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A Self-Consistent Multiscale Theory of Internal-Wave Mean-Flow Interactions in the Ocean

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Abstract

This is the final report of a three-year, Laboratory Directed Research and Development (LDRD) project at Los Alamos National Laboratory (LANL). The research reported here produced new effective ways to solve multiscale problems in nonlinear fluid dynamics, such as turbulent flows and global ocean circulation. This was accomplished by first developing new methods for averaging over random or rapidly varying phases in nonlinear systems at multiple scales. We then used these methods to derive new equations for analyzing the mean behavior of fluctuation processes coupled self consistently to nonlinear fluid dynamics. This project extends a technology base relevant to a variety of multiscale problems in fluid dynamics of interest to the Laboratory and applies this technology to those problems. The project's theoretical and mathematical developments also help advance our understanding of the scientific principles underlying the control of complex behavior in fluid dynamical systems with strong spatial and temporal internal variability.

Background and Research Objectives

Multiscale problems span an enormous range of physical phenomena and applications, including many that are central to the Laboratory's mission. Our main research objective was to produce effective new ways to solve multiscale problems in nonlinear fluid dynamics, such as turbulent flows and global ocean circulation. This was accomplished by developing new methods for averaging over random, or rapidly varying,

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phases in nonlinear systems at multiple scales, and using these methods to derive new equations for analyzing the mean behavior of fluctuation processes coupled self consistently to nonlinear fluid dynamics.

For this purpose, we began by studying internal-wave mean-flow (WMF) interaction in ocean models. In our approach, fine scale descriptions of physical phenomena were replaced by effective average, or statistical, descriptions of the phenomena at the coarser and slower scales that make numerical computations feasible.

Internal waves in the ocean were a good starting point for a variety of reasons. First, they account for a significant amount of the observed high frequency variability of the ocean. Second, internal waves transfer momentum and hence exert a stress on larger scale motions. Third, internal waves cause mixing and redistribution of buoyancy by sporadic overturning and breaking -- the process of convective adjustment. Thus, the full understanding of these transfer and mixing processes and their proper parameterization is essential in modeling the variability of the ocean's general circulation.

Internal waves are a high-frequency phenomenon, compared to the mean oceanic flow. The spatial scales of ocean internal waves, however, lie in the entire range between the planetary scales where variability of the velocity and density fields is generated by atmospheric forcing, and the microscales where these contrasts are dissipated by molecular processes. It is a central problem in ocean modeling to model how energy and entrophy are cascaded from the large generation scales to the small scales at which dissipation occurs. Internal waves provide an important link in this energy cascade, since they have the unique ability to convert two-dimensional motions that are prevalent at large scales to three-dimensional motions that are prevalent at small scales.

In interactions between internal waves and the mean flow there is a separation of time scales and a resonant interaction. In principle, capturing this type of wave mean-flow interaction requires subgrid modeling and parameterization.

We approached this problem by first treating the resolved scales using spatial averaging, as well as time averaging, in order to capture the dynamics of an internal wave packet whose slowly varying envelope interacts self consistently with the mean flow at the resolved scales.

In the first year of this project, we derived new WMF equations (for the rectified effects of a modulated single frequency wave) as self-consistent dynamical equations for the wave action density and wave vector fields, coupled to the mean fluid motion at the resolved space and time scales. These self-consistent equations showed, for example, that a steady wave train with nonzero vorticity can exert stress on the mean flow without dissipation, in contrast with the classical Charney-Drazin nonacceleration theorem for potential disturbances without vorticity.

During the subsequent two years of the project, we followed the implications of these new WMF equations for more general fluctuations in fluids. In particular, we generalized our theory to apply to: (1) mesoscale eddies in ocean circulation; and (2) turbulent fluctuations.

Thus, our LDRD project addressed fundamental research in establishing "mesoscopic bridges" between microscopic and macroscopic modeling and determining statistically predictive scientific principles underlying the nonlinear, nonequilibrium dynamics of mesoscale complexity. There is a growing need to understand strong spatial and temporal internal variability, which typically appears at mesoscopic scales, a need shared by many other disciplines including biology, fluids, and geophysics.

