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CONTROL MECHANISMS FOR A NONLINEAR MODEL OF INTERNATIONAL RELATIONS*

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Abstract

Some issues of control in complex dynamical systems are considered. We discuss two control mechanisms, namely: a short range, reactive control based on the chaos control idea and a long-term strategic control based on an optimal control algorithm. We apply these control ideas to simple examples in a discrete nonlinear model of a multi-nation arms race.

1 Introduction

The field of nonlinear dynamics offers the possibility to describe high dimensional complex systems - which hitherto have only been modeled stochastically or heuristically - in terms of nonlinear low dimensional deterministic models. The appeal of such models stems from two apparently opposing features. Indeed, on one hand the models - described by a system of either coupled maps (discrete evolution) or ordinary differential equations (continuous evolution) - are simple enough to warrant a rigorous and extensive analysis while on the other hand they display an extremely rich dynamical behavior that captures some of the complexity of the systems they were supposed to model. Due to these features, low dimensional nonlinear models have been extensively applied to ecologic, neuromorphic, societal, and military systems. Recently the focus of these applications has shifted from analysis and exhaustive exploration of the phase and parameter spaces to *control and prediction* [15].

In this paper we discuss the application of two conceptually different control mechanisms to simple discrete dynamical systems of Richardson type used to model arms races. [8, 13]. A short description of these models will be given in Section 2. In Section 3 we present our first control strategy to arms races and international relations, namely a *short range, reactive control* designed to *react* to changes in international relations based on immediate feedback utilizing the natural dynamics of the system. In this case policy changes are typically planned only one (time)step ahead. At each time step control is revised based on some feedback that reflects the actual state of the system and does not take into account any long-term strategic thinking. The main advantage of this *short term crisis management* is that typically it requires only very small changes in the system parameters. Indeed, due to the inherently nonlinear and potentially unstable character of the dynamics, this control is based on a chaos control algorithm [12, 13, 14] and it is used to stabilize unstable periodic orbits or fixed points of the dynamics by applying only small perturbations to (one of) the system parameters. The basic strategy is to perturb the system in such way to drive it to the unstable manifold of the desired orbit. On this manifold the natural dynamics of the system will bring the trajectory closer and closer to the desired fixed point or limit cycle.

The second procedure, called *long range strategic control* reflects a longer term planning of policy changes that are implemented consistently throughout the whole evolution. The long term strategic control is achieved by an optimal control algorithm [6] that we briefly discuss in Section 4. In the optimal control framework the perturbations (controls) are evaluated in order to minimize or maximize some objective functional that depends on the state and the controls. The actual form of this functional essentially depends on the goal we want to achieve and explicitly includes the reward and cost of that goal. The main advantage of such planning is that the changes one has to implement are known over the whole time span of the system's evolution and provide an optimal solution (an optimal cost effectiveness is realized for attaining the desired goal). The basic disadvantage of this method is that modeling errors, inherent perturbations, and unforeseen factors will usually lead to a trajectory different from the planned one.

2 Modified Richardson Model

To illustrate the control algorithms we choose a very simple example related to the socio-political field. Recently, Sulcoski and Miller have presented a modification of the Richardson model (MRM) describing an N-nation arms race, in the form of a discrete, nonlinear set of coupled dynamical equations [8]. This complex spatio-temporal system can be simply represented by a coupled map lattice (CML). Specifically, the model is given by

$$x_{i+1}^a = x_i^a + (x_{max}^a - x_i^a)\left[k_{aa}(x_{min}^a - x_i^a) + \sum_b k_{ab} x_i^b\right] \tag{1}$$

where x_i^a is the "total military capability potential" (TMCP) of country a at time i, x_{max}^a and x_{min}^a are maximum/minimum values for the TCMP sustainable by the respective country a, and k_{aa} and k_{ab} are coupling constants, representing internal threats and external alliances, respectively. The internal instability index $I_a = 2k_{aa}/(x_{max}^a - x_{min}^a)$ determines whether country a behaves like a democratic ($I_a > 0$) or totalitarian ($I_a < 0$) nation [8]. Another interpretation classifies these as non-militaristic and militaristic, respectively. For particular parameter values, iteration of this mapping produces a 2D landscape (1 spatial, 1 time) whose topology describes the qualitative dynamics of the political situations of the N countries. Being globally coupled and nonlinear, this model has the potential for rather rich behavior, and so far they have identified stationary, periodic, and chaotic parameter regimes. The relevance of this model, as for the original Richardson model, is that it may have solutions which yield considerable insight into the possible dynamical regimes for the current multi-sphere world order, allowing qualitative analysis and prediction.

