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Transverse Cooling in the Muon Collider

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Abstract. Ionization cooling is the preferred method for reducing the emittance of muon beams in a muon collider. The method described here uses passive liquid hydrogen absorbers and *rf* acceleration in an alternating lattice of solenoids. We consider the basic principles of ionization cooling, indicating our reasons for selecting various parameters. Tracking simulations are used to make detailed examinations of effects on the beam, such as transmission losses, transverse cooling, bunch lengthening, and introduction of energy spread. The system reduces the overall 6-dimensional emittance to 44% of its initial value.

IONIZATION COOLING

The most likely process for achieving a large reduction in the emittance of muon beams in a future muon collider (1-5) is ionization cooling (6-8).[1] In this process the muon loses transverse and longitudinal momentum by dE/dx in a material, and then has the longitudinal momentum (but not the transverse momentum) restored in a subsequent *rf* cavity. The combined effect is to reduce the beam divergence and thus the emittance of the beam. The process is complicated by the simultaneous presence of multiple scattering in the material, which acts as a source of "heat" and increases the emittance. The cooling effect can dominate for low Z materials in the presence of strong focusing fields. A lattice of solenoids with alternating direction is the method considered here for achieving the required focusing.

A charged particle traversing matter loses energy because of electromagnetic interactions with the atomic electrons. The energy loss falls dramatically as the particle energy increases, reaching a minimum value for muons with energy around 300 MeV.

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[1] Conventional forms of cooling, such as electron cooling or stochastic cooling, are too slow compared to the muon lifetime.

Above this is the region of relativistic rise where the energy loss increases logarithmically. Consider a diffuse beam of muons focused onto a block of material. Muons traveling at an angle through the material lose both transverse and longitudinal components of momentum. Operation at any part of the dE/dx curve could be used for transverse cooling.

In order to cool the longitudinal emittance the higher energy particles in the beam must lose more energy than the lower energy particles. On the dE/dx curve this only occurs for muon energies greater than ≈ 500 MeV (9). This natural longitudinal cooling is very inefficient. A more practical idea is to introduce dispersion into the beam, so that the muons receive a transverse displacement proportional to their deviation from the mean momentum. Then a wedge shaped absorber can be used to cause the higher momentum muons to lose more energy, and thus reduce the momentum spread in the beam. However, this system results in a corresponding increase in the transverse direction, and thus only exchanges longitudinal for transverse emittance.

The theory of ionization cooling predicts (7,8) that the *rms* normalized transverse emittance is changed by traveling a step dz into a material at the rate

$$\frac{d\varepsilon_{xN}}{dz} = -\frac{1}{\beta^2}\frac{\varepsilon_{xN}}{E}\left|\frac{dE}{dz}\right| + \frac{\beta_\perp}{2}\frac{E_S^2}{\beta^3 E m c^2}\frac{1}{L_R} \tag{1}$$

where β is the relativistic velocity factor, E is the energy of the muon, dE/dz is the local value of the ionization energy loss function, $\beta_\perp$ is the betatron focusing parameter, $E_S \approx 15$ MeV is a characteristic energy from multiple scattering theory, m is the mass of the muon, c is the velocity of light, and L_R is the radiation length of the material.

The minimum achievable normalized emittance, reached when the cooling rate equals the heating rate, is

$$\min\ \epsilon_{xN} = \frac{\beta_\perp E_s^2}{2\beta m c^2 L_R \left|\frac{dE}{dz}\right|} \ . \tag{2}$$

Thus cooling to small emittance values requires a combination of strong focusing (small $\beta_\perp$) together with the choice of a material with a large value for the product L_R dE/dz.

The cooling method described here uses solenoidal focusing, which has a betatron function given by

$$\beta_{\perp} = \frac{2p_Z}{cB_Z} \tag{3}$$

We can obtain small values of $\beta_{\perp}$ by using a low momentum or by using a strong magnetic field.