Experimental advances in remote sensing for geophysics now enable us to image increasingly detailed structures and even begin following their dynamics in some cases, such as in the TOPEX-POSEIDON satellite imaging of the ocean's variable surface height on Earth. However, the majority of these problems are characterized by measurements producing only sparse data sets. Consequently, interpretative frameworks are urgently needed for statistical predictions of phenomena that we may observe only on sparse data sets.

The results of our LDRD project form a technological base for some of the future needs in this area. The scientific needs that will dictate technology in the next decade are:

(i) Understanding the roles of nonlinearity and stochastic processes in controlling internal variability of nonequilibrium physical phenomena; i.e., the nonlinear interrelation of system and environmental degrees of freedom, and its effects on the statistical properties of internal variability. Ultimately, this must determine the limits of predictability in nonlinear systems.

(ii) Assessing predictability -- learning how to quantify sparse measurements in ways that are relevant to specific macroscopic properties; and

(iii) Understanding "upscaling," i.e., how specific types of sparse measurements of complex systems can be used to predict statistical macroscopic properties. An understanding of upscaling will provide constructive schemes for including effects of microstructure in; e.g., kinetic coefficients such as an eddy viscosity coefficient due to interaction of the mean flow with the internal wave field viewed as a "thermodynamic" reservoir.

Importance to LANL's Science and Technology Base and National R&D Needs

Multiscale problems are a common thread in all of the grand challenge problems of modern science and technology. Progress in any of these grand challenge problems has the potential for immediate payoffs in many applications. The classical multiscale problems include turbulence, flow through porous and fractured media, and wave propagation in heterogeneous domains. Progress in any of these classical problems can stimulate related progress in many applications, such as climate modeling, oil recovery, or remote seismic sensing.

The research reported here has both specific impact in the field of ocean modeling and more general impact in computational physics. Specifically, we have created parameterizations for turbulent fluctuations which when incorporated into numerical simulation codes will improve the predictability of global ocean models, especially in the long time limit appropriate to climate simulations. More generally, our theory of the coupling of temporally unresolved internal waves to the resolved mean flow will be applicable to other problems of physical modeling in which there are multiple time scales, especially in fluid turbulence problems.

For example, modeling turbulence is central in many of the Laboratory's research efforts, particularly the US DOE programs CHAMMP (Computer Hardware Advanced Mathematic and Modeling Physics), CCPP (Climate Change Prediction Program) and impending ACPI (Advanced Climate Prediction Initiative). These programs address global ocean circulation, coupled to the atmosphere for the purpose of long time climate modeling and prediction. Future applications may also involve, for example, mix parameterizations in weapons codes that are designed to assure responsibility for stockpile stewardship.

Scientific Approach and Accomplishments

The separation in frequencies between internal gravity waves and the resolved mean-flow dynamics of the ocean (and atmosphere) suggested to us an approach involving the "two-timing" and averaging methods of modern applied mathematics. With applications in mind to oceanic (and atmospheric) dynamics characterized by multiple time and space scales, we used this approach to develop a new set of wave mean-flow (WMF) equations during the first year of the project. These equations describe the slow-time dynamics of the slowly modulated envelope of an internal wave packet interacting with a mean flow. Our strategy in deriving the WMF equations was based on a decomposition of the displacement of a Lagrangian fluid parcel into a slowly varying component due to the mean flow and a rapidly varying component due to the gravity wave. We inserted this decomposition of the fluid trajectory into Hamilton's principle for the combined motion of mean flow and internal wave variations in a continuously stratified incompressible fluid moving in a three-dimensional rotating domain. We formulated an asymptotic expansion of Hamilton's principle in two small parameters: the ratio of time scales and the wave amplitude. We then averaged over the rapidly varying phase of the slowly modulated wave train, before taking constrained variations and using our Euler-Poincaré theory to obtain the resulting self-consistent mean flow equations.

The new WMF equations derived by this approach describe the rectified nonlinear effects of the interactions of a modulated wave train with the mean flow of the fluid. Many examples appear in nature in which rapid oscillations produce organized motion -- two familiar examples are parametric resonance -- as in the inverted pendulum -- and viscous streaming, the organized flows driven by sound waves discussed by Kelvin and by Lighthill. In previous theories of these effects, the rapid displacement due to the wave driving had to be prescribed. An example is the theory of Craik and Leibovich [1], which is designed to describe Langmuir circulations (wind generated longitudinal vortices) in the mix layer of the ocean.

