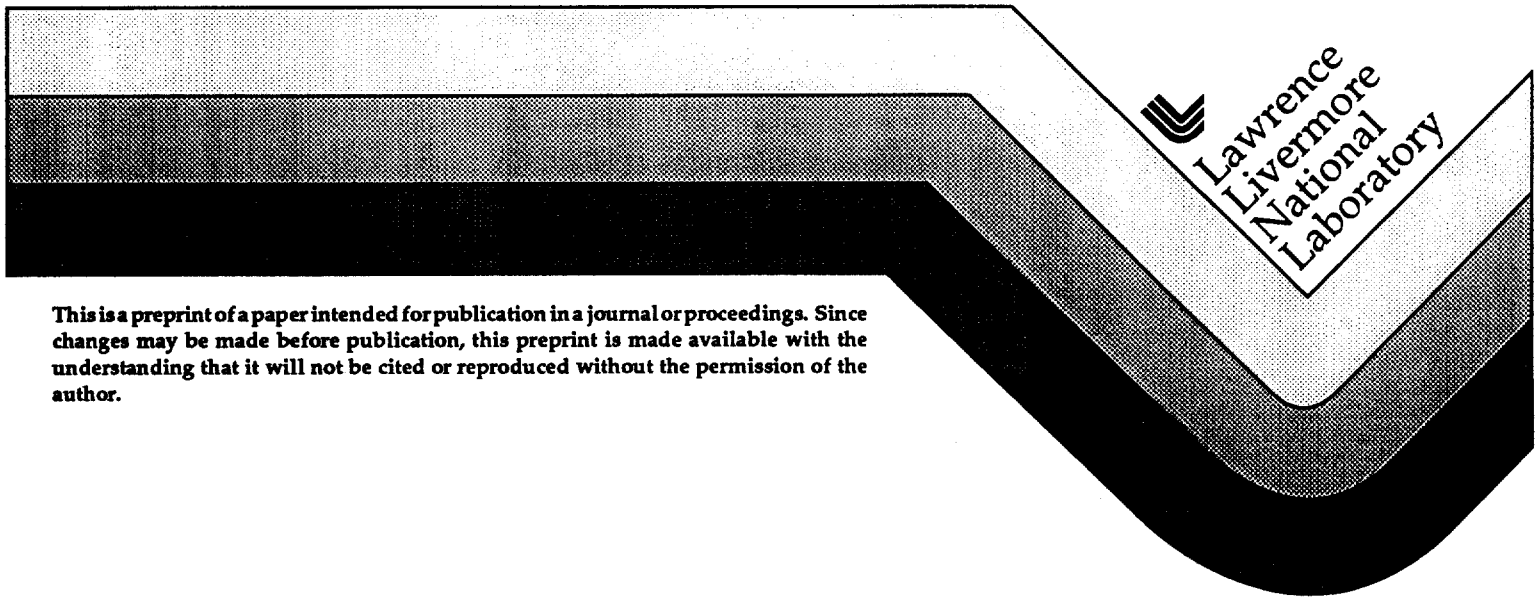
UCRL-JC-127496
PREPRINT

Dispersion Relationship for Solid State Instability Growth and Sensitivity to Equation of State

J. D. Colvin
L. G. Wiley
E. A. Chandler
B. A. Remington
D. H. Kalantar

This paper was prepared for submittal to the
6th International Workshop on the Physics of Compressible Turbulent Mixing
Marseille, France
June 18-21, 1997

June 12, 1997



Dispersion Relationship for Solid State Instability Growth and Sensitivity to Equation of State

J. D. Colvin, L. G. Wiley, E. A. Chandler, B.A. Remington, and D. H. Kalantar
Lawrence Livermore National Laboratory
Livermore, California, USA 94550

