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Performance (Extended Abstract)

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Selection of Sites for Monitoring Network Performance (Extended Abstract)

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Introduction. We apply the ideas from optimal design theory to the very specific area of monitoring large computer networks. The behavior of these networks is so complex and uncertain that it is quite natural to use the statistical methods of experimental design developed for situations in which the random character of phenomena is a crucial component and systems are too complicated to be described by any reasonable deterministic model. We want to emphasize that only the first steps have been completed, and relatively simple underlying concepts about network functioning have been used. Our immediate goal is to initiate studies focused on developing efficient experimental design techniques which can be used by practitioners working with large networks operating and evolving in a random environment.

In most cases a computer network can be represented as a graph with a given and fixed number of nodes (vertices, or sites) and with edges (links, communication channels). Possible objectives of experiment(s) may include: delays on a given subset (subset of interest) of edges, processing times at a given subset of nodes, traveling times from one subset of nodes to another, et al. The existing software and hardware allow the measurement (see, for instance, Monk and Claffy (1996), Claffy (1996) of a large variety of network performance indicators, i.e., in general our "measurement" is a vector. Types of measurement strategies may be very different. For instance, a meter can be installed at any chosen node and measure input and output flows; a device is located at a host node, and a preselected set of nodes or edges can be monitored; a practitioner can cooperate with others (i.e., there are multiple host nodes) to monitor a network. Thus, if we have an opportunity to plan (design) experiments, we may look for the best subset

of nodes where devices must be allocated, find the most informative subset of nodes and edges to be monitored by a given host, select the most effective group of host nodes and match them with the set of nodes and edges to be monitored.

Our approach assumes that only the correlation structure of the network is known (either theoretically or empirically) and is essentially based on the theory of optimal experimental design for correlated observations (see Fedorov (1996), for the survey of main results and further references).

Main assumptions. Let there be S nodes $X = (x_1, \dots, x_s)$ of interest. At each of these nodes we can observe a few response variables, such as flow rates, delays in various types of processing, queue lengths, etc. To keep notations simple we consider only the univariate case. We admit the possibility of repeated observations. For instance, a selected node may be interrogated several times during a relatively short period. If the long term trends are neglected or properly eliminated then the following model may be applied

$$y_{jk}(x_i) = u_j(x_i) + \varepsilon_{jk}(x_i), \quad (1)$$

where $u_j(x_i)$ describes the i -th node at moment j , and $\varepsilon_{jk}(x_i)$ is the corresponding observational error, $k = 1, \dots, r_{ij}$. All components in (1) are assumed to be random variables. The first one, $u(x_i)$ describes the random behavior of the network, while the second one is related to observational errors or short time disturbances. The same characters are used both for random variables and their realizations. The latter ones are standardly marked by additional indices: i.e., $u(x_i)$ stands for the random variable, and $u_j(x_i)$ is its realization.

Let the vector $U = (u(x_1), \dots, u(x_S))^T$ describe the network performance, and $E_u(U) = U_0$, $Var_u(U) = E[(U - U_0)(U - U_0)^T] = K$, where the $S \times 1$ vector U_0 and the $S \times S$ covariance matrix K are given. The subscript u (or ε) means that expectation or variance is taken with respect to u (or ε). The obvious transform $U \rightarrow U - U_0$ zeroes the expectation of U and, therefore, in what follows we assume that $E_u(U) = 0$. The observational errors are assumed to have zero means and to be uncorrelated: $E_\varepsilon(\varepsilon_j(x_i)) \equiv 0$, $E_\varepsilon(\varepsilon_j(x_i)\varepsilon_{j'}(x'_i)) \equiv \sigma^2\delta_{ii'}\delta_{jj'}$. Introduction of σ^2 depending on x does not lead to any significant changes and is not considered here.

