CONF-970497--Absts

Final Report on the Copper Mountain Conference on Multigrid Methods

Copper Mountain, Colorado
April 6-11, 1997

The Copper Mountain Conference on Multigrid Methods was held on April 6-11, 1997. It took the same format used in the previous Copper Mountain Conferences on Multigrid Method conferences. Over 87 mathematicians from all over the world attended the meeting (see attached list of participants). In the following paragraphs, special features of the Conference are briefly described.

Talks and Workshops

No matter how well organized, a conference is only as good as the talks presented. Under that measure this meeting was a great success. The quality and diversity of the talks was superb. During the five day meeting, 56 half-hour talks on current research topics were presented (see the Final Program). Talks with similar content were organized into sessions. Session topics included:

- Fluids
- Domain Decomposition
- Iterative Methods
- Basics
- Adaptive Methods
- Non-Linear Filtering
- CFD
- Applications
- Transport
- Algebraic Solvers
- Supercomputing
- Student Paper Winners

Late evening sessions included Informal Workshops and a Circus Session.

DISCLAIMER

Abstracts and Special Journal Issues

At registration, each participant was provided with a schedule and a list of abstracts from each of the speakers. Speakers were then urged to submit a journal quality version of their paper to E.T.N.A., which are now in the refereeing process.

Student Support

Student participation in the Conference was stimulated by holding a Student Paper Competition. Students were urged to submit a ten page paper containing original research. Eleven students responded. A panel of judges made up of members of the Program Committee selected seven winners. The winners were: Boris Diskin/Weizmann Institute of Science, Jule Kouatchou/George Washington University, Marc Kuether/University of Freiburg, Xavier Vassuer/Ecole Centrale de Nantes, Erik Sterner/Department of Scientific Computing, Hubertus Oswald/University of Heidelburg, and Wing-Lok Wan/University of California Los Angeles.

The Conference provided travel expenses for the winners as well as lodging expense for other students who participated in the competition. In addition, the conference provided lodging support for a number of other students, women and minorities , several of whom presented talks at the Conference. Following is a list of students supported:

Mark Adams
James Forsythe
Travis Austin
Markus Berndt
Brian Bloechle
Tim Chartier
Boris Diskin
Serguei Maliassov
David Gines
Chris Higginson
Jules Kouatchou
Marc Kuether
Barry Lee
Hugh McMillian
Jerry Miranda
Hubertus Oswald
Erik Sterner
Xavier Vasseur
Wing-Lok Wan
Jan Peters Weem

Conference Questionnaire

An informal survey of the participants at the conference indicated that a majority were satisfied with the conference organization and would be interested in attending a similar conference in the future.