Derivation and Analysis of Dispersion Relationship

We have derived an approximate analytical dispersion relation for solid state instability growth following the method of Mikaelian.[1] He starts with the general eigenvalue equation for the velocity of a perturbation on a finite-thickness fluid layer with surface tension and viscosity, and derives an exact solution numerically from det(M)=0, where M is an 8x8 matrix. He then derives an approximate solution analytically by substituting the inviscid eigenfunctions into the exact eigenvalue equation. The integrations yield a dispersion relation which is a polynomial in the growth rate γ_s. Adapting the same method to a finite-thickness solid layer with shear strength G and elasto-plastic viscosity ν, we find:

$$\gamma_s^2 + 2k^2\nu\gamma_s + k\cdot\tanh(kh)\cdot(kG/\rho - Aa) = 0, \qquad (1)$$

where k is the wavenumber of the perturbation, h the layer thickness, ρ the layer density, A the Atwood number, and a the acceleration. The solution of this second-order equation for the growth rate is

$$\gamma_s = \nu k[(1 - (C/\nu^2k^3))^{1/2} - 1], \qquad (2)$$

where $C=k\cdot\tanh(kh)\cdot(kG/\rho - Aa)$.

It is easy to see that the perturbation is unstable and grows ($\gamma_s>0$) only for $k<k_c$ or wavelength of the perturbation $\lambda>\lambda_c=2\pi G/Aa\rho$. That is, the perturbation does not grow if the shear wave can transmit the restoring force of the lattice across the perturbation in less time $(1/k(G/\rho)^{1/2})$ than the characteristic growth time of the perturbation $(1/(kAa)^{1/2})$. Thus, the material shear strength limits the range of unstable wavelengths.

Equation (2) was evaluated for the conditions of the Nova laser experiments described by Kalantar et al.[2] In those experiments we create an x-ray drive inside a cylindrical gold hohlraum using about 22 kJ of energy in eight beams of the Nova laser at Lawrence Livermore National Laboratory. We developed a temporally shaped laser pulse (ps56) that produces a temporally shaped radiation drive pulse in a hohlraum that reaches a peak radiation temperature of 97 eV at 6.5 ns. This drive pulse provides a low-adiabat compression by a factor of about 1.5 to drive a metal foil to a peak pressure of about 3 Mbar while leaving it solid. Using this pulseshape we observed that perturbations with wavelengths of 20 μm and 50 μm on 15-μm-thick Cu and Mo foils do not begin to grow until after 10 ns, more than 4 ns later than for classical fluid growth, ostensibly a demonstration of material strength stabilization.

In order to evaluate equation (2) we calculated P(t), ρ(t), T(t), and a(t) for the Nova foils using the radiation-hydrodynamics code LASNEX[3] in 1-D and the measured radiation drive of ps56. This drive pulse compresses the Mo foil to a peak density of 15.8

gm cm^{-3} and a peak pressure of 3.1 Mbar at 7.0 ns. Both the Cu and the Mo foils remain solid at all times when driven by ps56, with the melt temperature computed from the Lindemann law. The plastic ablator/foil interface, on which the perturbations have been machined, begins to move at 3.8 ns when the first shock reaches it, and the interface has moved about 20 μm in 10 ns. In evaluating equation (2) we accounted for the increase in material shear strength with pressure (strength increases as the lattice potential energy increases), according to the material strength model of Steinberg et al.[4] When the pressure exceeds the yield strength, the material enters the plastic flow regime and the lattice keeps rearranging itself into a minimum energy state, i.e., the atoms "slide over" each other with a characteristic viscosity that depends on pressure P, ρ, and the strain rate dε/dt. The strain rate was calculated from its dependence on P, G, and the bulk modulus K, accounting for the pressure dependence of G and K. It is easy to show that, in compression, the plastic strain rate is always greater than the elastic strain rate.