It is important that any focusing scheme proposed for ionization cooling be able to transmit large angle (non-paraxial) particle trajectories. The required angular acceptance of the lattice is given by

$$\theta_{acc} = j \left\{ \frac{n E_S^2}{2\beta^2 \gamma mc^2 L_R \left| \frac{dE}{dx} \right|} \right\}^{1/2} \tag{4}$$

where j is the required number of standard deviations for the angular acceptance, i.e. $\theta_{acc}=j\,\sigma_{\theta,}$ and n is the size of the initial transverse normalized emittance compared to the equilibrium value, i.e. $\epsilon_N=n\,\epsilon_N^{eq}$. As an example for j=n=4, 186 MeV/c muons in liquid hydrogen absorber require a lattice with an angular acceptance of 0.39 radians. Higher Z materials would require a larger acceptance.

The normalized longitudinal emittance also changes in a material according to

$$\frac{d}{dz}\epsilon_{zN} \approx \frac{\sigma_{P_Z}}{mc}\frac{d}{dz}\sigma_Z + \frac{\sigma_Z\sigma_E}{\beta mc^2}\frac{d}{dE}\left(\frac{dE}{dz}\right) + \frac{K_S\sigma_Z}{2\beta mc^2\sigma_E}\gamma^2(1 - ½\beta^2) \tag{5}$$

The terms on the right side of the equation come from bunch lengthening, the curvature of the dE/dx relation with energy, and straggling, respectively. The coefficient K_S of the straggling term is given by

$$K_S = 4\pi(r_e mc^2)^2 \frac{N_A Z\rho}{A} \tag{6}$$

where r_e is the classical radius of the electron, N_A is Avogadro's number, and {Z, ρ, A} are the {atomic number, density, atomic mass} of the material.

CHOICE OF PARAMETERS

Many conditions must be simultaneously satisfied in order to produce a useful cooling system. We have chosen to use liquid hydrogen as the absorber since it provides the lowest possible minimum achievable emittance and makes the least demands on angular acceptance of the focusing lattice. We have chosen to use high field solenoids for the focusing since they have large angular acceptance, they focus both transverse dimensions simultaneously and the technology exists for achieving high field strengths. The space between absorbers must be filled with *rf* cavities to replace the energy lost in the absorbers. The particular cooling section described here is designed to reduce the transverse emittance of the muon beam. Because of straggling and path length variations, the longitudinal emittance will be simultaneously growing. We use the 6-dimensional normalized emittance (ϵ_6) as the figure of merit for the overall design. We seek a design that rapidly decreases the value of ϵ_6. We continue the channel until the longitudinal growth becomes excessive and the ϵ_6 cooling rate begins to saturate.

We have chosen 186 MeV/c as the reference momentum for the muon beam, corresponding to a velocity factor $\beta = 0.87$. There are at least three advantages in choosing a low working momentum. 1) We see from Eq. 3 that this helps achieve a smaller value of $\beta_\perp$. 2) We see from Eq. 1 that, ignoring heating, the fractional change in emittance is equal to the fractional loss of energy. Thus the absolute amount of *rf* re-acceleration required for the same fractional cooling is higher at larger momenta. 3) The amount of straggling, which is proportional to γ^2, is less at lower momenta. However, if one chooses too small a momentum (<100 MeV/c), one has to deal with the unfavorable curvature of the dE/dx curve, which produces very large longitudinal heating.

One of the main, non-intuitive design decisions was to periodically alternate the direction of the solenoid field. Simulations using a constant solenoid channel gave worse cooling performance than the alternating system proposed here[2]. In addition the alternating field arrangement is cheaper, since the high field solenoids are only placed over the absorber regions. A third reason for alternating the field direction concerns the build up of angular momentum in the channel.

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[2] The simulation looked at cooling inside a 20 m long solenoid channel with a constant field of 15 T. The initial beam distributions, liquid hydrogen absorber and *rf* cavity structure were identical to that used for the alternating cooling system described in this paper. The transverse cooling factor saturated at 79% after 11 m. The losses at 15 m were ~3 times worse and the longitudinal phase space was ~50% worse than the alternating solenoid example.