1997 Copper Mountain Attendee List (E-mail Addresses)

FIRST NAME	LAST NAME	E-MAIL
Mark	Adams	madams@ce.berkeley.edu
Travis	Austin	austin@magellan.colorado.edu
Alda	Behie	behiea@cuug.ab.ca
Markus	Berndt	berndt@newton.colorado.edu
Marco	Bittencourt	mlb@fem.unicamp.br
Brian	Bloechle	bloechle@colorado.edu
Pavel	Bochev	bochev@utamat.uta.edu
Achi	Brandt	mabrandt@weizmann.weizmann.ac.il
Susanne	Brenner	brenner@math.sc.edu
Walter	Buell	buehlwm@corning.com
Hans-Joachim	Bungartz	bungartz@informatik.tu-muenchen.de
Zhiqiang	Cai	zcai@math.purdue.edu
Tim	Chartier	chartier@newton.colorado.edu
Zhangxin	Chen	zchen@dragon.math.smu.edu
Nicholas	Coult	nicolas.coult@colorado.edu
Wenlong	Dai	wenlong@lcse.umn.edu
Joel	Dendy	jed@lanl.gov
Boris	Diskin	diskin@wisdom.weizmann.ac.il
Craig C.	Douglas	douglas@ccs.uky.edu
Laura	Dutto	dutto@cfdlab.concordia.ca
Jonathan	Dym	jdym@cams.usc.edu
Rob	Falgout	rfalgout@llnl.gov
Paul	Farrell	farrell@mcs.kent.edu
Diederik	Fokkema	fokkema@ise.ch
James	Forsythe	jforsythe@twsu.campus.mci.net
Paul	Frederickson	pfrederi@admin.ed.umuc.edu
Scott	Fulton	fulton@sun.mcs.clarkson.edu
Michael	Gildea	gildeamb@corning.com
David	Gines	gines%rococo@colorado.edu
Michael	Griebel	griebel@iam.uni-bonn.de
Van	Henson	vhenson@nps.navy.mil
Chris	Higginson	higginson@colorado.edu
Ralf	Hiptmair	hiptmair@math.uni-augsburg.de
Jim	Jones	jjones@icase.edu
Kirk	Jordan	kjordan@vnet.ibm.com
Leonard	Kent	kentlr@corning.com
Sang Dong	Kim	sangdong@newton.colorado.edu
Dana	Knoll	nol@lanl.gov
Andrew	Knyazev	aknyazev@math.cudenver.edu
Jules	Kouatchou	kouatchou@math.gwu.edu
Marc	Kuether	marc@mathematik.uni-freiburg.de
Reinard	Kulke	kulke@imst.de
Barry	Lee	blee@newton.colorado.edu
Ira	Livshits	livshits@wisdom.weizmann.ac.il
Rudolph	Lorentz	lorentz@gmd.de
Hugh	Mac Millan	macmilln@newton.colorado.edu
Sachit	Malhotra	malhotra@@sca.com
Serguei	Maliassov	malyasov@ima.umn.edu
Tom	Manteuffel	tmanteuf@boulder.colorado.edu
Demitri	Mavriplis	demitri@icase.edu
Steve	McCormick	steve@boulder.colorado.edu

1997 Copper Mountain Attendee List (E-mail Addresses)

FIRST NAME	LAST NAME	E-MAIL
Steven	McKay	mckay@math.byu.edu
A.J.	Meir	ajm@math.auburn.edu
Jerry	Miranda	
William	Mitchell	william.mitchell@nist.gov
David	Moulton	moulton@lanl.gov
Hubertus	Oswald	hubertus.oswald@IWR.uni-heidelberg.de
Rossen	Pareshkovov	rossen@newton
Seymour	Parter	parter@cs.wisc.edu
Joseph	Pasciak	pasciak@jep.isc.tamu.edu
Martin	Peters	peters@springer.de
Bill	Rider	wjr@lanl.gov
Ulrich	Ruede	ruede@math.uni-augsburg.de
John	Ruge	jruge@newton.colorado.edu
Arden	Rutten	ruttan@mcs.kent.edu
Marcus	Sarkis	msarkis@schwartz.cs.colorado.edu
Ken	Senior	ksenior@bgnet.bgsu.edu
John	Shadid	jshandi@cs.sandia.gov
Yair	Shapira	yairs@lanl.gov
William	Spotz	spotz@ucar.edu
Gerhard	Starke	starke@mathematik.uni-karlsruhe.de
Erik	Sterner	erik@tdb.uu.se
William	Stewart	billy@csc.ncsu.edu
Rune	Teigland	rune@cmr.no
Clemens	Thole	thole@gmd.de
Xavier	Vasseur	xavier.vasseur@ec-nantes.fr
Gordon	Wade	gwade@math.bgsu.edu
Wing-Lok	Wan	wlwan@math.ucla.edu
Takumi	Washio	washio@ccrl-nece.technopark.gmd.de
Kristian	Witsch	witsch@numerik.uni-duesseldorf.de
Dexuan	Xie	xie@monod.biomath.nyu.edu
Irad	Yavneh	irad@csa.cs.techion.ac.il
Huilong	Zhang	hzhang@math.u-bordeaux.fr
Xuejun	Zhang	xzhang@math.tamu.edu

EIGHTH COPPER MOUNTAIN CONFERENCE ON MULTIGRID METHODS

APRIL 6-11, 1997

Organized by

The University of Colorado
The Society for Industrial and Applied Mathematics
The Institute for Algorithms and Scientific Computing of the GMD