The result of all these calculations is that the cutoff wavelength λ_c, which starts at a very high value when the foil first begins to accelerate, asymptotes to about 250 μm after 10 ns. Thus, the 50-μm and 20-μm perturbations of the Nova experiments should be stable at all times. At the wavelength of maximum growth, which is twice the cutoff wavelength or ~500 μm, growth is significantly suppressed in thin foils, as shown in Figure 1. This figure shows the perturbation growth at the wavelength of maximum growth as a function of time for Mo foils of thickness 15 μm and 150 μm driven by pulseshape 56. Figure 2 shows the dispersion curves for these two cases compared to the dispersion curve for classical growth.

Instability Analysis and Transition to Plasticity

The perturbation growth of the Nova experiments has also been simulated in 2-D with LASNEX and with another radiation-hydrodynamics code, both including the Steinberg et al. material strength model, and these simulations predict little difference from classical growth.[5] The codes, however, do not account for the rate dependence of the transition to plasticity. In the code simulations, wherever and whenever the stress exceeds the material strength (which at atmospheric pressure is 1.2 kbar for Cu and 16 kbar for Mo), the material is assumed to be instantaneously plastic, with a much lower shear strength G. The observed late-time growth in the experiments, therefore, is explained as the perturbation crossing an instability boundary, i.e., the material takes a few nanoseconds to transition to a plastic, when it has a much lower but non-zero shear strength, at the high strain rates of the Nova experiments (few x 10^7 s^{-1}). It is not known how the shear strength in actual metals decreases in the transition to plastic flow, so this is not accounted for in the dispersion curve analysis. In this analysis, instability growth depends only on perturbation wavelength, foil thickness, and material strength.

In recent work by Lebedev at al.[6] stability boundaries are derived that depend also on perturbation amplitude, but they do not derive a dispersion relation. Using their formulation, we calculate that for a 15-μm-thick Mo foil driven by ps56, the initial amplitude is larger than the threshold amplitude for instability after the shock emerges from the back of the foil, ~6.5 ns. Thus, the perturbation should become unstable after that time. Additionally, we calculate that the Cu foil melts at the interface after 10 ns if the ps56 "foot" temperature is too low, thus allowing the second shock to catch up to the first before it reaches the interface, effectively shocking the material too hard initially. The broad-band filter diode array that measures the radiation flux from which the drive temperature is inferred can determine only an upper limit to the foot temperature. Thus, the observation of

late-time growth in the Nova experiments is consistent with the dispersion curve analysis (which predicts no growth), and not with the 2-D code simulations (which predict near-classical growth) if the material melts at the interface after 10 ns because the foot drive is too low, and/or if the perturbation crosses an instability boundary (transitions to plasticity) after 10 ns. The near-term focus of the experimental campaign is to measure the foot temperature by measuring the time of first shock breakout with a new "visar" technique, and to determine the state of the material by a Bragg diffraction measurement.[2]

Sensitivity to Equation of State

Melt temperatures are generally a function of compression, increasing with increasing compression. Likewise, material pressure is dominated by cold lattice pressure, which is a function only of compression. Hence, the treatment of equation of state (EOS) is important in characterizing the material state. Three different EOS models were used in the LASNEX calculations: a tabular EOS (EOP) based on a Thomas-Fermi atomic model; the analytic Quotidien EOS[7] (QEOS); and a Gruneisen EOS for which we derived the internal energies from the pressure formulation of Steinberg et al.[4] The derivation of the Gruneisen EOS takes into account both the cold lattice energy and the ion thermal energy, but ignores the negligibly small electron thermal contribution. In the pressure-density regime of interest the Gruneisen EOS is softer than both EOP and QEOS. With the softer Gruneisen EOS Cu compresses to higher density and lower temperature, for the same drive pressure, than with EOP. Thus, in principle, a pulse shape can be designed to melt the metal with EOP and not with Gruneisen. We note that both Cu and Mo stay solid at all times with ps56. We have, in addition, designed another pulse shape, a higher-temperature variant of ps56 (ps 55) that melts Cu at 8.8 ns if EOP is used, but does not melt it if the Gruneisen EOS is used. This pulseshape can thus serve as an EOS model discriminator, using an experimental technique that can measure melt time. The Bragg diffraction diagnostic technique being developed for these experiments[2] can provide these measurements.

