plane stretching by setting p = 0.0.

The 3D-FEA EQS model is shown in Figure 3. The perforated plate region in the EQS model is treated as a solid region whose properties are adjusted to account for the penetrations. Also, for the bending solution, the pressure applied to the equivalent solid portion of the model is reduced by the ratio of the explicit surface area to the equivalent solid surface area. The elastic properties are taken from Slot and O'Donnell (1971) and are $E^*/E = 0.30348$, $\nu^* = 0.34331$, $E_Z^*/E = 0.57735$, and $G_Z^*/G = 0.40569$. The elastic-perfectly plastic EQS method is based upon the collapse surface developed by Gordon et al. [3] for a ligament efficiency (h/P) of 0.32 so it is directly applicable to this study. The same yield strength and elastic properties of the base material were used for both the EQS models and for the explicit models.

A highly refined 3D-FEA mesh is used in each model so that the results will accurately represent the near surface ligament deformation that contributes to the thickness effects. There are 86,560 and 11,136 20-node reduced integration hexagonal elements in the explicit and EQS models, respectively. Both the EQS and explicit models were run with a small displacement, small strain (linear geometry) formulation. Young's modulus and Poisson's ratio were taken to be 27.0E+6 psi (207E+3 MPa) and 0.3, respectively. The yield strength was taken to be 0.001639*E. Since linear geometry assumptions are chosen, the limit loads are proportional to yield strength and the actual value chosen is not important.

## RESULTS

### a. In-plane Loads

Figure 4 shows load-deflection curves for the in-plane loading case from the equivalent and explicit solution for $t/P = 21.33$. For this geometry, the limit loads from the explicit model and the EQS model are identical. This is expected since the yield surface for the EQS flow rule is developed from in-plane loadings of relatively thick plates.

Figure 5 compares the explicit and EQS limit load results for in-plane loads for all t/P cases which were investigated. For $t > 2P$, the solutions are within 0.1%. For thinner plates, the EQS limit load is greater than that of the explicit solution. The thinnest plate studied was $t = P/2$. For this plate, the EQS limit load is 8% greater than the limit load for the explicit model.

### b. Bending Loads

Figure 6 shows the transverse pressure versus transverse deflection curve from both analyses for the plate loaded by transverse pressure for $t/P = 21.33$. For this geometry, the limit load from the EQS-EPP model is about 4% below that of the explicit model. This is expected since the collapse surface for the EQS flow rule was developed from in-plane loadings of very thick plates and included a conservative treatment of the effect of transverse stresses on the collapse behavior of the perforated material.

Figure 7 shows the limit load due to transverse pressure as a function of t/P and compares the results from the explicit model with the EQS model. For $t > 2P$, the EQS solutions are a small percentage below the explicit plate solutions. For thinner plates, the EQS limit load is about 5% above that of the explicit model. The solutions are about equal when $t = 2P$.

## DISCUSSION

These studies show that the effect of plate thickness on limit load is not the same for bending and stretching modes of loading. For the in-plane loading case, yielding through the section is achieved at the same location whether the plate is thick or thin. This is simply the difference between plane stress and plane strain yielding. For bending, the failure mode changes from shear at the solid interface for the thick plate (Figure 8-a) to bending at the center of the perforated region for the thin plate (Figure 8-b). For both loading cases and for $t > 2P$, it is conservative to use the EQS model to compute limit loads. That is, the limit load calculated by the EQS method will either be nearly equal to or less than the limit load calculated by an explicit model. This conclusion is not true for plates thinner than 2P.

Most applications in heat exchanger design involve consideration of circular plates subjected to combined bending and tension loads. Also, most materials used in pressure vessel construction exhibit significant work hardening capability. The work here would suggest that limit loads achieved by the EQS method would be conservative for application to such structures.

## CONCLUSIONS

Solutions for limit loads of perforated plates are presented for tension and bending loads for a ligament efficiency of 0.32. It is shown that it is conservative to use the EQS-EPP method to compute limit loads for plates with thickness greater than twice the pitch. For thinner plates, the EQS method may give somewhat higher limit loads than those calculated by an explicit model.

