

LA-UR-97-1315

Approved for public release;  
distribution is unlimited.

CONF-970760--5

Title: On Predicting the Transition to  
Turbulence in Stably Stratified Fluids

Author(s): D.O.ReVelle

RECEIVED

JUL 14 1997

OSTI

Submitted to: 12th AMS Symposium on Boundary Layers  
and Turbulence, Vancouver, B.C., Canada,  
July 28 - August 1, 1997

MASTER

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Los Alamos  
NATIONAL LABORATORY

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by the University of California for the U.S. Department of Energy under contract W-7405-ENG-36. By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. The Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

**DISCLAIMER**

**Portions of this document may be illegible  
in electronic image products. Images are  
produced from the best available original  
document.**

## P6.7 ON PREDICTING THE TRANSITION TO TURBULENCE IN STABLY STRATIFIED FLUIDS

D.O. ReVelle

Earth and Environmental Sciences Division, Los Alamos National Laboratory  
Los Alamos, New Mexico 87545

### I. INTRODUCTION AND OVERVIEW

The development of turbulence in stratified fluids has historically been studied using the flux,  $Ri_f$ , and the gradient Richardson no.,  $Ri$ , whereas the simpler shear flow transition in homogeneous fluids has been studied using the Reynolds no. A complete dimensional analysis of the relevant linearized conservation equations in the Boussinesq approximation predicts that the physical processes in stably stratified boundary layers should depend on as many as five dimensionless parameters, namely, the Rayleigh no.,  $Ra$ , the Reynolds no.,  $Re$ , the Taylor no.,  $Ta$ , the Prandtl no.,  $Pr$ , and the Radiation no.,  $Rd$ . The Radiation no. is very similar to  $Pr$ , but includes thermal radiative transfer instead of molecular heat conduction.

### II. PRELIMINARY EVALUATION OF $Ri_b$

It is possible to directly evaluate the bulk (finite difference form) of the gradient Richardson no.,  $Ri_b$ , in a non-rotating, stratified fluid using an expression developed by Asai (1970) for convective flows or using a relation derived by Sutton (1977) for the case of stably stratified flows. The individual terms in both expressions for  $Ri_b$  for the non-rotating flow case (small Taylor no.) include only the Prandtl no., the Reynolds no. and the Rayleigh no., consistent with our dimensional analysis.

Using the critical values,  $Ra_c = 1.128 \cdot 10^4$  and  $Re_c = 125$  for a Prandtl no. of 0.714 (at 273.13 K) after correcting for the appropriate imposed boundary conditions, we have evaluated  $Ri_b$  for the onset of turbulence from a state of laminar flow using either the theoretical or experimental values for the individual terms in Sutton's relationship. This yields critical transition  $Ri_b$  values in the range from 0.51-1.00. These  $Ri_b$  values are in good agreement with the reevaluation of the classical result of Chandrasekhar (1961) by Miles (1986), who found a critical value,  $Ri_c$ , of unity. Our result also agrees with Townsend (1958), who found that the inclusion of thermal radiation significantly increased  $Ri_c$  above the small

amplitude, transitional semi-infinite fluid value, 0.25. Although interesting, our initial evaluation of  $Ri_b$  does not allow insight into the detailed dynamics of flow transition.

### III. STABLY STRATIFIED FLUID BEHAVIOR

#### A. MOTIVATION

In the case of the turbulence transition in the stably stratified atmosphere or ocean, the transitional  $Ri_b$  value can be used to predict the onset of "bursting" or breakdown of the turbulent boundary layer to laminar flow. Using a semi-empirical approach, ReVelle and Coulter (1995) and ReVelle, Nilsson and Kulmala (1997) carried out bursting analyses and explored transitional  $Ri_b$  values in the range from about 0.30-1.0, including the possibility of hysteresis effects using prescribed transitional  $Ri_b$  values for the laminar/turbulent/laminar flow reversals.

#### B. ANALYSIS OF TRANSITIONAL BEHAVIOR: REEVALUATION OF THE CRITICAL $Ri_b$

We can evaluate additional details of the flow transition process, since  $dRi/dt$  has been observed, e.g., by ReVelle (1993) to change sign during transition, i.e., the time series of  $Ri_b$  was observed to be oscillatory, resembling that of a frictionally damped, transient oscillation. Prior to the turbulence transition  $dRi_b/dt < 0$ , but it then switches sign after transition. Thus, using the fact that  $dRi_b/dt = 0$  is the limiting value between the flow behavior changes, we can show that the product of the Brunt-Vaisalla frequency,  $\omega_{BV}$ , and the depth of the boundary layer flow,  $\delta$ , divided by the wind speed at the top of the layer,  $V(\delta)$ , is a constant during transition. This constant is just the square root of  $Ri_b$ , indicating the constancy of a critical transitional  $Ri_b$  value,  $Ri_b|_c$ . Using the semi-empirical boundary layer model of ReVelle and Coulter (1995), we have evaluated the possible values of this constant during periods of nocturnal boundary layer flow where bursting was evident. Using this approach, we have determined the constant to be about 0.5 - 1.0 depending on the external flow forcing. As

expected, the square of the above range of values agrees very well with the range of  $Ri_b|_c$  values prescribed during testing of the model.