The canonical angular momentum is given by

$$L_Z^C = r p_\phi - \frac{e}{2} \left| B_Z(s) \right| r^2 \qquad \textbf{(7)}$$

Consider first a particle with 0 angular momentum starting outside a solenoid that is devoid of absorbing material. As a consequence of the symmetry the canonical angular momentum is a conserved quantity with an initial value of 0. As the particle crosses the fringe field entering the solenoid, a rotational angular momentum is developed represented by the first term in Eq. 7. However, the canonical angular momentum is still 0 since the second term cancels the first. This follows because the azimuthal momentum given to the particle by the fringe field is

$$p_\phi = \frac{e}{2} B_Z r \quad . \qquad \textbf{(8)}$$

When the particle leaves the solenoid the exit fringe field stops the rotational angular momentum. Now consider the case when absorbing material is present inside the solenoid. Energy loss causes the particle to suddenly obtain a p_ϕ that does not satisfy Eq. 8. As a result the canonical angular momentum develops a non-zero value. The problem with this is that when the particle finally leaves the solenoid, the first term in Eq. 7 no longer vanishes and the particle beam has to diverge from the end of the magnet. However, we have found that breaking the solenoid up into an alternating sequence of shorter solenoids prevents the build up of canonical angular momentum, as shown in Fig. 1.

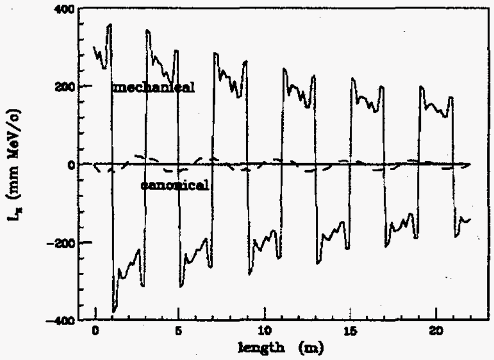

FIGURE 1. The mechanical (rotational) angular momentum and canonical angular momentum vs. length in the alternating solenoid system.

One cell of our proposed cooling arrangement is shown in Fig. 2a. The magnetic field peaks at a magnitude of 15 T in the center of the absorbers. The field falls to 0 and alternates direction in the center of the *rf* cavities. Matching the betatron focusing between cells is a very important issue in an alternating solenoid lattice. Without careful matching unacceptable particle losses occur after the zero field crossing points. The matching was accomplished by using three independent sets of solenoidal coils in each cell. Let us refer to these as the absorber, matching, and bucking coils. The axial field produced by these coils is shown in Fig. 2b. High field solenoids are used over the absorber region to provide the low $\beta_\perp$ required for efficient cooling. Immediately following the absorber the betatron function begins growing rapidly, as shown in Fig. 2c. We adjust the matching coils so that $\beta_\perp$ rises linearly in this region, since this gives the best matching and the largest momentum acceptance. We want to do the field reversal in a region with high $\beta_\perp$, where the amount of modulation in the betatron function from the rapidly varying magnetic field is minimized. The bucking coil was adjusted to symmetrize the betatron function across the 0-crossing point. The matching coils on opposite sides must have different currents in order to account for the energy gain of the particles going through the *rf* cavities.

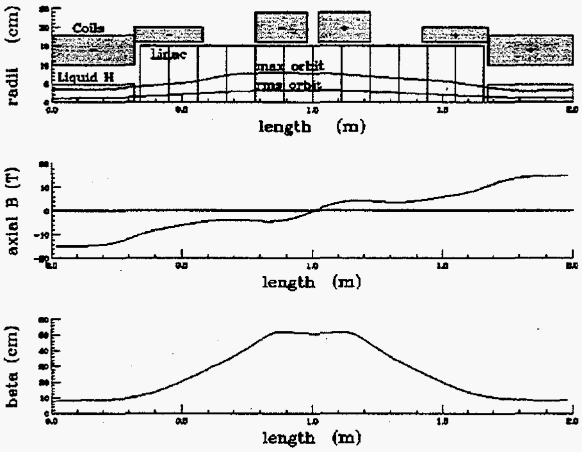

FIGURE 2. (a) Cross section of one cell of the alternating solenoid cooling system; (b) axial magnetic field vs. length; (c) betatron function vs. length.