References

1. K. Mikaelian, *Phys. Rev. E 54*, 3676, 1996.

2. D. H. Kalantar, *et al.*, these Proceedings.

3. G. B. Zimmerman and W. L. Kruer, *Comments Plasma Phys. Controlled Fusion* 2, 51 (1975).

4. D. J. Steinberg, S. G. Cochran, and M. W. Guinan, *J. Appl. Phys. 51*, 1498 (1980).

5. S. V. Weber, *et al.*, these Proceedings.

6. A. I. Lebedev, P. N. Nisovtsev, and V. A. Rayevsky, in the *Proceedings of the 4th International Workshop on the Physics of Compressible Turbulent Mixing*, 29 March - 1 April 1993, Cambridge, England (Cambridge University Press, 1993), pg. 81.

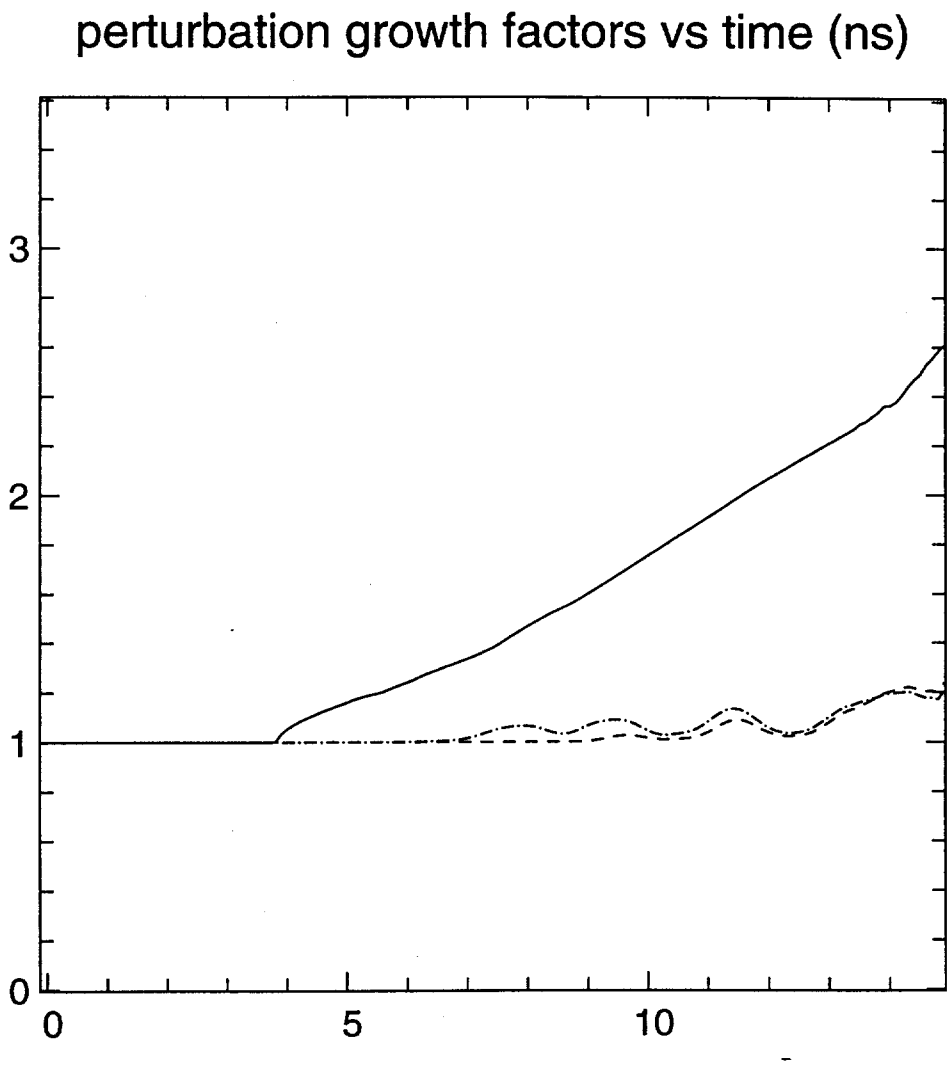
7. R. M. More, K. H. Warren, D. A. Young, and G. B. Zimmerman, *Phys. Fluids 31*, 3059 (1988).