## ACKNOWLEDGEMENT

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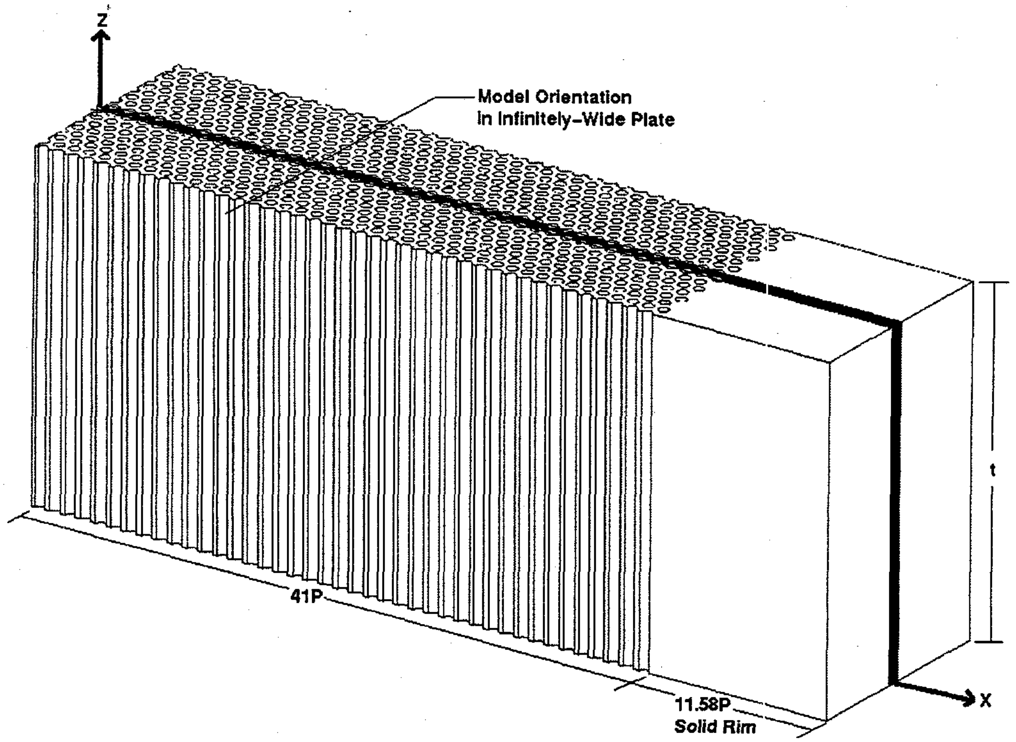