Sponsored by

DOE, NSF, and IBM

CONFERENCE CHAIRS

Tom Manteuffel, University of Colorado
Steve McCormick, University of Colorado

PROGRAM COMMITTEE

Joel Dendy , Los Alamos
Van Henson , Naval Postgraduate School
Jim Jones , ICASE
Kirk Jordan, IBM
Jan Mandel, CU-Denver
Duane Melson, NASA Langley
Seymour Parter, Univ. Wisconsin
Joseph Pasciak , Texas A & M
John Ruge, CU-Denver
Klaus Stueben, GMD, Germany
Ulrich Trottenberg, GMD, Germany
Pieter Wesseling , Univ. of Delft
Olof Widlund , Courant Institute
Irad Yavneh , Technion

CIRCUS CHAIR

Craig Douglas , Kentucky/Yale

WORKSHOP CHAIR

Paul Frederickson , Univ. Maryland, Eur. Div.

SPECIAL FEATURES

* Circus--Forum for Late-Breaking Results
* Workshops--Informal Topical Discussions
* Student Papers and Travel Support
* MGNet Virtual Proceedings
* Special Issue of ETNA
* ETNA CD Sponsored by IBM

FORMAT

Morning and afternoon sessions will consist of individual talks of approximately 25 minutes each. Workshop and Circus sessions will be held in the evenings.

VIRTUAL PRE-PROCEEDINGS

To obtain papers from the Virtual Proceedings, access MGNet. Also available are the 1993 and 1995 virtual proceedings.

For submission of a paper to the MGNet Virtual Proceedings:

Instead of having to cram your paper into a few pages, you have as many pages as you wish, preferably in an easy to read font. Please submit a PostScript file using the directions for a MGNet preprint submission. Then send e-mail with a message that you submitted a pre-proceedings paper.

SPECIAL ISSUE OF ETNA

A special issue of ETNA (Electronic Transactions on Numerical Analysis) will be dedicated to this conference. The tentative time-table for submission to ETNA is May 15, 1997. Final versions of all papers (including changes determined in the refereeing process) must be in the hands of ETNA by Dec. 15, 1997. Expected publication (posting) as an ETNA volume: February 15, 1998.

ETNA CD SPONSORED BY IBM

Thanks to generous support by IBM, each conference participant will be provided with a CD containing the special issue of ETNA as soon as it becomes available.

WORKSHOPS AND CIRCUS SESSIONS

The program will include special time set aside for Workshops and Circuses. The Workshop Chairman will encourage and support informal workshops on special topics of interest. The Circus Chairman, for each of the three planned circus sessions, will encourage participation and contributions and will organize, set the schedule, and oversee its progress.

Program

This program is preliminary and subject to modifications.
***Times* given here are only estimates, and are likely to change.**
We will *try* to limit most schedule changes to within the given day.
Participants and speakers should monitor this page and the bulletin board at the conference.

Speakers should plan their talks to be at most 20 minutes.
Program Chairs will adhere strictly to this schedule.

We will have a slide projector and an overhead projector for speaker use.
We will also provide an LCD panel (ViewFrame Spectra), which plugs into standard video outlets on a Mac or PC to project its image onto the screen.
This equipment will also be available for the Circus and Workshop sessions.

Wednesday night's cash bar & banquet is in Victoria B in the Pavillon.
All other activities will be held in the Village Square conference facilities.

Sunday, April 6

7:00-9:00 PM Reception

Monday, April 7

SESSION I	Chair: Seymour Parter
8:00am	Hans-Joachim Bungartz A Multigrid Algorithm for Higher Order Finite Elements on Sparse Grids
8:25am	Zhangxin Chen The Analysis of Intergrid Transfer Operators and Nonconforming Multigrid Methods
8:50am	Nicholas Coult Multiresolution Reduction of Elliptic PDE's and Eigenvalue Problems
9:15am	Joel Dendy Some Applications of Black Box Multigrid to Systems
9:40am	COFFEE BREAK
10:10am	Jonathan Dym Multilevel Inversion of the Radon Transform
10:35am	David Gines A Wavelet-based Direct Multilevel Solver
11:00am	Ralf Hiptmair Multigrid Method for *H(div)* in Three Dimensions
11:25am	D.A. Knoll Multigrid Preconditioning of Newton-Krylov Methods
11:50am	Steven M. McKay Solution of a Convolution Equation with Unbounded Domain via Multigrid