Finally, we have also evaluated this constant by direct calculations using the fundamental dimensionless parameters identified earlier for laminar boundary flows utilizing the displacement thickness as the operational definition of the boundary layer depth. We have summarized our method of calculation, below:

- 1) Solve the expressions for  $Ra = Ra_c$  and for  $Re = Re_c$  for  $\delta$ , equate them and solve for  $V(\delta)$ .
- 2) Solve these same expressions again for the kinematic viscosity ( $\mu/\rho$ ), equate them and solve for  $\delta$ .
- 3) Having computed  $\delta$  and  $V(\delta)$ , compute  $\omega_{BV}$  and  $Ri_b|_c$  (where  $V(\delta) \approx \omega_{BV} \cdot \delta$ ).

The results of these temperature dependent calculations (due to the molecular shear viscosity and thermal conductivity variations with temperature) are indicated in Table 1 below. The values of  $Ra_c$  and of  $Re_c$  noted above were used in this evaluation. In Table 1,  $\Delta T$  is the prescribed temperature difference across the layer. Thus, from these comparisons, it appears that the bursting process is a transition from a thick, turbulent surface boundary layer (depth of  $\approx 10$  m in middle latitudes for a roughness length  $> 0.1$  m and a geostrophic wind between 1.5 m/s and 3.5 m/s) to a thin, laminar boundary layer of depth generally  $< 1$  m. In addition, we have determined that  $Ri_b|_c$  varies slightly over the range of input  $\Delta T$  values, but that it generally has a value close to unity. For small  $\Delta T$ ,  $V(\delta)$  becomes very small and bursting is far less likely. Thus, in this region  $Ri_b|_c$  values are progressively less meaningful. Finally, as can readily be seen from Table 1, as the stability increases, the depth decreases as the predicted wind speed steadily increases and vice versa.

#### IV. SUMMARY AND CONCLUSIONS

We have attempted to evaluate details of the flow transition process. We first evaluated the critical  $Ri_b$  value using a simple dimensionless equation developed by earlier workers. This agreed in magnitude with earlier results, but did not provide any details on the transition process or on the boundary

layer depth, wind speed, etc. A second analysis was performed using a more detailed approach. This allowed a re-evaluation of the critical  $Ri_b$  value and also predicted explicit properties of the laminar boundary layer as well. It also predicted a critical  $Ri_b$  value whose magnitude agreed with earlier work, but which varied slightly with the prescribed  $\Delta T$  across the layer.

#### V. REFERENCES

- Asai, T., 1970, Stability of a Plane Parallel Flow with Variable Vertical Shear and Unstable Stratification, *J. Meteor. Soc. Japan*, **48**, 129-139.
- Chandrasekhar, S., 1961, *Hydrodynamic and Hydromagnetic Stability*, Oxford University Press, London.
- Miles, J., 1986, Richardson's Criterion for the Stability of Stratified Shear Flow, *Phys. Fluids*, **29**, 3470-3471.
- ReVelle, D.O. and R.L. Coulter, 1995, Bursting in the Near-surface Boundary Layer: Comparisons between Realistic Models and Observations, Preprints of the 11th AMS Symposium on Boundary Layers and Turbulence, Paper 17.1, 560-563, Charlotte, NC.
- ReVelle, D.O., E.D. Nilsson and M. Kulmala, 1997, Modeling of the Arctic Boundary Layer: Comparisons with Measurements from the Arctic Ocean Expedition 1996, Paper P6.6, 12th AMS Symposium on Boundary Layers and Turbulence, Vancouver, B.C., Canada.
- Sutton, O.G., *Micrometeorology*, 1977, Second Edition, Krieger Pub. Co., Malabar, Florida.
- Townsend, A.A., 1958, The Effects of Radiative Transfer on Turbulent Flow of a Stratified Fluid, *J. Fluid Mech.*, **4**, 361-375.

TABLE 1.

Simultaneous solution of the dimensionless fundamental parameters for the temperature dependent, laminar boundary layer properties. (for  $Pr = 0.714$  at  $T = 273.13$  K)

$\Delta T$ : K	$\delta$ : m	$V(\delta)$ : m/s	$\omega_{BV}$ : 1/s	$Ri_b _c$	Const
0.001	0.427	.0039	0.021	5.222	2.285
0.005	0.249	.0067	0.033	1.503	1.226
0.01	0.198	.0084	0.047	1.206	1.098
0.05	0.116	0.014	0.126	1.034	1.017
0.10	0.092	0.018	0.199	1.020	1.010
0.50	0.054	0.031	0.578	1.012	1.006
1.0	0.043	0.039	0.918	1.011	1.006