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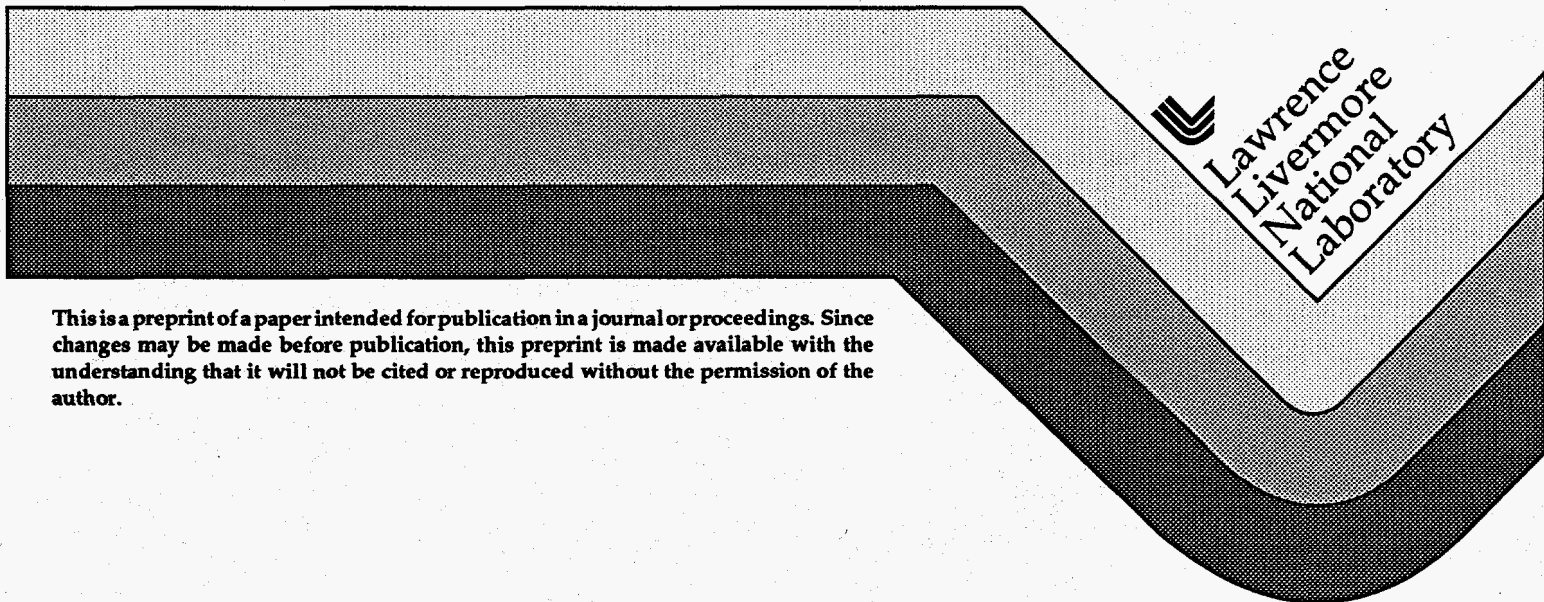
UCRL-JC-119862
PREPRINT

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B. N. Maker

This paper was prepared for submittal to the
Fifth Annual Conference on Numerical Methods
in Industrial Forming Processes
Ithaca, New York
June 18-21, 1995

January 25, 1995



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Rigid Bodies for Metal Forming Analysis with NIKE3D

Bradley N. Maker

Lawrence Livermore National Laboratory, Livermore, California, USA

ABSTRACT: Perhaps the most common approximation in engineering is that, relative to its neighbors, a system component is structurally rigid. This paper presents a development of the rigid assumption for use in nonlinear, implicit finite element codes. In this method, computational economy is gained by condensing the size of the associated linear system of equations, eliminating the processing of rigid elements, and reducing the overall nonlinearity of the problem.