SESSION II	Chair: Kirk Jordan
4:45pm	Steve McCormick First-Order System Least Squares (FOSLS) for PDEs: Basic Tools
5:10pm	Tom Manteuffel First-Order System Least Squares (FOSLS) for Transport Problems
5:35pm	Markus Berndt Adaptive Mesh Refinement for First-Order System Least Squares (FOSLS)
6:00pm	REFRESHMENTS SPONSORED BY IBM
6:45pm	Pavel Bochev Negative Norm Least-squares for the Velocity-flux Navier-Stokes Equations
7:10pm	Sang Dong Kim A FOSLS Approach for Three-Dimensional Pure Traction Linear Elasticity
7:35pm	Barry Lee First-Order System Least-Squares: Numerical Experiments

Tuesday, April 8

SESSION III	Chair: Irad Yavneh
8:00am	Zhiqiang Cai An Analytic Basis for Multigrid for Stabilized Finite Element Methods for Stokes
8:25am	Boris Diskin Multigrid Algorithm with Conditional Coarsening for the Non-aligned Sonic Flow
8:50am	Scott R. Fulton A Comparison of Multilevel Adaptive Methods for Hurricane Track Prediction
9:15am	Jim E. Jones A Note on Multi-block Relaxation Schemes for Multigrid Solvers
9:40am	COFFEE BREAK
10:10am	Dimitri J. Mavriplis Directional Coarsening and Smoothing for Anisotropic Navier-Stokes Problems
10:35am	A. J. Meir A Two-Level Discretization Method for the MHD Equations
11:00am	Erik Sterner A Multigrid Smoother for High Reynolds Number Flows
11:25am	Xavier Vasseur FMG-FAS for the Navier-Stokes Equations on Cell-Centered Colocated Grids
11:50am	Achi Brandt Multiscale Methods: 1996 Review

CIRCUS	Chair: Craig Douglas
7:00pm	**Everyone is welcome to talk and/or listen. The program will be determined at the start by polling the participants. Contact Craig for suggestions and information, or just show up and be ready to talk, interact, or just listen.**

Wednesday, April 9

SESSION IV	Chair: Jim Jones
8:00am	Susanne C. Brenner Multigrid Methods for Stress Intensity Factors and Singular Solutions
8:25am	Joseph Pasciak Inexact Uzawa Algorithms for Nonsymmetric Saddle Point Problems
8:50am	Marcus Sarkis Overlapping Non-matching Grids Mortar Element Methods for Elliptic Problems
9:15am	Gordon Wade A Comparison of Multilevel Methods for Total Variation Regularization
9:40am	COFFEE BREAK
10:10am	Wing-Lok Wan An Energy-Minimizing Interpolation for Multigrid Methods
10:35am	Takumi Washio Krylov Subspace Acceleration Method for Nonlinear Multigrid Schemes
11:00am	Kristian Witsch Multilevel Evaluation of Point Forces
11:25am	Irad Yavneh Stranger Than Fiction: On Smoothing Symmetric Nine-point Operators
11:50am	Xuejun Zhang A Multigrid Analysis for an Anisotropic Problem

SESSION V	Chair: Joe Pasciak
4:40pm	Michael Griebel An Adaptive Multilevel Method for Sparse Grid Finite Difference Discretizations
5:05pm	Ulrich Rüde On the Efficient Implementation of Adaptive Multigrid Methods
5:30pm	Yair Shapira Multigrid for Locally Refined Meshes
5:30pm	Huilong Zhang Object Oriented Programming and FAC in Numerical Reservoir Simulation
6:20pm	Ira Livshits Multigrid Methods for Standing Wave Equations