Figure Captions

1. Growth factors of a 500-μm perturbation as a function of time on Mo foils of a) 15 μm thickness and b) 150 μm thickness driven by pulseshape 56. The dashed curve shows the growth factors calculated with accounting for the pressure dependence of the shear strength, and the dash-dot curve without. These growth factors are compared to the growth factors of the Mo foil if it were liquid (solid curve).

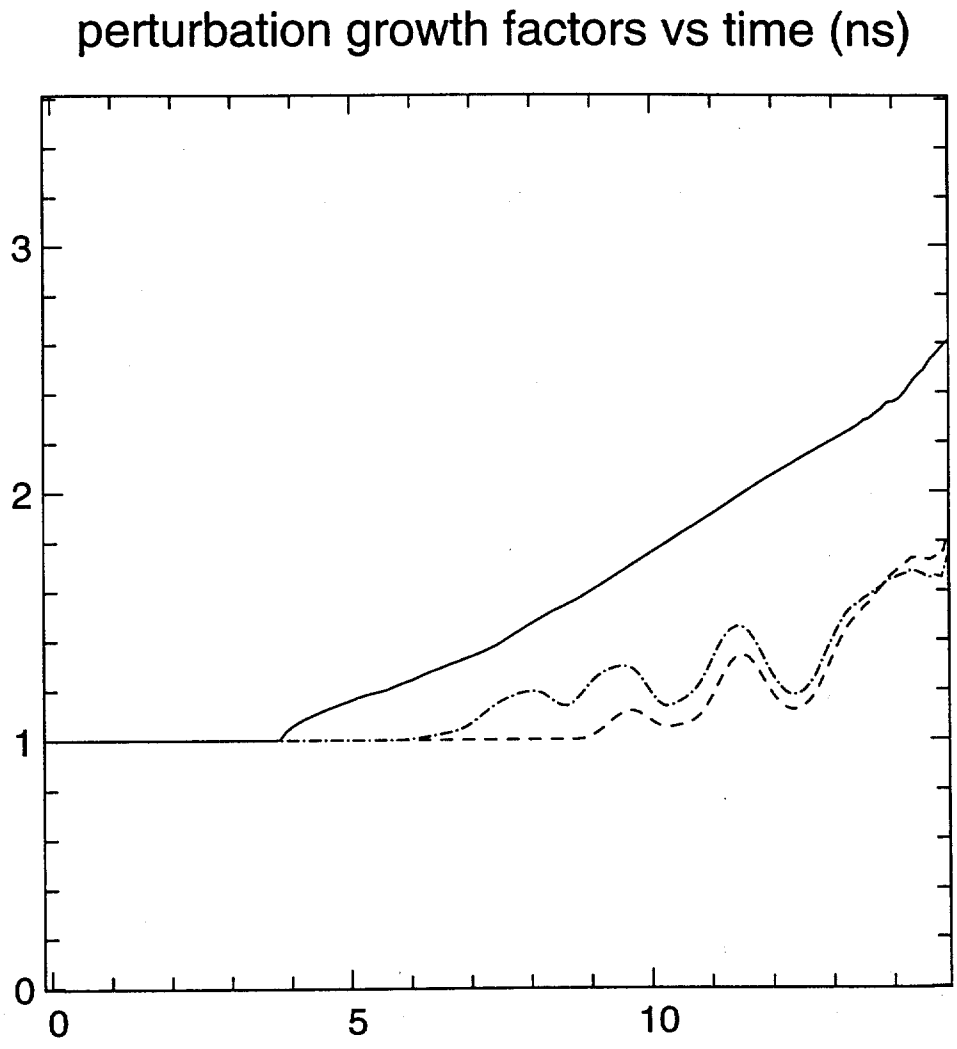
2. Dispersion curves for Mo foils of a) 15 μm thickness and b) 150 μm thickness driven by pulseshape 56. The dashed curve shows the strength-stabilized dispersion curve, and is compared to the dispersion curve of the Mo foil if it were liquid (solid curve).