1. INTRODUCTION

Finite element methods have traditionally been applied to problems where complexities in geometry or material behavior preclude closed form analytic solutions. The power of the finite element approximation has allowed engineers to include considerable detail in their structural response calculations, potentially eliminating the need for simplifying assumptions. However as engineers investigate processes with nonlinear material and geometric behavior, computational resources are quickly exhausted, and the rigid assumption once again becomes valuable. An excellent example is in sheet metal forming, where the assumption of rigid tooling is very reasonable.

Finite elements may be rendered rigid in several ways. The simplest method is to apply very large elastic moduli to an otherwise deformable element. This "brute force" method requires no code modifications, yet has several shortcomings. Rigid elements still need to be processed during stress, strain, and stiffness evaluations. Nodal degrees of freedom associated with rigid elements remain

independently active. And large disparities in material properties lead to numerical errors.

This paper presents a rigid material implementation based on rigid body mechanics. Following the convention Benson and Hallquist [1986] used for explicit time integration, a group of finite elements may be defined to be a rigid body. The center of mass is computed, and six degrees of freedom are assigned to the body - three for translation and three for rotation. Nodal loads and boundary conditions are resolved onto the center of mass coordinates.

In this implicit implementation, motion of the rigid body is determined by solving a coupled set of equilibrium equations for the rigid body and any other deformable elements in the model. Finally, the current coordinates of rigid body nodes are updated using the center of mass coordinates and the rigid body assumption. The rotational motion is characterized using quaternion parameters, which also find application in computer graphics literature (Burger and Gillies [1989], Glassner [1990]).

This approach results in a method where stress, strain, and stiffness computations for rigid elements

Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract W-7405-Eng-48

may be skipped. Numerical errors from disparate material properties are avoided. And most importantly, the total number of independent degrees of freedom in the model is reduced. This allows very large numbers of elements to be included in rigid bodies without adding significant computing cost to the model, and allows existing models to be executed more quickly by invoking the rigid assumption where appropriate.

2. FORMULATION

The following presents an integration of rigid body mechanics with the updated Lagrangian finite element formulation using implicit time integration. The procedure is applied to the inner-most loop of the nonlinear equilibrium solver, during stiffness and force computation, and after the linear equation solving procedure.

2.1. Kinematics

The position vector \mathbf{x} of a finite element node point may be written as

$$\mathbf{x} = \mathbf{X} + \mathbf{u} \quad (2.1)$$

where \mathbf{X} is the initial coordinate of the point and \mathbf{u} is its displacement vector. If the point is associated with a rigid body, then

$$\mathbf{x} = \overset{cm}{\mathbf{X}} + \mathbf{a} \quad (2.2)$$

where $\overset{cm}{\mathbf{X}}$ is the current position of the center of mass, and \mathbf{a} is the current vector from the center of mass to the point. The vector \mathbf{a} may be written in terms of \mathbf{a}_0 , its value in the undeformed or reference state, and a rotation matrix Λ :

$$\mathbf{a} = \Lambda \mathbf{a}_0 \quad (2.3)$$

The incremental displacement relationships used in implicit finite element formulations are obtained by linearization of the above expressions:

$$\Delta \mathbf{u} = \Delta \overset{cm}{\mathbf{X}} + \Delta \Lambda \mathbf{a}_0 \quad (2.4)$$

Linearization of the rotation matrix Λ , presented by Simo [1988], results in expressions of a convenient form:

$$\Delta \Lambda \mathbf{a}_0 = \mathbf{R} \Delta \boldsymbol{\theta} = \begin{bmatrix} 0 & a_3 & -a_2 \\ -a_3 & 0 & a_1 \\ a_2 & -a_1 & 0 \end{bmatrix} \begin{Bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \Delta \theta_3 \end{Bmatrix} \quad (2.5)$$

where $\mathbf{a} = \{a_1, a_2, a_3\}^T$, and $\{\Delta \theta_1, \Delta \theta_2, \Delta \theta_3\}$ are the components of the rigid body rotation increment expressed in global coordinates.