Case A) A four-nation model, consisting of a weak democratic nation (1), a weak totalitarian nation (2), a stronger totalitarian nation (3), and a strong democratic nation (4). These countries form two alliances, between the totalitarian and democratic nations, respectively, which are relatively strong. The parameter values are: $k_{11} = 1.0$, $k_{22} = -0.1$, $k_{33} = -0.3$, $k_{44} = 2.0$, $k_{12} = k_{21} = 0.1$, $k_{13} = k_{31} = 3.0$, $k_{14} = k_{41} = -0.1$, $k_{23} =$

$k_{32} = -0.1$, $k_{24} = k_{42} = 0.1$, $k_{34} = k_{43} = 0.1$, $x^1_{max} = 0.7$, $x^2_{max} = 1.5$, $x^3_{max} = 0.7$, $x^4_{max} = 2.0$, $x^1_{min} = 0.2$, $x^2_{min} = 0.5$, $x^3_{min} = 0.3$, $x^4_{min} = 0.5$. As initial conditions (armament levels) we choose $x^1_0 = 0.4$, $x^2_0 = 1.4$, $x^3_0 = 0.6$ and $x^4_0 = 1.6$. Figure 1(a) shows the resulting time evolutions of this system which indicate exponentially growing behavior (which is artificially truncated after some time for illustrative purposes).

Case B) A three-nation model, with the following parameter values: $k_{11} = 0.1$, $k_{22} = -0.9$, $k_{33} = 0.3$, $k_{12} = k_{21} = 0.4$, $k_{23} = k_{32} = 0.7$, $k_{13} = k_{31} = 0.4$, $x^1_{max} = 0.6$, $x^2_{max} = 1.5$, $x^3_{max} = 0.7$, $x^1_{min} = 0.2$, $x^2_{min} = 0.3$, $x^3_{min} = 0.5$. As initial conditions (armament levels) we choose $x^1_0 = 0.3$, $x^2_0 = 1.1$, and $x^3_0 = 0.4$. This corresponds to a system with two weak democratic nations (1 and 3) and a stronger totalitarian nation (2) which is destabilized by an internal security threat. The resulting complex time evolution of this system is shown in Fig. 2(a), which indicates chaotic evolution of the totalitarian nation's TMCP.

In this Section, we presented two examples indicating that in many cases instabilities in the MRM lead to fast oscillations or chaotic evolution of the TMCP, that may be interpreted as a transition to war or insurgency [7] (c.f. Figs. 1(a),2(a)). The basic question we try to answer in the next Section is whether or not such instabilities arising at some stage of the dynamics of the interaction of the nations can be controlled, by actively changing the international or internal relations, and thus avoid the onset of "war". The question is difficult and here we only address some simple examples. We will examine the MRM dynamics and sketch some conclusions about controllability, as follows from the MRM (1).

For this discussion, we will assume that the model equations and most estimated coefficients are known, and the only parameters that can be effectively changed are those related with internal affairs (k_{aa}) and international relations (k_{ab}). Interpreted in the context of international relations, we assume that a given government can attempt to strengthen or weaken its military potential by changing the internal relations within its own country, or by carefully tailored international alliances with any of the other countries. The other parameters of Eq. (1), the maximum and minimum sustainable capability x^a_{max} and x^a_{min}, are assumed difficult to change on short time-scales as they reflect economic structure, culture, demographics, technological development, governmental structure, etc.

3 Short Range Reactive Control

Recently nonlinear dynamics has provided us with a series of methods designed to control unstable saddle points of complex systems, with only *small* changes in one or a few of the experimentally accessible parameters. These methods assume an active monitoring of the system and evaluation of the new control parameters at each time step based on the actual state of the system (active feedback, "closed-loop control"). Although here we assume perfect knowledge of the governing equations, in practical cases this is not necessary, as most of the information needed to control the system can be extracted by observing the evolution of the system and the effect of parameter variations (c.f. Ref. [12, 13]). We have to emphasize that such an approach to stabilize international relations may not be regarded as a long-term solution, but rather as a short term *crisis management.*

Here we will consider the method of Ott, Grebogi and Yorke [12], which is one of the most widely accepted chaos control algorithms, and which has already been applied to a variety of physical systems, such as magneto-elastic ribbon, chaotic laser, semiconductor devices, chemical reactions etc. The method applies only small perturbations to drive the system to the curve in the phase space along which the saddle point can be exactly reached, called the stable manifold. Thus it takes advantage of the naturally attractive dynamics along the stable manifold. The reader is referred to Ref. [12, 13] for a detailed description of the method as well as to Ref. [14] for an extension to higher dimensional systems.