Figure 1. Slice Model Location in Infinite Plate

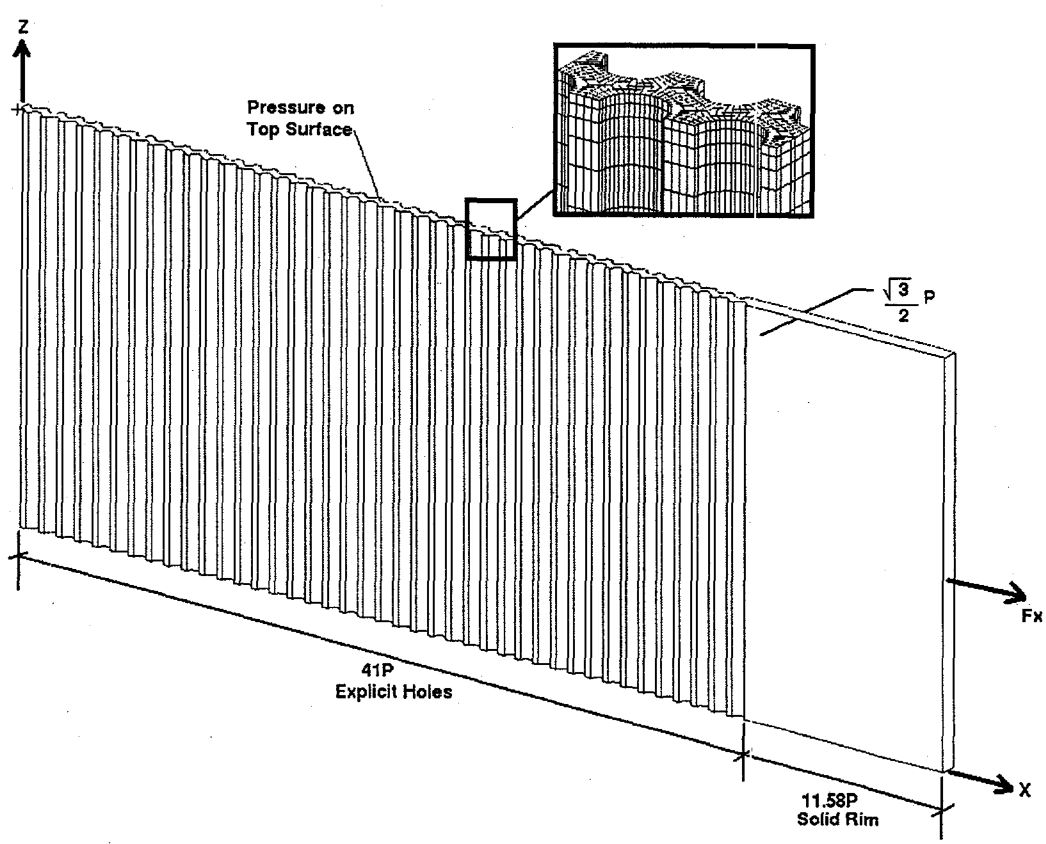


Figure 2. Clamped Plate Geometry

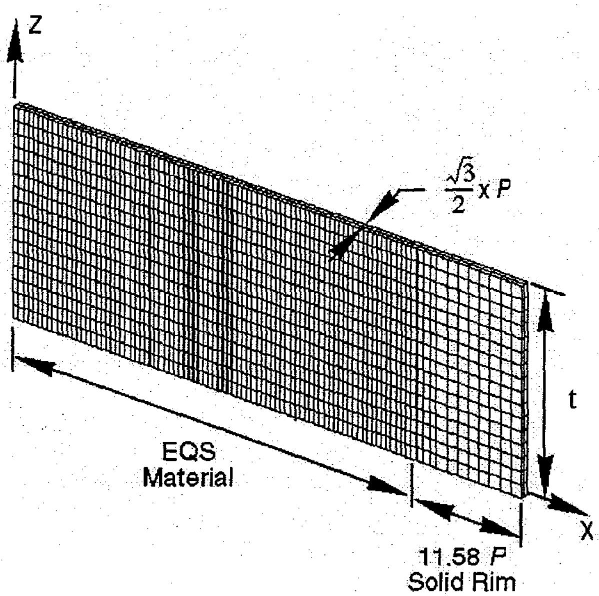


Figure 3. Equivalent Solid Plate Model

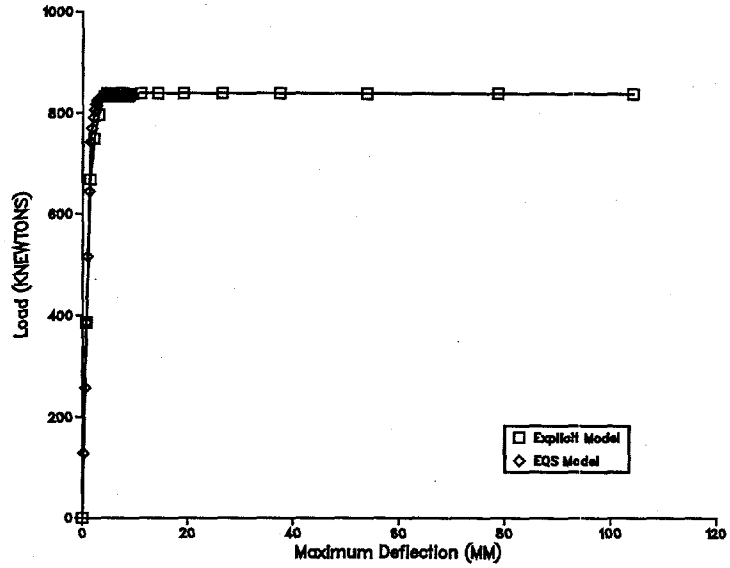