For a model containing both deformable and rigid components, the nodal degrees of freedom may be grouped, and the above expressions used to obtain a condensed set of unknowns:

$$\begin{Bmatrix} \Delta \mathbf{u}^D \\ \Delta \mathbf{u}^R \end{Bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{R} \end{bmatrix} \begin{Bmatrix} \Delta \mathbf{u}^D \\ \Delta \overset{cm}{\mathbf{X}} \\ \Delta \boldsymbol{\theta} \end{Bmatrix} \Rightarrow \Delta \mathbf{u} = \mathbf{A} \Delta \hat{\mathbf{u}} \quad (2.6)$$

where the $(\hat{\cdot})$ superscript denotes a condensed degree of freedom vector. Substituting this expression into the discrete form of the principle of virtual work, we obtain expressions for the condensed finite element stiffness matrix and residual vector for the coupled deformable/rigid system:

$$\hat{\mathbf{K}} \Delta \hat{\mathbf{u}} = \hat{\mathbf{F}}, \quad \hat{\mathbf{K}} = \mathbf{A}^T \mathbf{K} \mathbf{A}, \quad \hat{\mathbf{F}} = \mathbf{A}^T \mathbf{F} \quad (2.7)$$

This condensed system is passed to a standard linear equation solver, returning rigid body translation and rotation increments $\Delta \overset{cm}{\mathbf{X}}$ and $\Delta \boldsymbol{\theta}$. These are used to update the center of mass, and finally the nodal coordinates.

2.2. Center of mass update

The update phase proceeds by first updating the center of mass degrees of freedom, and then the positions of each node point in the rigid body. The center of mass update consists of a translational and rotational part. The translational degrees of freedom are trivially updated:

$$\overset{cm}{\mathbf{X}}_{(n+1)} = \overset{cm}{\mathbf{X}}_{(n)} + \Delta \overset{cm}{\mathbf{X}} \quad (2.8)$$

We employ quaternion parameters to characterize the rotational configuration of the rigid body, allowing finite rotation increments in a single step without inducing deformation. These are updated as follows. First, the incremental quaternion parameters (q_0, \mathbf{q}) are computed from the rotation increments $\Delta\theta$:

$$q_0 = \cos \alpha \quad , \quad \mathbf{q} = \beta \Delta\theta \quad (2.9)$$

where

$$\alpha = \frac{1}{2}(\Delta\theta_1^2 + \Delta\theta_2^2 + \Delta\theta_3^2)^{\frac{1}{2}} \quad (2.10)$$

$$\beta = \begin{cases} \frac{1}{2\alpha} \sin \alpha & , \quad \alpha \geq 10^{-5} \\ \frac{1}{2} \left(1 - \frac{\alpha^2}{6} + \frac{\alpha^4}{120} - \frac{\alpha^6}{5040} \right) & , \quad \alpha < 10^{-5} \end{cases}$$

These are normalized according to

$$\gamma = (q_0^2 + q_1^2 + q_2^2 + q_3^2)^{\frac{1}{2}} \quad , \quad q_i = q_i / \gamma \quad (2.11)$$

The total quaternion parameters (x_0, \mathbf{x}) and (y_0, \mathbf{y}) characterize the total rotation of the rigid body from the undeformed or reference state to states (n) and $(n+1)$, respectively. The update is performed using quaternion algebra:

$$(y_0, \mathbf{y}) = (q_0, \mathbf{q}) \circ (x_0, \mathbf{x}) \quad (2.12)$$

where

$$y_0 = q_0 x_0 - \mathbf{q} \cdot \mathbf{x} \quad (2.13)$$

$$\mathbf{y} = q_0 \mathbf{x} + x_0 \mathbf{q} + \mathbf{q} \times \mathbf{x}$$

The total quaternions are then normalized using the aforementioned procedure. From these, the current rotation matrix Λ may be computed:

$$\Lambda = \begin{bmatrix} (y_0^2 + y_1^2 - \frac{1}{2}) & (y_1 y_2 - y_3 y_0) & (y_1 y_3 + y_2 y_0) \\ (y_2 y_1 + y_3 y_0) & (y_0^2 + y_2^2 - \frac{1}{2}) & (y_2 y_3 - y_1 y_0) \\ (y_3 y_1 - y_2 y_0) & (y_3 y_2 + y_1 y_0) & (y_0^2 + y_3^2 - \frac{1}{2}) \end{bmatrix} \quad (2.14)$$

2.3. Nodal update

For consistency with the updated Lagrangian finite element formulation, the displacement and rotation vectors of each rigid body node must be determined.

With the center of mass degrees of freedom updated, the nodal displacements may be computed using (2.2), (2.3), (2.8), and (2.14) simply as

$$\Delta \mathbf{u} = \mathbf{x}_{(n+1)} - \mathbf{x}_{(n)} \quad (2.15)$$

The nodal rotation increments are equal to the center of mass rotation increments. These are computed similarly by differencing the total rotation vectors $\theta_{(n+1)}$ and $\theta_{(n)}$. The rotation vectors are extracted from the total quaternions using:

$$\theta = \beta \mathbf{y} \quad (2.16)$$

where

$$\alpha = (y_1^2 + y_2^2 + y_3^2)^{\frac{1}{2}} \quad (2.17)$$

$$\beta = \begin{cases} \frac{2}{\alpha} \sin^{-1} \alpha & , \quad \alpha \geq 10^{-6} \\ 0 & , \quad \alpha < 10^{-6} \end{cases}$$

3. IMPLEMENTATION

The rigid body capability was implemented as a material attribute in the finite element code NIKE3D (Maker, Ferencz, and Hallquist [1991]). This proves convenient for users, since each part in a finite element model is typically assigned a unique material number as the model is constructed. Rigid bodies may be constructed from any of the element types available in NIKE3D - beams, shells, and/or solids.

The condensation procedure (2.7) is implemented at the element level. Processing of elements assigned rigid material types is skipped entirely. The element stiffness and internal force vector for all other elements are formed by the usual procedure. Next, deformable elements containing one or more rigid body nodes are passed to a condensation routine. This routine uses (2.5) and (2.6) to modify the element stiffness and internal force data prior to assembly.

Although the condensation procedure reduces the number of global unknowns in virtually every model, the element level matrices may in some cases be expanded. This occurs, for example, when

a single node from a trilinear continuum element is associated with a rigid body. The three (translational) degrees of freedom of that node are then replaced by the six (translational and rotational) degrees of freedom of the rigid body, increasing the dimension of the element matrices by three. However, if several of the element nodes belong to the same rigid body, the net size of the element system is indeed reduced. In general, provision must be made for assembly of a variable sized element system. In the worst case of an eight node hexahedral element where each node is attached to a different rigid body, the element system expands from 24 to 48 degrees of freedom.

All of the boundary conditions in NIKE3D are available for use with rigid bodies. These include prescribed displacement, force, pressure, and body forces. In the case of prescribed displacements, the user must prescribe the motion of the center of mass of the rigid body. Nodal displacement boundary conditions are ignored. Slide surface or contact boundary conditions are also available for use with rigid bodies. Penalty parameters for these interfaces are chosen using reasonable values of elastic moduli which are entered as rigid body material properties.

4. APPLICATIONS

The following examples demonstrate the validity of the implementation, and some of the potential cost savings from the rigid assumption.

4.1 *Rotation of rigid block*

To test the accuracy of the rotational update, a simple block of solid elements was rotated a total of 360 degrees. Several simulations were performed, using from one to ten steps to complete the rotation. Figure 1 shows the model in several intermediate configurations. Deformation induced by errors in the update was measured by comparing initial and final geometry of the block. In all cases, normalized error was less than 10^{-6} .