7:30pm	CASH BAR AND BANQUET

Thursday, April 10

SESSION VI	Chair: Van Henson
8:00am	M.L Bittencourt Non-nested Multigrid Methods in Finite Element Linear Structural Analysis
8:25am	Serguei Maliassov Multilevel Fictitious Space Preconditioner for Nonconforming Unstructured Grids
8:50am	Wenlong Dai Iterative Schemes for Nonlinear Heat Transfer in Systems of Multi-materials
9:15am	Andrew Knyazev Uniform Wellposedness of a Mixed Formulation of Symmetric Problems
9:40am	COFFEE BREAK
10:10am	R. Kulke Multigrid for the Laplace Equation in 2D-Electrostatic Structures
10:35am	Paul A. Farrell Multigrid Methods for the Solution of Singularly Perturbed Differential Equations
11:00am	Jules Kouatchou Asymptotic Stability of 9-Point Multigrid for Convection-Diffusion
11:25am	Marc Küther Exponentially Fitted Hierarchical Bases Multigrid for Convection Diffusion
11:50am	Gerhard Starke Multilevel Methods for Exponential Spline Discretizations of Convection Diffusion

7:00pm	DINNER BREAK
WORKSHOP	Chair: Paul Frederickson
8:30pm	Topics to be arranged. Contact Paul for suggestions and information.

Friday, April 11

SESSION VII	Chair: Joel Dendy
8:00am	John Ruge Recent Developments in Algebraic Multigrid Methods
8:25am	Van Emden Henson Towards a Fully-Parallelizable Algebraic Multigrid
8:50am	Craig C. Douglas Cache Based Multigrid Algorithms
9:15am	Laura C. Dutto An Algebraic Multilevel Parallelizable Preconditioner for Large-Scale Computations
9:40am	COFFEE BREAK
10:10am	Sachit Malhotra Accelerating ADI on Parallel Processors including Multigrid with an ADI Smoother
10:35am	William F. Mitchell A Parallel Adaptive Multilevel Method Using the Full Domain Partition
11:00am	Hubertus Oswald Parallel Multilevel Algorithms for Incompressible Navier-Stokes
11:25am	Clemens-August Thole Fast Solution of MSC/Nastran Sparse Matrix Problems Via a Multilevel Approach
11:50am	Dexuan Xie Parallel U-Cycle Multigrid Method
12:15pm	Rudolph Lorentz Computationally Efficient Wavelets in Sobolev Spaces
12:40pm	William Spotz Stable Multigrid for Convection-Diffusion Using High Order Compact Discretization

Computationally Efficient Wavelets in Sobolev Spaces

Rudolph A. Lorentz
GMD - German National Research Center for Information Technology
Schloß Birlinghoven
D-53754 Sankt Augustin
Germany

We show that there do not exist any compactly supported prewavelets with respect to Sobolev seminorms for the multiresolution analysis generated by box splines in d-dimensional Euclidean space for $d > 2$. Neither do there exist compactly supported prewavelets with respect to the full norm, nor if we use one Sobolev norm to define the semiorthogonality and another for the Riesz basis property of the prewavelets.

The case $d = 1$ is an exception, since it is already known that compactly supported univariate spline prewavelets exist for integer s.

This is work done together with P. Oswald.

A Stable Multigrid Strategy for Convection-Diffusion Using High Order Compact Discretization

William F. Spotz
NCAR, SCD
P. O. Box 3000
Boulder, CO 80307

Multigrid schemes based on high order compact discretization are developed for convection-diffusion problems which circumvent numerical oscillations and instability, while also yielding higher accuracy. These instabilities are typically exacerbated by the coarser grids in multigrid calculations. Our approach incorporates a 4th order compact formulation for the discretization, while also constructing a consistent multigrid restriction scheme to preserve the accuracy of the fine-to-coarse grid projections. Numerical results demonstrating the higher accuracy and robustness of this approach are presented for representative 2D convection-diffusion problems. These calculations also confirm that our numerical algorithms exhibit the typical multigrid efficiency and mesh-independent convergence properties.

Parallel multilevel algorithms for solving the incompressible Navier-Stokes equations with nonconforming finite elements in three dimensions.