Figure 4. Deflection Curve for In-plane Load for t/P=21.33

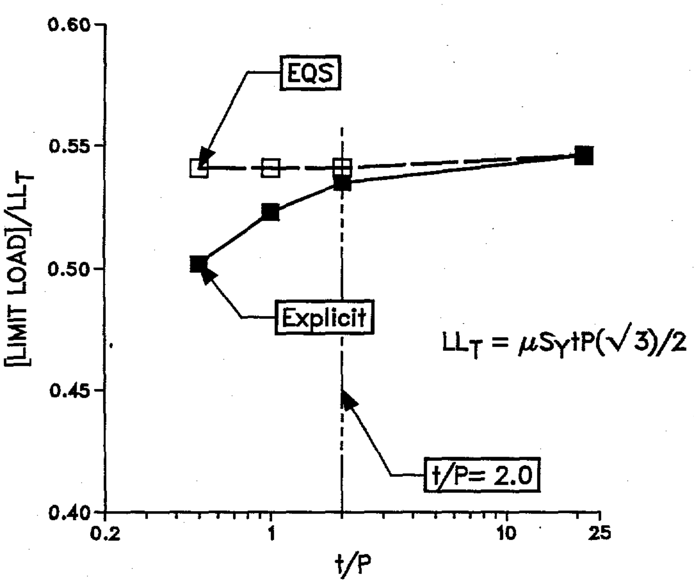


Figure 5. Limit-Load versus Thickness for In-plane Load

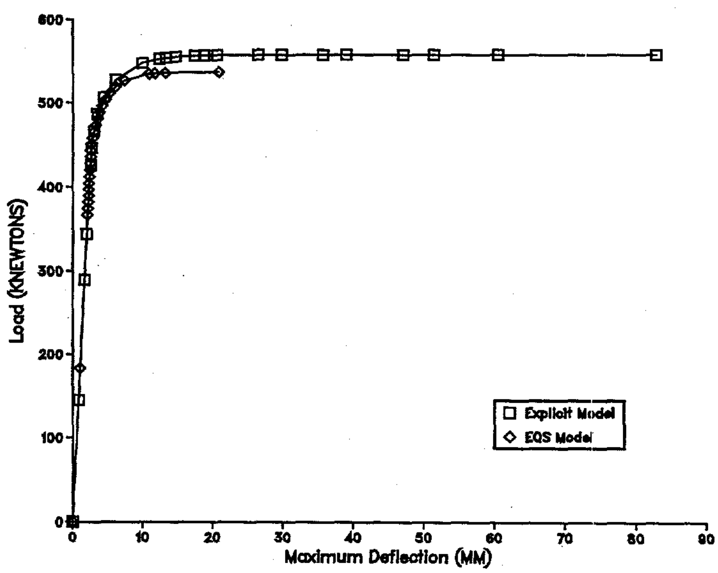