4.2 *Elastic / rigid beam*

A simple cantilevered beam loaded with a tip moment is shown in Figure 2. The beam is discretized using ten elements. The four center elements are defined as rigid, while the remaining elements have elastic properties. Load is applied in ten increments. The beam centerline is shown for several stages during the loading. The rigidity of the center section of the beam through finite translation and rotation is apparent.

4.3 *Hydroformed sheet metal cover*

A hydroforming simulation first presented in [Maker, 1988] was repeated using the rigid material. Figure 3 shows the model used to form an aluminum sheet into a three dimensional pan. In the hydroform process, the male tooling emerges from an initially flat blank holder surface, while subsequent external pressure drives the sheet to conform to the tooling contour. The tooling and blank holder were modeled using one layer of solid eight node elements.

Table 1 shows the cost breakdown associated with two NIKE3D simulations: one with deformable tooling, the other with rigid tooling. The size of the linear system in the deformable tooling analysis necessitated the use of NIKE3D's EBE iterative linear equation solver [Ferencz, 1989], which dramatically reduces storage requirements by operating at the element level. Condensing the linear system using the rigid material assumption allowed the use of a traditional direct solver.

Applying the rigid material decreased the total simulation time by over 90%, from 500 down to 37 cpu minutes on our Cray Y/MP machine. A large portion of this savings was realized by skipping the processing of the solid elements which comprise the tooling, saving nearly 250 cpu minutes. The decreased size of the condensed linear system accounted for a savings of nearly 200 cpu minutes. In addition, the rigid tooling simplified enforcement of the contact constraint between the sheet and tooling, since only one side of this interface was deformable. This resulted in fewer iterations in the

nonlinear equation solver while searching for equilibrium during each load step.

Model Characteristics:

sheet: 1650 shell elements
 tooling: 1550 brick elements

	<i>deformable</i>	<i>rigid</i>
Linear System:		
equations	16000	10000
storage (Mword)	17.0	3.2
(EBE)	6.6	
CPU Cost Breakdown		
initialization	0.5%	2.0%
slide surfaces	1%	53%
brick elements	47%	0%
shell elements	12%	21%
linear solver (EBE)	40%	21%
 Cray Y/MP cost	 500 min.	 37 min.

Table 1: Cost of hydroforming simulation using deformable or rigid tooling. The EBE iterative linear equation solver was required with the deformable tooling model to fit the linear system into available core memory. The tooling (brick elements) consumed nearly half of the cost in the first model, and virtually none when the rigid material was used.

5. CONCLUSIONS AND FUTURE WORK

The rigid body capability has been implemented in NIKE3D for static analyses, and validated on a number of test problems. Significant cost savings have been obtained by reducing the number of

degrees of freedom in the linear system, eliminating element processing for rigid elements, and by reducing the complexity of the contact problem. Additional work is required to extend the capability to dynamic problems, namely the solution of an additional (rotational) equilibrium equation, and the resulting (non-symmetric) set of linear equations.