The advantage of the theory we created by using this approach lies in its **self consistency**. The forces due to the rapidly oscillating waves in our initial theory are not prescribed, as in other theories. Instead, the wave modulations have their own dynamics, which are coupled to the dynamics of the mean flow self consistently at the slow time scale, and the evolution of these modulations leaves invariant the original decomposition of the Lagrangian fluid trajectory into its slowly and rapidly varying components. No previous theory of mix-layer dynamics had possessed this self consistency. We also generalized our initial WMF equations to describe the interactions of the mean flow with a spectral distribution of waves.

As described above, the theory we created and report on here incorporates the rectified mean effects of the unresolved waves. However the time-averaged theory cannot describe the effects of instantaneous rapidly varying fluctuations. These fluctuations may have importance in long-term ocean modeling, being the mechanism for precipitating transitions from one equilibrium state to another. In the future we plan to extend our rectified mean theory by deriving fluctuation-dissipation relations for the WMF interactions based on assuming that the wave-wave interactions enforce a certain type of statistical equilibrium. This will form the basis for representing the statistical effects of the wave fluctuations in computer simulations in terms of kinetic coefficients, such as an eddy viscosity coefficient due to interaction with the internal wave field viewed as a thermodynamic reservoir.

Scientific and Technical Results

The main results of this project were:

- A self consistent continuum theory of internal-wave mean-flow interactions.
- New equations for self-consistent mix-layer dynamics for the interaction of the mean ocean flow with rapidly fluctuating surface waves.
- New Euler-Poincaré formulations of ideal continuum dynamics.
- New dynamically self-consistent turbulence-closure equations describing the interaction dynamics of the mean and fluctuating components of a turbulent flow at the mean time scale.
- The comparisons of the solutions of these new turbulence closure equations with experimental data and numerical simulations.

These results are reported in detail in 25 journal publications (see **Publications** list). Here we give a chronological summary of our progress.

The project's first year -- New Wave, Mean-Flow Interaction (WMFI) Equations. (Darryl D. Holm, T-7; Ivan Gjaja, CNLS/T-8)

In the first year of this project, we derived a hierarchy of approximate models of wave, mean-flow interaction (WMFI) for modeling ocean dynamics by using asymptotic expansions. One small parameter for these expansions is the ratio of time scales between internal waves at most wavenumbers and the mesoscale mean flow of an inviscid stratified rotating fluid. This "adiabatic ratio" is small and is comparable to the ratio of space scales for the class of initial conditions that support internal waves. Another small parameter available in these expansions is the ratio of the internal wave amplitude to its wavelength. The new self-consistent WMFI equations were derived in two ways: first, by requiring Euler's equations to preserve the wave, mean-flow decomposition to linear order in the wave amplitude; and second, by substituting this decomposition into Hamilton's principle for the Euler's equations and applying asymptotic expansions and phase averaging. The derivation from Hamilton's principle showed that the resulting equations possess a Kelvin circulation theorem, conserve a potential vorticity, and are Lie-Poisson Hamiltonian dynamical systems in the Eulerian variables. The derivation from Euler's equations confirms the validity of the derivation from Hamilton's principle. Passage to the Lie-Poisson Hamiltonian formulation brought the WMFI theory into a framework in which formal and nonlinear stability analysis may be applied as in [2]. We also found the relations of these results to the non-acceleration theorem [3], averaging [4,5], WKB stability theory [6], and Lagrangian-mean fluid equations for prescribed wave displacements such as those of [7].