We assume that for all j moments the same nodes are interrogated or observed:

$$\xi_n = \{p_i, x_i\}_1^n, \quad p_i = r_i/N, \quad N = \sum_{i=1}^n r_i, \quad x_i \in X, \quad n \leq S.$$

The set ξ_n is called an experimental design in the standard statistical setting, and p_i is called weight of the node x_i . Let $K(\xi_n)$ be a submatrix of K , which corresponds to the nodes x_1, \dots, x_n ; $K(x, \xi_n)$ be a column vector of covariances between $u(x)$ and $u(x_1), \dots, u(x_n)$; the matrix $W(\xi_n)$ be diagonal with the elements $W_{ii} = N\sigma^{-2}p_i$. We also use the matrices $K(Z, \xi_n) = (K(x_1, \xi_n), \dots, K(x_q, \xi_n))$, where $x_1, \dots, x_q \in Z \subset X$, and $K(Z)$ is a submatrix of K corresponding to the nodes from Z . The set Z may be considered as a set of nodes where the response variable must be predicted.

Estimation and optimality criteria. Let $Y_j(\xi_n)$ be the vector of averaged observations

$$Y_j^T(\xi_n) = \left(\frac{1}{r_1} \sum_{k=1}^{r_1} y_{jk}(x_1), \dots, \frac{1}{r_n} \sum_{k=1}^{r_n} y_{jk}(x_n) \right). \quad (2)$$

The estimator

$$\hat{U}(Z) = K^T(Z, \xi_n) \left(K(\xi_n) + W^{-1}(\xi_n) \right)^{-1} Y_j(\xi_n) \quad (3)$$

minimizes the matrix of expected squared residuals

$$D(\xi_n, \tilde{U}(Z)) = E_{u, \epsilon} \left[\left(\tilde{U}(Z) - U(Z) \right) \left(\tilde{U}(Z) - U(Z) \right)^T \right]$$

among all linear estimators $\tilde{U}(Z)$ such that $E_{u, \epsilon} [\tilde{U}(Z) - U(Z)] = 0$.

Minimization must be understood in the sense of ordering of nonnegative definite matrices. From (1) and (3) we can derive that

$$D(\xi_n) = K(Z) - K^T(Z, \xi_n) \left(K(\xi_n) + W^{-1}(\xi_n) \right)^{-1} K(Z, \xi_n) \quad (4)$$

The objective of this study is to provide some methods which allow the minimization of some given functions of the matrix $D(\xi_n)$, for instance, $\text{tr}D(\xi_n)$, $\ln |D(\xi_n)|$, $\max_i D_{ii}(\xi_n)$, etc. See Cook and Fedorov (1995) for details about optimality criteria and further references. Thus, we have to consider the following optimization problem

$$\xi_n^* = \arg \min_{\xi_n} \Psi [D(\xi_n)], \quad (5)$$

where Ψ is a selected objective function.

Properties of optimal designs. Optimization problem (5) may be simplified both theoretically and numerically if we allow weights to be continuous, so that $0 \leq p_i \leq 1$, $\sum_{i=1}^n p_i = 1$, and make $n = S$. Zero weights may necessitate using the limit transition in (4). When $n = S$ and Z coincides with X

then from (4) and the identity $(A + B)^{-1} = A^{-1} - A^{-1}(A^{-1} + B^{-1})^{-1}A^{-1}$ it follows that

$$D(\xi_n) = \left(K^{-1} + W(\xi_n) \right)^{-1}. \quad (6)$$

Of course, the regularity of the matrix K is assumed in the latter formula. The subscript n will be skipped if it does not lead to ambiguity. Let us assume that the function $\Psi(D)$ is convex and has a directional derivative $\phi(\xi^*, \xi)$ for any $\bar{\xi} = (1 - \alpha)\xi^* + \alpha\xi$ and $0 \leq \alpha < 1$.

Then a necessary and sufficient condition for a design ξ^* to be optimal is fulfillment of the inequality

$$\phi(\xi^*, \xi) \geq 0, \quad (7)$$

where ξ is any other design. This result is widely used in experimental design theory (c.f. Cook and Fedorov (1995)).