Our main conclusions about controllability can be summarized as follows: a) although in the mathematical model the control can be sustained for arbitrary long time with only small perturbations, in practical cases this procedure may fail after some time as the model can become invalid; b) with the exception a few special cases it is not possible to control nations at their maximum capability potential; c) when the nations are strongly coupled and have nontrivial fixed points one has to employ a higher dimensional control algorithm, and the controllability condition should be also tested in a higher dimensional framework as presented in Ref. [14].

This control crisis management is most effective when at least two of the nations have saddle points that differ from their maximum sustainable potential. Such an example is provided by Case A. This model results in a fixed point at $\mathbf{r}^* = (0.6422, 1.5, 0.7, 0.5779)$. This shows the presence of two strongly democratic nations, one of them at the edge of internal stability ($I_4 = 1.5$), (similar behavior is observed at slightly lower I_4 values).

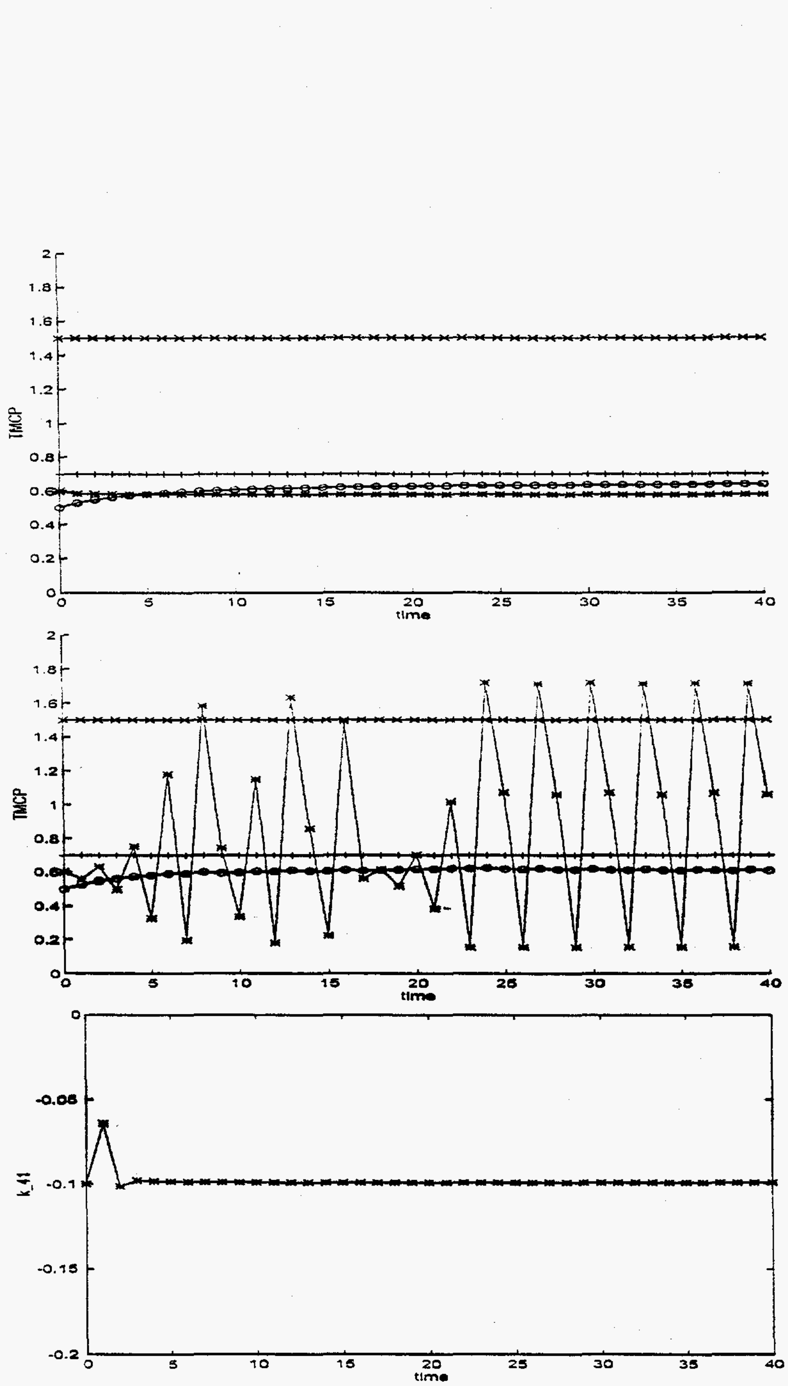

Figure 1: Time evolution of the TMCP for the uncontrolled (a) and controlled (b) case. Here the short range reactive control have been employed. Fig. (c) shows the time evolution of the control parameter. Note that the except the first time step the applied perturbation is very small.