The project's second year -- New Euler-Poincaré Formulations of Ideal Continuum Dynamics. (Darryl D. Holm, T-7; Jerrold E. Marsden, Caltech; and Tudor Ratiu, UC Santa Cruz)

In the project's second year, we studied the general theory of reduction of variational principles with respect to their invariance groups. These group-reduced variational principles are mathematically interesting since they involve constraints on the allowed variations analogous to what one finds in the theory of nonholonomic systems with the Lagrange d'Alembert principle. These equations generalize earlier work by Poincaré for dynamics on a Lie algebra, in that they depend on a parameter and this parameter in fluid dynamics applications has the interpretation of being advected, or Lie dragged, as with the density in compressible fluid flow. In addition, we derived the basic abstract theorem about fluid circulation for these variational equations, which we call the Kelvin-Noether theorem. We also derived a series of approximate models of ocean circulation dynamics by using asymptotic expansions of Hamilton's principle for the most accurate theory in the small dimensionless parameters that typically appear in the large-scale, rapidly rotating situations that commonly occur in oceanographic applications. Our approach in deriving these approximate models preserves the invariance properties of the action principle that are responsible for the Kelvin-Noether theorem. Thus, the approximate model equations we derived for internal wave, mean-flow interactions in the ocean each possesses its own circulation theorem and its own attendant conservation law for potential vorticity. This conservation law is an important and useful central concept in geophysical fluid dynamics at every level of approximation.

Hamiltonian reduction of classical mechanics on Lie groups reduces the phase space to the corresponding Lie algebra. In our previous work, we developed a theory of Hamiltonian reduction for semidirect product groups. This theory applies to ideal (nondissipative) fluid dynamical systems that are governed by Lie-Poisson type equations, such as compressible fluids, magnetohydrodynamics, and some ocean circulation models. In this previous work, we also used the Hamiltonian setting to develop a powerful method of establishing explicit nonlinear stability conditions for ideal fluid and plasma equlibria. We applied this method to obtain explicit basic nonlinear stability results for a number of fundamental theories of fluid and plasma dynamics in, for example, Refs. [2] and [8].

In support of the theoretical base for our project, during the second year we studied Lagrangian reduction; that is, the reduction of variational principles with respect to their invariance groups. These group-reduced variational principles are mathematically interesting in their own right since they involve constraints on the allowed variations, analogous to what one finds in the theory of nonholonomic systems with the Lagrange d'Alembert principle. We call the resulting variational equations the Euler-Poincaré equations, since Poincaré [9] came rather close to this general picture in his work in 1901. These equations generalize Poincaré's version of the Euler-Poincaré equations on a Lie algebra in that they depend on a parameter, and this parameter in examples has the interpretation of being advected, or Lie dragged, as with the density in compressible fluid flow.

In addition, we derived the basic abstract theorem about fluid circulation for these variational equations, which we call the Kelvin-Noether theorem. We also derived a series of approximate models of ocean circulation dynamics by using asymptotic expansions of Hamilton's principle for the most accurate theory in the small dimensionless parameters, which typically appear in the large-scale rapidly rotating situations that commonly occur in oceanographic applications. Our approach in deriving these approximate models preserves the invariance properties of the action principle that are responsible for the Kelvin-Noether theorem. Thus, the approximate model equations we derived for internal-wave mean-flow interactions in the ocean each possesses its own circulation theorem and its own attendant conservation law for potential vorticity. This conservation law is an important and useful central concept in geophysical fluid dynamics at every level of approximation.

We also studied Euler-Poincaré systems (i.e., the Lagrangian analog of Lie-Poisson Hamiltonian systems) defined on semidirect product Lie algebras. We first gave a derivation of the Euler-Poincaré equations for a parameter-dependent Lagrangian by using a variational principle of Lagrange d'Alembert type. Then we derived an abstract Kelvin-Noether theorem for these equations and determined how to apply them to the analysis of continuum mechanics and fluid dynamics. We used our Euler-Poincaré theory to rederive various known, ideal continuum models including our new closure model for WMFI. This was done by using asymptotic expansions, two-timing, and averaging in Hamilton's principle for an ideal incompressible fluid, then introducing viscosity semi-empirically as diffusion of an appropriate momentum. This work prepared us for our derivation of new turbulence models during the project's third year.