Inequality (7) leads to constructive results only when some simple presentation of $\phi(\xi^*, \xi)$ exists. For instance, for the D-criterion (entropy criterion in the case of normally distributed u and ε):

$$\Psi(D) = \ln |D| \quad \text{and} \quad \phi(\xi^*, \xi) = \text{tr} D(\xi^*) (W(\xi^*) - W(\xi)). \quad (8)$$

Combining (7) and (8) we can get

Theorem 1 *A necessary and sufficient condition for ξ^* to be D-optimal is that*

$$\max_i D_{ii}(\xi^*) \leq \sum_{i=1}^S p_i^* D_{ii}(\xi^*),$$

and equality holds for all nodes with $p_i^ > 0$. A D-optimal design also minimizes the maximal variance of prediction:*

$$\xi^* = \arg \min_{\xi} \max_i D_{ii}(\xi).$$

In this theorem and in what follows \max_i means maximization over all points from X , i.e., $1 \leq i \leq S$.

On an intuitive level this theorem leads to a very transparent method of selecting of sites: we have to take measurements at nodes where the variance of prediction may be worst.

For the averaged variance of prediction

$$\Psi(D) = \text{tr} D \quad \text{and} \quad \phi(\xi^*, \xi) = \text{tr} (W(\xi^*) - W(\xi)) D^2(\xi^*),$$

and the following result holds.

Theorem 2 A necessary and sufficient condition for ξ^* to be optimal is that

$$\max_i \sum_{j=1}^S D_{ij}^2(\xi^*) \leq \sum_{j=1}^S p_j^* D_{ij}^2(\xi^*),$$

and the equality holds for all nodes with $p_i^* > 0$.

If the set of interest Z does not coincide with the whole network X then instead of directly solving (4) one may introduce matrix $q \times S \{A\}_{ij} = \delta_{ij}$, if $x_i \in Z$, and $\{A_{ik}\} \equiv 0$, otherwise, and minimize $\Psi(ADA^T)$. For instance, for the variance of prediction averaged over Z $\Psi(ADA^T) = \text{tr}ADA^T$, a necessary and sufficient condition for ξ^* to be optimal is that

$$\max_i \sum_{j \in I(Z)} D_{ij}^2(\xi^*) \leq \sum_{i=1}^S \sum_{j \in I(Z)} p_i^* D_{ij}^2(\xi^*),$$

where $I(Z)$ is the set of all indices corresponding nodes from Z , and the equality holds for all nodes with $p_i^* > 0$.

First order algorithms. The above theorems help to develop and analyze various first order algorithms for the construction of optimal designs. Large matrices are required for processing and calculating applications to computer networks. It is, therefore, especially important to use recursions which are computationally simple and stable. The most convenient in this sense are algorithms similar to the first order exchange type algorithms (see, for instance, Mitchell (1974)). Here we formulate the simplest version of that kind of algorithm for D-criterion.

Let the initial design ξ_0 be such that all weights $p_{0i} = b_i \alpha_0$, where b_i is an integer and $\sum_{i=1}^S p_{0i} = 1$.

- a) Given ξ and $D(\xi)$, find $a = \arg \max_i D_{ii}(\xi)$. Add the weight α_s to point x_a to construct ξ_s^+ and $D(\xi_s^+)$.
- b) Find $d = \arg \min_{i \in I_t} D_{ii}(\xi_t^+)$, where I_t is a set of all supporting points of ξ_t , i.e., points with nonzero weights. Delete α_t from the weight of point x_d to construct ξ_{t+1} .
- c) If $|D(\xi_{t+1})| / |D(\xi_t)| < 1 - \gamma$, where γ is a small positive number, put $\alpha_{t+1} = \alpha_t$ and go to (a). Otherwise $\alpha_{s+1} = \alpha_s / 2$ and then go to (a).

Computations may be stopped when α_s is sufficiently small. The following recursions

$$|D(\xi_t^+)| = \frac{|D(\xi_t)|}{1 + \zeta_t D_{aa}(\xi_t)}, \quad D(\xi_t^+) = D(\xi_t) - \frac{\zeta_t C^+(\xi_t)}{1 + \zeta_t D_{aa}(\xi_t)}, \quad (9)$$

where $C_{ij}^+(\xi_t) = D_{ai}(\xi_t) D_{aj}(\xi_t)$ and $\zeta_t = \sigma^{-2} N \alpha_t$, make computations simpler. The versions of (9) for the deleting procedure are obvious.