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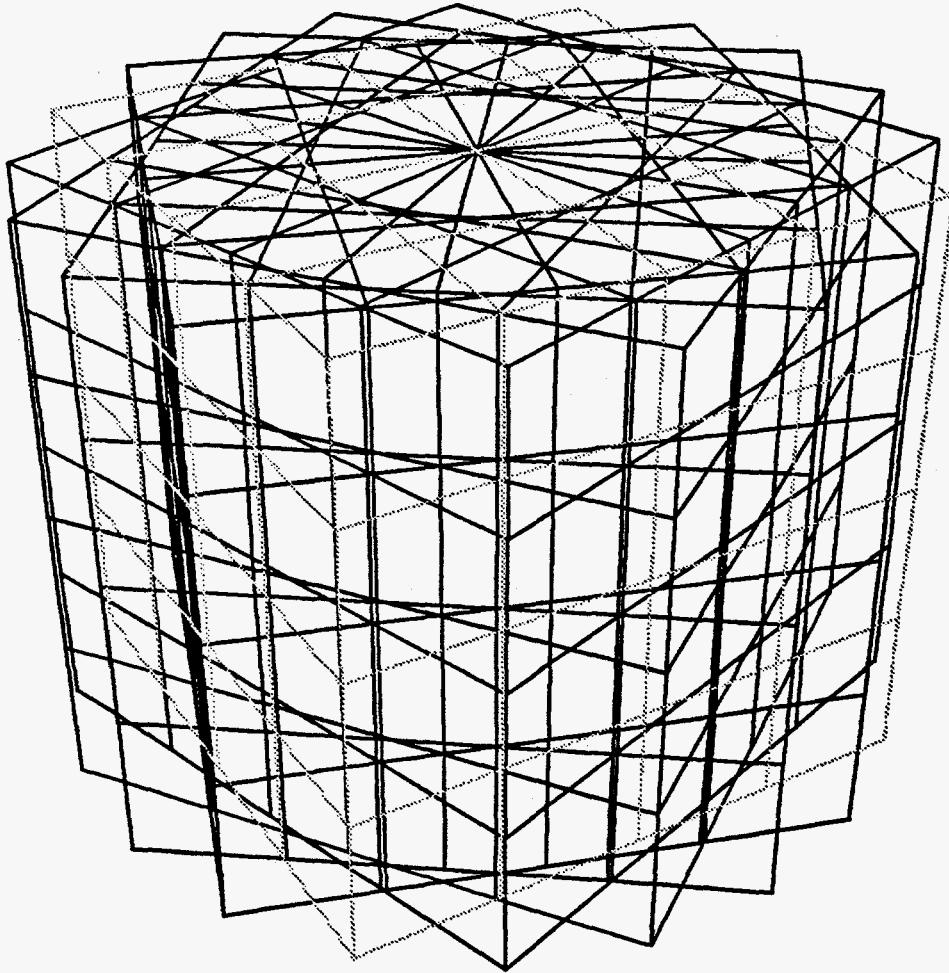


Figure 1 - In a test of the rigid body rotational update, a rigid block of elements is rotated 360 degrees using from one to ten steps. In all cases, normalized deformation of the body was less than 10^{-6} .