The project's third year -- New Turbulence Models. (Darryl D. Holm, T-7; Shiyi Chen, CNLS; Ciprian Foias, U Indiana; Eric Olson, U Indiana; Edriss Titi, UC Irvine; and Shannon Wynne, UC Irvine)

The energy in a turbulent fluid cascades to ever smaller scales. That is, the fluid energy in the large "integral" scales that are resolvable in a computational simulation transfers "ballistically" to the smaller "dissipation" scales that eventually become unresolvable in a computational simulation at low dissipation (high Reynolds number). Turbulence modeling describes the nonlinear interplay between the resolved and unresolved scales (or subgrid scales, abbreviated as SGS). The need for modeling the effects of the small scales upon the larger ones in the presence of this turbulent energy cascade introduces a statistical and probabilistic element into the prediction and simulation of turbulent fluid motion. Characterizing this element is called the "parameterization" of turbulence. Historical or traditional approaches to turbulence modeling introduced an ensemble mean description. The derivation of this description involved ensemble averaging the fluid motion equation after first introducing the "Reynolds decomposition" of the fluid velocity into its ensemble mean and fluctuating parts. Substituting this decomposition into the motion equation and taking the ensemble mean produces "Reynolds stress" terms whose dynamics cannot be expressed in closed form without introducing further approximations. Modeling the dynamics of the Reynolds stress terms is called the "turbulence closure problem" and it is the outstanding problem of classical physics. Analytical methods for the development of Reynolds stress closures for turbulence are reviewed in Speziale [10] and references therein.

Based on our work on WMF, in the project's third year we took a nontraditional approach to the turbulence modeling problem that yielded different, but related dynamical closure equations for turbulence. Our approach was developed in response to the Laboratory's need in modeling the ocean circulation component of the Earth's climate. Instead of wave trains, in global ocean circulation dynamics the fluctuations are identified physically as "mesoscale eddies." These are confined patches of potential vorticity that move with the mean flow and also act back upon it. The nonlinearity in this interaction between resolvable and unresolvable scales in global ocean circulation dynamics raises the issue of self consistency between the dynamics of the mean and fluctuating components of the flow at the mean time scale. In our approach, as with our WMF work, dynamical self consistency of the turbulence closure (or mesoscale eddy parameterization, as it is called in geophysical fluid dynamics) is established by applying decompositions of Reynolds type not in the fluid motion equations, but instead in Hamilton's principle for these motion equations. Our approach may be applied at any level of fluid description, from the equations for the incompressible motion of a homogeneous fluid, to the equations of global ocean circulation dynamics. Thus, our earlier work on wave mean-flow interaction in [11], followed by our mathematical development of the Euler-Poincaré equations of continuum dynamics in [12], provided the technological basis needed for developing new and effective turbulence closure models.

We also applied our approach to the modeling of turbulent flows in pipes and channels at high Reynolds numbers in Chen et al. [13-16]. Our approach to the turbulence closure problem began by substituting the Reynolds decomposition into the Lagrangian for

Hamilton's principle for ideal fluid motion. We also invoked Taylor's hypothesis [17] to relate the space and time derivatives of the fluctuating quantities. We then averaged the fluid Lagrangian over the short time scale of the small rapid fluctuations, before taking variations of resulting mean quantities to obtain the equations of motion for these mean quantities as Euler-Poincaré equations. Because they arise from Hamilton's principle, our motion equations for these mean quantities are dynamically self consistent.

The mean variables in these equations are defined by taking averages over fast time in the Lagrangian, either at fixed position in space or at fixed fluid parcel label. The first type of average is called the Eulerian mean. The second type is called the Lagrangian mean. (The explicit form of the Taylor's hypothesis we employ in Hamilton's principle depends on whether we take the Eulerian mean or the Lagrangian mean.)

In summary, after understanding the mathematical structure of the first motivating example of WMFI, in the third year of this project we extended its methodology to derive new dynamical equations for incompressible turbulence in three dimensions. The steady solutions of this model compared well with experimental data for mean fluid velocity profiles in pipes and channels at high Reynolds numbers. We also interpreted the resulting equations as either a Large Eddy Simulation (LES) model, or equivalently a one-point turbulence closure model. Finally, we used the same Euler-Poincaré method to develop a new second-moment closure model for three-dimensional incompressible turbulence. This model acts like an adaptive LES model and gives a dynamical equation for how the Taylor diffusivity tensor responds to shear forcing. We also used this approach in formulating a new second-moment closure model of three-dimensional oceanic turbulence for the Laboratory's climate modeling efforts.