The channel was constructed from a series of identical 2 m long cells. The length of each cell was chosen to avoid resonances that were known to seriously affect cooling in an earlier scheme, known as the FOFO lattice (1,4). These resonances can lead to large particle losses when the betatron wavelength equals the period of the magnetic field or the synchrotron oscillation wavelength (~14 m).

The total length of liquid hydrogen absorber was chosen to be 64 cm per cell. The liquid hydrogen was assumed to be contained in a vessel with thin end windows. Studies of the effects of window material and thickness show that 2 mil stainless steel or 5 mil Al windows have negligible effects on the cooling performance. The absorber is centered under the high field portion of the solenoidal field, so that the low $\beta_\perp$

region is in the absorber. The length was chosen so that the increase in $\beta_\perp$ near the ends of the vessel was not significant. The mean energy loss (including small contributions from the vessel and *rf* windows) is 20.6 MeV. For modeling purposes the cell was chosen to begin at the center of the absorber region, as shown in Fig. 2a.

Reacceleration is done by a series of pillbox *rf* cavities. In order to get simultaneous acceleration and phase stability the *rf* cavity wavelength must be sufficiently long to totally include the bunch within a quarter *rf* wavelength, i.e. $\sim 6\sigma_Z < \lambda_{rf}$. For the initial bunch length given in Table 1, this matches an *rf* frequency of 805 MHz. The cavity phase was selected so that the center of the bunch lies near the center of the good *rf* quarter wavelength. The exact phase of 39° up from the 0-crossing point of the electric field was chosen to optimize performance. The peak gradient of 36 MV/m was chosen to be as large as possible, consistent with expectations for breakdown. Finally, the cavity length was chosen to provide sufficient acceleration to match the energy loss in the liquid hydrogen. Once the reference momentum and *rf* frequency were specified, the cavity cell length was determined to be 8.1 cm and the total cavity length was broken into 16 cells. The transit time factor is 0.90. The *rf* was modeled using exact fields for a TM_{010} cylindrical pillbox cavity. This turns out to an excellent approximation in this case, since the cavity design for the muon collider includes thin beryllium end windows. (10) Studies of the effects of window material and thickness show that 5 mil Be windows have negligible effects on the cooling performance. After the initial 32 cm of liquid hydrogen, the *rf* drives the reference particle momentum up beyond what is required by the loss in the initial liquid hydrogen, and then it gets restored to the nominal value by the subsequent 32 cm of liquid hydrogen.

COOLING PERFORMANCE

The Monte Carlo simulation was done using the program ICOOL. Energy loss was modeled using the Vavilov distribution function, while multiple scattering was modeled using the Moliere distribution.

The properties of the initial beam used in the simulation are shown in Table 1. All the initial phase space dimensions are assumed to be gaussian except P_Z. Several correlations must be imposed on the variables of the initial particles. 1) Since we begin the simulation inside a strong solenoidal field, each particle must start with the angular momentum the particle would have received from crossing the entrance solenoid fringe field. 2) We have seen that the lattice must transmit large angle tracks for efficient cooling. However, large angle tracks have a significantly longer path length in a solenoidal channel than paraxial tracks. This would lead to significant bunch lengthening unless a correlation between the particle's longitudinal velocity and its transverse amplitude is imposed. We believe this correlation could be introduced from energy loss in a radially varying absorber, placed in the cooling channel immediately preceding the alternating cooling section. We then introduce this correlation in the initial P_Z distribution and this leads to a non-gaussian, high momentum tail. 3) Other possible correlations, such as a longitudinal emittance distribution matched to the asymmetric shaped (alpha) *rf* bucket, have not yet been introduced into the simulation.