Figure 6. Load Deflection Curve for Bending Load

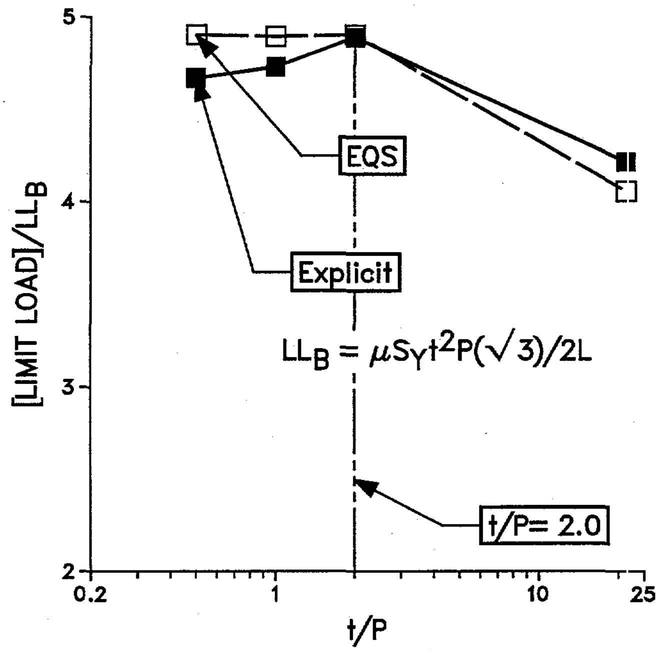


Figure 7. Limit-Load versus Thickness
for Bending Load

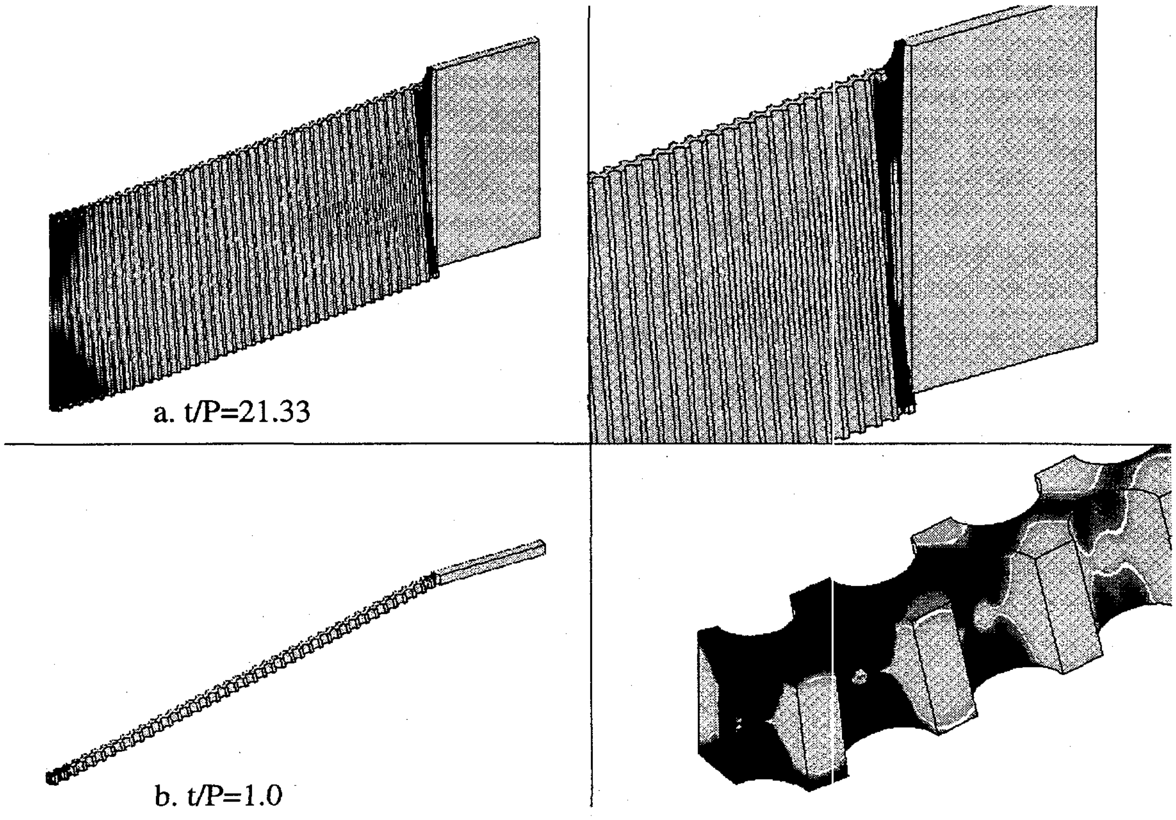


Figure 8. Fringe Plots of Equivalent Plastic Strain on Deformed Geometry