Publications:

1. Aceves, A., Holm, D. D., Kovacic, G. and Timofeyev, I., "Homoclinic Orbits and Chaos in a Second-Harmonic Generating Optical Cavity," Phys. Lett. A **233**, 203-208 (1997).

2. Alber, M. S., Camassa, R., Holm, D. D., and Marsden, J. E., "The geometry of new classes of weak billiard solutions of nonlinear PDE's," Submitted to Commun. Math. Phys. (1999).

3. Allen, J. S. and Holm, D. D., "Extended-geostrophic Hamiltonian models for rotating shallow water motion," Physica D, **98**, 229-248 (1996).

4. Allen, J. S., Holm, D. D. and Gent, P. R., "A note on Kelvin waves in balance models," J. Phys. Ocean. **27**, 2060-2063 (1997).

5. Allen, J.S., Holm, D. D., and Newberger, P.A., "Toward an extended-geostrophic model Euler--Poincaré model for mesoscale oceanographic flow," in Proceedings of the Isaac Newton Institute Programme on the Mathematics of Atmospheric and Ocean Dynamics, Cambridge University Press, to appear (1999).

6. Camassa, R., Holm, D. D. and Levermore, C.D., "Long-time effects of bottom topography in shallow water," Physica D, **98**, 258-286 (1996).

7. Camassa, R., Holm, D. D. and Levermore, C.D., "Long-time shallow-water equations with a varying bottom," J. Fluid Mech., **349**, 173-189 (1997).

8. Cendra, H., Holm, D. D., Hoyle, M. J. W. and Marsden, J. E., "The Maxwell-Vlasov equations in Euler-Poincaré form," J. Math. Phys., **39**, 3138-3157 (1998).

9. Cendra, H., Holm, D. D., Hoyle, M. J. W. and Marsden, J. E., "Lagrangian Reduction, the Euler-Poincaré equations, and semidirect products," in Arnol'd Festschrift Volume II, Am. Math. Soc. Translations Series 2, **186**, (1999).

10. Chen, S., Foias, C., Holm, D. D., Olson, E.J., Titi, E.S. and Wynne, S., "The Camassa-Holm equations as a closure model for turbulent channel and pipe flows," Phys. Rev. Lett., **81**, 5338-5341 (1998).

11. Chen, S., Foias, C., Holm, D. D., Olson, E.J., Titi, E.S. and Wynne, S., "A connection between the Camassa-Holm equations and turbulent flows in pipes and channels," Phys. Fluids, to appear, (1999).

12. Chen, S., Foias, C., Holm, D. D., Olson, E.J., Titi, E.S. and Wynne, S., "The Camassa-Holm equations and turbulence," Physica D, to appear, (1999).

13. Chen, S., Holm, D. D., Margolin, L. G., and Zhang, R., "Direct numerical simulations of the Navier-Stokes alpha model," Physica D, to appear, (1999).

14. Duan, J., Holm, D. D., and Li, K., "Variational methods and nonlinear quasigeostrophic waves," Phys. Fluids, to appear (1999).

15. Fabijonas, B., Holm, D. D. and Lifschitz, A. "Secondary instabilities of flows with elliptic streamlines," Phys. Rev. Lett., **78**, 1900-1903 (1997).

16. Gjaja, I. and Holm, D. D., "Self-consistent wave-mean flow interaction dynamics and its Hamiltonian formulation for a rotating stratified incompressible fluid," Physica D, **98**, 343-378 (1996).

17. Holm, D. D., "Hamiltonian balance equations," Physica D, **98**, 379-414 (1996).

18. Holm, D. D., "The ideal Craik-Leibovich equations," Physica D, **98**, 415-441 (1996).

19. Holm, D. D., "Fluctuation effects on 3D Lagrangian mean and Eulerian mean fluid motion," Physica D, to appear (1999).

20. Holm, D. D., and Fringer, O., "Integrable vs nonintegrable geodesic soliton behavior," Submitted to Physica D, January, 1999.

21. Holm, D. D., Kouranbaeva, S., Marsden, J. E., Ratiu, T. and Shkoller, S., "A nonlinear analysis of the averaged Euler equations," in Arnol'd Festschrift Volume II, Am. Math. Soc. Translations Series 2, **186**, (1999).