*This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.



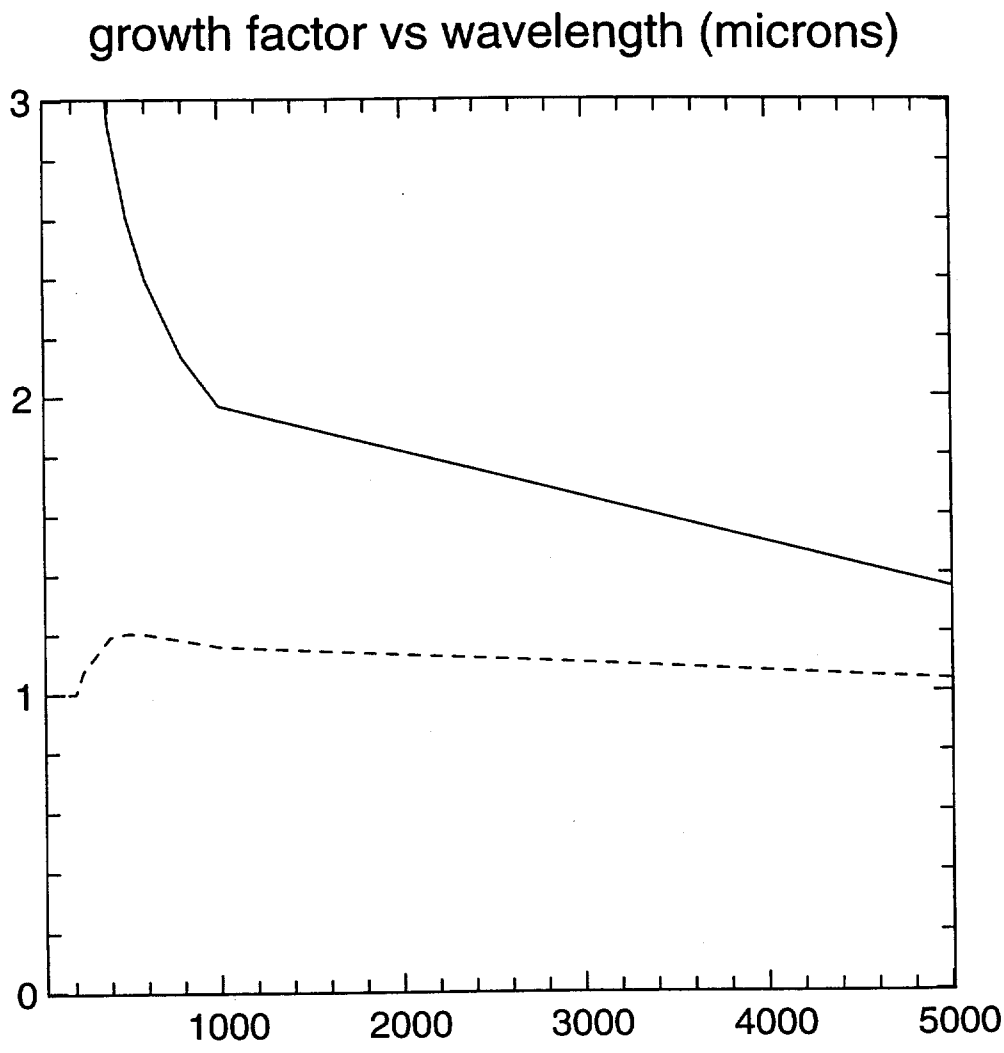
/rhy004, lambda = 500 microns
classical
----- strength-stabilized with G(P)
-.-.- strength-stabilized with G0