The strongly democratic nation has a large maximum sustainable potential ($x^4_{max} = 2.0$) but has a low armament level. Figure 1(a) shows the time evolution with initial TMCP values $x^1_0 = 0.5$, $x^2_0 = 1.5$, $x^3_0 = 0.7$ and $x^4_0 = 0.6$. One can observe that while the weak democratic and the totalitarian nations quickly reach a steady TMCP level, the strong democratic nation's TMCP oscillates periodically after some chaotic transients. Our strategy is now to actively change one of the coupling coefficients k_{41} between the two democratic nations. Figure 1(b) shows the time evolution during the control procedure and Fig. 1(c) the evolution of the corresponding coupling parameter k_{41}.

4 Long Term Strategic Control

This approach is based on the optimal control method applied to discrete systems [6]. The optimal control method can be simply summarized as follows. The system is described by a scalar or vector *state function* depending on a number of independent variables that take values in the *phase space* of the system. The state satisfies a (usually nonlinear) dynamical scalar or vector equation that depends also on some parameters that take values in the parameter space. One or more of these parameters are considered external controls to be adjusted at will. The goal is to optimize a given objective functional that depends on the state and on the control(s). From the state equation and objective functional one constructs, in a canonical way, a non-homogeneous *adjoint equation* for a (scalar or vector) adjoint variable. The original state equation together with the adjoint equation form the *optimality system* (OS). The optimality condition yields an explicit, analytical formula for the optimal control in terms of the solutions of the OS. By replacing this expression of the control in the OS and solving it, one obtains the optimal state and adjoint variable and therefore the optimal control. The general framework for optimal control for general nonlinear systems as developed by J.-L. Lions [9] was recently applied to competitive systems of social and military interest [11, 10].

As an example for this control strategy we chose the three-nation model of Case B described in Section 2. The overall goal is to minimize the oscillations of the totalitarian nation's TMCP by changing his internal stability coefficient (k_{22}). Also we want to achieve this goal at an optimal cost, and by simultaneously keeping the the democratic nations stable. The corre-

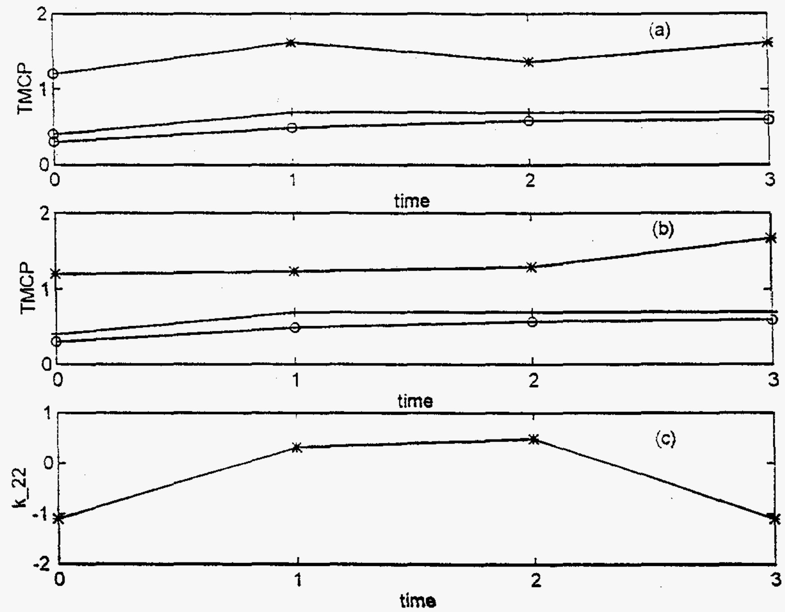

Figure 2: Time evolution of the TMCP for the uncontrolled (a) and controlled (b) case. The long range strategic control is applied to minimize the oscillations of the totalitarian nation's TMCP (*) at a minimal cost. The parameters used are $a_1 = 0.2, a_2 = 1.0, a_3 = 0.2$ and $\gamma = 0.01$. Fig. (c) shows the time evolution of the control parameter.