22. Holm, D. D., Marsden, J. E. and Ratiu, T. S., "The Euler--Poincaré equations and semidirect products with applications to continuum theories," Adv. in Math., **137**, 1-81 (1998).

23. Holm, D. D., Marsden, J. E. and Ratiu, T. S., "Euler--Poincaré models of ideal fluids with nonlinear dispersion," Phys. Rev. Lett., **80**, 4173-4177 (1998).

24. Holm, D. D., Marsden, J. E. and Ratiu, T. S., "The Euler--Poincaré equations in geophysical fluid dynamics," in Proceedings of the Isaac Newton Institute Programme on the Mathematics of Atmospheric and Ocean Dynamics, Cambridge University Press, to appear (1999).

25. Holm, D. D. and Zeitlin, V., "Hamilton's principle for quasigeostrophic motion," Phys. Fluids A, **10**, 800-806 (1998).

References

[1] Craik, A. D. D., and Leibovich, S., "A rational model for Langmuir circulations," J. Fluid Mech., **73**, 401-426 (1976).

[2] Holm, D. D., Marsden, J. E. and Ratiu, T. S., "Nonlinear Stability of Fluid and Plasma Equilibria," Physics Reports, **123**, 1--116 (1985).

[3] Charney, J. G., and Drazin, P.G., "Propagation of planetary-scale disturbances from the lower into the upper atmosphere," J. Geophys. Res., **66**, 83--109 (1961).

[4] Whitham, G. B., "A general approach to linear and nonlinear dispersive waves using a Lagrangian," J. Fluid Mech., **22**, 273--283 (1965).

[5] Whitham, G. B., "Two-timing, variational principles and waves," J. Fluid Mech., **44**, 373--395 (1970).

[6] Fabijonas, B., Holm, D. D. and Lifschitz, A. "Secondary instabilities of flows with elliptic streamlines," Phys. Rev. Lett., **78**, 1900-1903 (1997).

[7] Andrews, D. G., and McIntyre, M. E., "An exact theory of nonlinear waves on a Lagrangian-mean flow," J. Fluid Mech., 89, 609--646 (1978).

[8] Abarbanel, H. D. I., Holm, D. D., Marsden, J. E. and Ratiu, T. S., "Nonlinear stability analysis of stratified ideal fluid equilibria," Phil Trans. Roy. Soc. (London) A, **318**, 349--409 (1986).

[9] Poincaré, H., "Sur une forme nouvelle des équations de la méchanique," C.R. Acad. Sci., **132**, 369--371 (1901).

[10] Speziale, C. G., "Analytical methods for the development of Reynolds stress closures for turbulence," Ann. Rev. Fluid Mech., **23**, 107-158 (1991).

[11] Gjaja, I. and Holm, D. D., "Self-consistent wave-mean flow interaction dynamics and its Hamiltonian formulation for a rotating stratified incompressible fluid," Physica D, **98**, 343-378 (1996).

[12] Holm, D. D., Marsden, J. E. and Ratiu, T. S., "The Euler--Poincaré equations and semidirect products with applications to continuum theories," Adv. in Math., **137**, 1-81 (1998).

[13] Chen, S., Foias, C., Holm, D. D., Olson, E.J., Titi, E.S. and Wynne, S., "The Camassa-Holm equations as a closure model for turbulent channel and pipe flows," Phys. Rev. Lett., **81**, 5338-5341 (1998).

[14] Chen, S., Foias, C., Holm, D. D., Olson, E.J., Titi, E.S. and Wynne, S., "A connection between the Camassa-Holm equations and turbulent flows in pipes and channels," Phys. Fluids, to appear, (1999).

[15] Chen, S., Foias, C., Holm, D. D., Olson, E.J., Titi, E.S. and Wynne, S., "The Camassa-Holm equations and turbulence," Physica D, to appear, (1999).

[16] Chen, S., Holm, D. D., Margolin, L. G., and Zhang, R., "Direct numerical simulations of the Navier-Stokes alpha model," Physica D, to appear, (1999).

[17] Taylor, G. I., "Diffusion by continuous movements," Proc. London Math. Soc., **20**, 196-211 (1921).