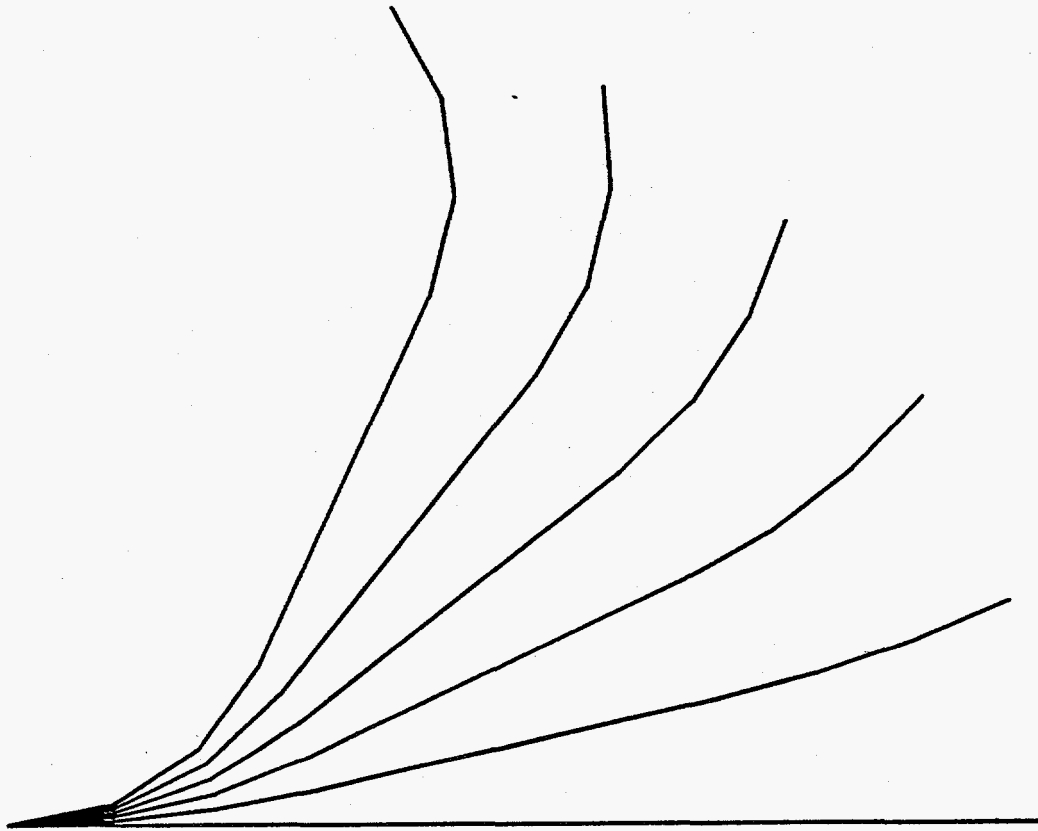


Figure 2 - The centerline of an elastic cantilevered beam loaded with an end moment is shown at several stages of deformation. The beam is discretized with ten elements, the central four elements being rigid. Load is applied in ten increments. The rigid behavior of the central portion of the beam through finite rigid body translation and rotation is demonstrated.