TABLE 1. Performance summary

		z = 0 m	z = 22 m
p	MeV/c	186	186
Transmission		1.000	0.990
$\sigma_X = \sigma_Y$	mm	7.92	5.96
$\sigma_{PX} = \sigma_{PY}$	MeV/c	25.4	17.3
σ_t	ps	43.4	68.1
σ_{PZ}	MeV/c	5.74	9.87
ϵ_{XN}	mm mr	1400	650
ϵ_{ZN}	mm mr	1000	2040
ϵ_6	$\times 10^{-12}$ (m rad)^3	1960	865

The *rms* beam size in the channel oscillates in response to the variations in the alternating magnetic field and has a maximum value of 2 cm. The maximum absolute radius seen by any particle is 7 cm. Fig. 3 shows a comparison of the transverse phase space at the beginning and end of the channel.

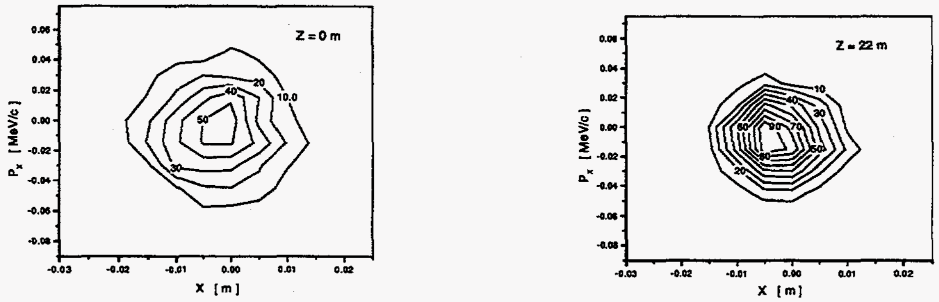

FIGURE 3. Contour plots of transverse normalized emittance at the beginning and at the end of the 22 m long channel.

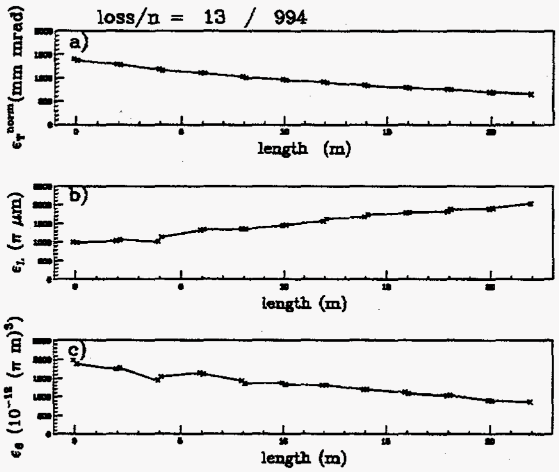

FIGURE 4. Normalized emittances vs. length in 11 alternating solenoid cells; (a) transverse emittance; (b) longitudinal emittance; (c) 6-dimensional emittance.

For our chosen parameters the equilibrium normalized transverse emittance from Eq. 3 is 420 mm mr. The results of the simulation are shown in Table 1 and in Fig 4. The transverse emittance in both the x and y planes is reduced to 46% of its initial value, while the longitudinal emittance grows by a factor of ~2. The overall 6-dimensional emittance is reduced to 44% of its initial value. Fig. 4c shows that the cooling rate for 6-dimensional emittance is noticeably smaller after 22 m. This happens because the longitudinal emittance has grown to fill the acceptance of the system. At this point significant numbers of particles begin to fall out of the *rf* bucket and get lost. In a real muon collider one would terminate the alternating solenoid cooling here and begin an emittance exchange system to bring the longitudinal emittance back down to a useable level. In addition to the transmission losses given in Table 1, the muon beam suffers decay losses of 1.9 % while traversing the 22 m long section. The major features of this simulation have been independently confirmed using DP-GEANT (11) and Parmela.

A similar cooling system, suitable for the final stage of a Higgs particle factory, has been designed using 31 T solenoids (5). The minimum $\beta_\perp$ for this case is 4 cm and the minimum equilibrium emittance is 195 mm mr. The simulation achieved a transverse emittance of 240 mm mr and a 6-dimensional emittance of 95 x 10^{-12} (m rad)^3, both within specifications for the Higgs factory design.

We anticipate making further refinements in this design, for example in the longitudinal matching of the initial beam to the channel. Another topic that we have only begun to study is the effects of space charge and wakefields on the cooling performance.

ACKNOWLEDGMENTS

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