sponding choice of the cost function is

$$G = \sum_{i=1}^{N} \left[a_1(x_i^1 - x_{i-1}^1)^2 + a_2(x_i^2 - x_{i-1}^2)^2 + a_3(x_i^3 - x_{i-1}^3)^2 - \gamma(k_{22,i} - k_{22})^2\right] \tag{2}$$

where a_1, a_2, a_3 and γ are positive constants weighting the relative importance of the "goals" and the "cost", respectively. In this example we choose a $N = 3$ time step planning, an correspondingly the control will be also projected three time steps ahead reflecting a longer term strategical objective. Fig. 2(a) shows the uncontrolled trajectory, while Fig. 2(b,c) the controlled version and the variation of the control parameter, respectively. According to the choice of the cost function the oscillations in the totalitarian nation's TMCP have been minimized at an optimal cost.

5 Conclusions

In principle, each of the above control methods can be used to achieve either stabilization or destabilization of the dynamics of MRM. The choice between the two methods may not be merely a political decision. In some cases one or another method may be more efficient, or one of these methods may not be suitable at all. One simple example is the problem of destabilizing stable fixed points of the dynamics. If the fixed point is elliptic, the short range reactive control does not work because small perturbations applied to the system are unable to perturb the trajectory from the elliptic point's neighborhood. In general, short range reactive control is better suited when the model of the system is incomplete and cannot take into account all relevant factors. Hence it is a good choice when one does not have a well-defined overall goal or strategy and in unforeseen crisis situations. The long range control is better suited when one has a good idea about global strategic objectives, the costs involved and, one wants to achieve a well-defined goal. Since the role of the user is essential in deciding what control strategy to use we see here an excellent example of applied semiotic analysis to international relations.

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References

[1] Kadtke, J. B., "Classification of highly noisy signals using global dynamical models." Phys. Lett. A **203** (1995): 196-202.

[2] Crutchfield, J., and McNamara, B., "Equations of motion from a data series." Complex Systems **1** (1987): 417-452.

[3] Cremers, J., and Hubler, A., "Construction of differential equations from experimental data." Z. Naturforsh. **42A** (1987): 797-802.

[4] Kadtke, J. B., and Kremliovsky, M., "Estimating Statistics for Detecting Determinism Using Global Dynamical Models", pre-print (1996).

[5] Richardson, L. F., *Arms and Insecurity*. Pittsburgh; Boxwood, 1960.

[6] Fan, L.-T., and Wang, C.-S., *The Discrete maximum Principle*. New York; John Wiley & Sons, 1964.

[7] Saperstein, A. M., "Chaos – a model for the outbreak of the war." Nature **309** (1984): 303-305.

[8] Miller, L. D., Sulcoski, M. F., and Farmer, B. A., "Discrete Richardson Model: A Paradigm for International Relations?", pre-print(1996).

[9] Lions, J.-L., *Optimal Control of Systems Governed by Partial Differential Equations*, New York; Springer Verlag, 1971.

[10] Lenhart, S., and Protopopescu, V., "Optimal Control for Competitive-Cooperative Systems: Modeling Flexible Coalitions in Tomorrow's Competitive World", paper presented at the 61st MORS Conference, Dayton, Ohio, June 22-24, 1993.

[11] Lenhart, S., Protopopescu, V., and Szpiro, A., A., "Optimal Control for Competing Coalitions", to appear in Nonlinear Analysis: Theory, methods, and Applications.

[12] Ott, E., Grebogi, C., and Yorke, J. A., "Controlling Chaos." Phys. Rev. Lett. **64** (1990): 1196-1199.

[13] J. Kadtke, M. Kremliovsky, and A. Pentek, "Dynamical Classification and Control of a Nonlinear Model of International Relations", Proceedings of the conference on Complex Systems, Las Cruces (1996).

[14] Pentek, A., Kadtke, J. B., and Toroczkai, Z., "Stabilizing chaotic vortex trajectories: an example of high-dimensional control." Phys. Lett. A in press (1996).

[15] Coombs, M., and Sulcoski, M., eds. *Proceedings of the 1996 International Workshop on Control mechanisms for Complex Systems: Issues of Measurement and Semiotic Analysis*, 8-12 December 1996, Las Cruces, New Mexico.