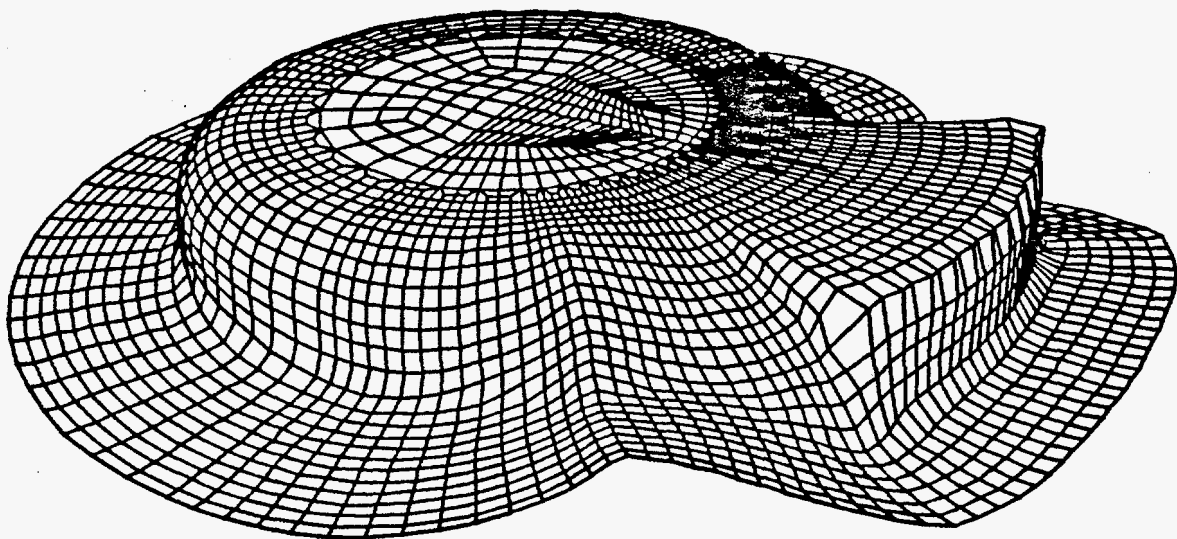
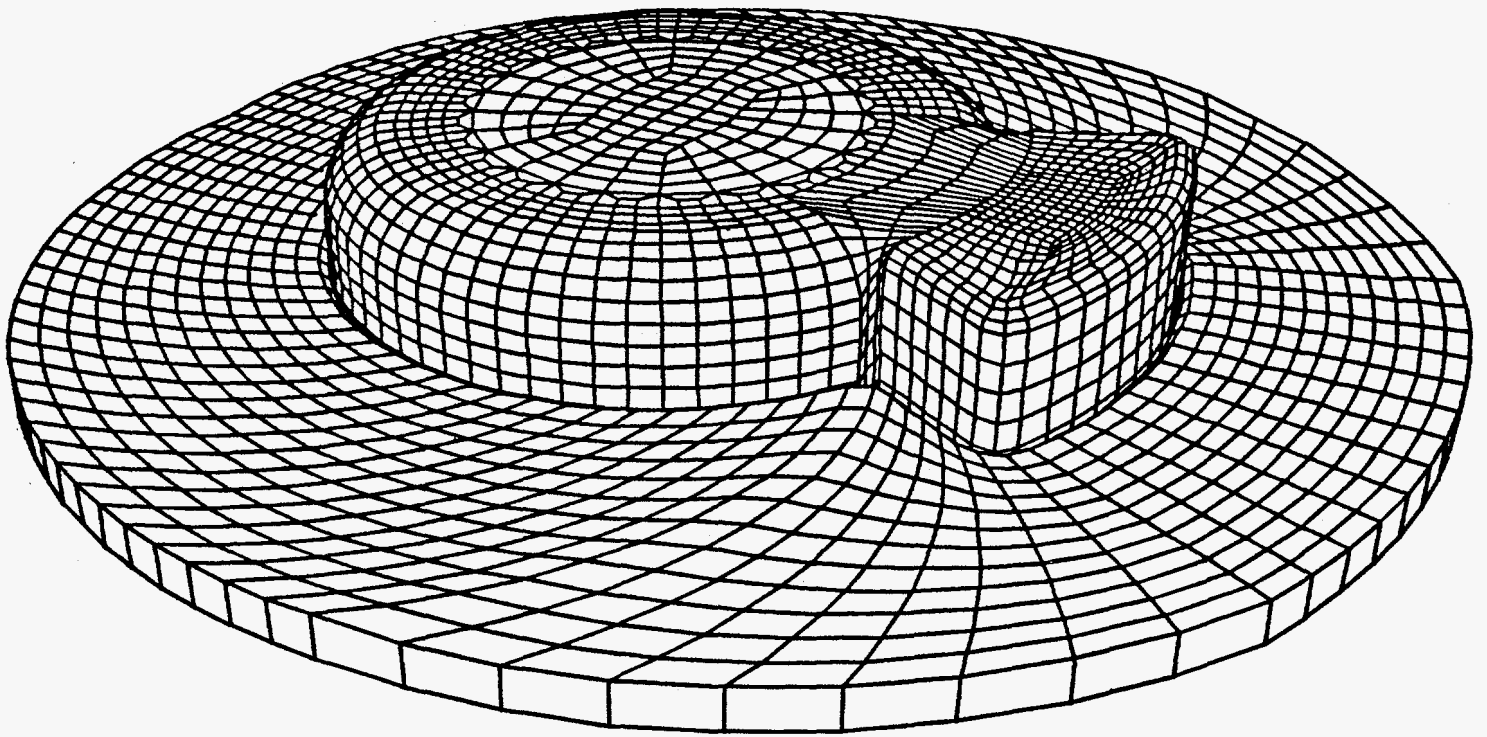


Figure 3 - Results from a NIKE3D hydroforming simulation show the tooling (top) and finished part (bottom). The tooling model is constructed using a single layer of eight node solid elements, making it an ideal candidate for the rigid material, which reduced runtime by over 90 percent.