fig. 1a



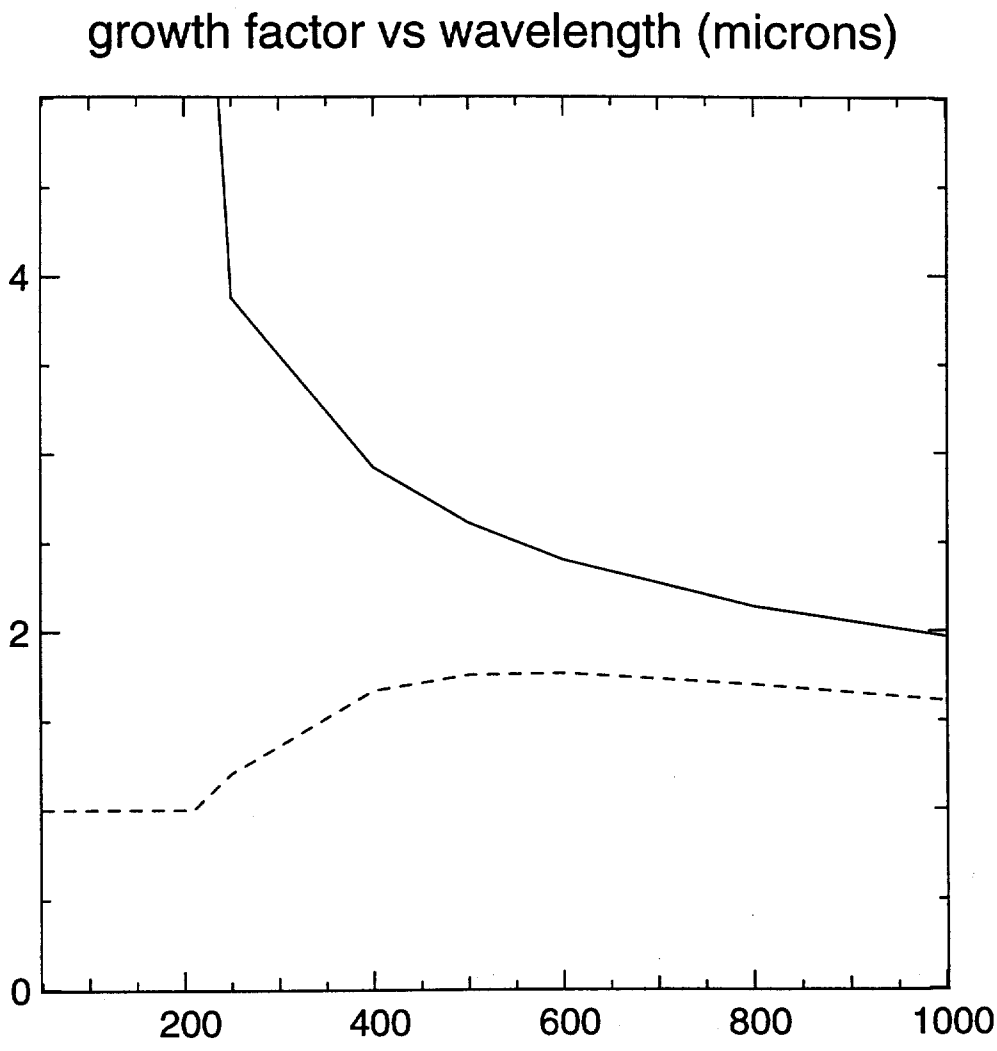
/rhy004, lambda = 500 microns
classical
----- strength-stabilized with G(P)
-.-.- strength-stabilized with G0

fig. 1b



Mo, ps56, h=15 microns

fig. 2a



Mo, ps56, h=150 microns

fig. 2b

Technical Information Department • **Lawrence Livermore National Laboratory**
University of California • Livermore, California 94551