Hubertus Oswald
INF 293, 69120 Heidelberg, Germany

This paper presents results of a numerical study for unsteady three-dimensional, incompressible flow. A finite element multigrid method is used in combination with a operator splitting technique and upwind discretization for the convective term. A nonconforming element pair, living on hexahedrons, which is of order O(h^2/h) for velocity and pressure, is used for the spatial discretization. The second order fractional-step-theta-scheme is employed for the time discretization. For this approach we present the parallel implementation of a multigrid code for MIMD computers with message passing and distributed memory. The parallelization uses the grid decomposition: The decomposition of the algebraic quantities is arranged according to the structure of the discretization. The grouping of gridpoints in subdomains automatically leads to a block structuring of the system matrix. Multiplicative multigrid methods as stand-alone iterations and additive multilevel methods as preconditioners are considered. We present a very efficient implementation of Gauss-Seidel resp. SOR smoothers, which have the same amount of communication as a Jacobi smoother.

Fast solution of MSC/Nastran sparse matrix problems using a multi-level approach

Clemens-August Thole, Alexander Supalov
GMD-SCAI.WR, Schloß Birlinghoven, D-53754 Sankt Augustin

As part of the European Esprit project EUROPORT, 38 commercial and industrial simulation codes were parallelized for distributed memory architectures. As a result of the project among others the CFD codes CFX4, PHOENICS, POLYFLOW and STAR-CD as well as the structural analysis codes LS-DYNA3D, MSC/NASTRAN, PAM-CRASH, PERMAS and SAMCEF are now available for parallel architectures.

During the project sparse matrix solvers turned out to be a major obstacle for high scalability of the parallel version of several codes. The European Commission therefore launched the PARASOL project to develop fast parallel direct solvers and to test parallel iterative solvers on their applicability and robustness in an industrial framework. The industrial codes involved in this project perform structural analysis (DNV-Sesame, MSC/NASTRAN), simulation of forming processes (ARC3D, INDEED) and viscous flow (POLYFLOW). Iterative numerical methods involved are domain decomposition methods and hierarchical methods, like multigrid. For each of the industrial test cases several solvers will be evaluated with respect to performance and robustness.

This paper presents initial results using a special multi-level method as preconditioner for matrices resulting from MSC/NASTRAN structural analysis for linear test cases in three dimensions. P-elements in MSC/NASTRAN allow to specify globally or for each element the polynomial degree of the elements. Solution depended adaptive "refinement" of the p-level can be selected. Based on this approach discretisations with lower p-level can be used

large (or small) the blocks must be in order for the data in the block to just fit into the processor's primary cache. By re-using the data in cache several times, a potential savings in run time can be predicted. This is analyzed for a set of examples. It is surprising to see how large a subdomain can fit into a relatively small, well designed cache (e.g., 256Kb). As caches continue to increase in size, the ideas here will extend quite nicely to three dimensional problems. In particular, machines with 1 - 4Mb caches are under design which will make three dimensional cache based multigrid a reality.

While most of the savings in time are with respect to the approximate solver, using a multiplicative or additive domain decomposition method saves even more time and allows us to use theoretical convergence rates from that field for our problems.

An automatic, low overhead software tool for determining a good cache based ordering for the multigrid components for general problems will also be discussed.

An Algebraic Multilevel Parallelizable Preconditioner for Large-Scale Computations

Laura C. DUTTO
CFD Lab, Department of Mechanical Engineering, Concordia University, Montreal, P.Quebec, CANADA H3G 1M8

An efficient parallelizable preconditioner for solving large-scale computations is presented. It is adapted to coarse-grain parallelism and can be used for both shared and distributed-memory parallel computers. The proposed preconditioner consists of several approximations of the system matrix. The first one is a block-diagonal, fully parallelizable approximation of the given system. The following matrices are coarser approximations of it, obtained trough an algebraic multi-grid strategy. The process is fully automatic and general and is solely based on information contained in the matrix and not on geometric domain decomposition techniques. The preconditioner is used to compute the steady solution of the compressible Navier-Stokes equations for subsonic laminar flows, on shared-memory computers, for a moderate number of processors. The multilevel preconditioner is robust and has a large potential for parallelism. Interesting savings in computational time for parallel computations are obtained when comparing the two-level preconditioner with the well-known incomplete Gaussian factorization.

Accelerating ADI Methods on Parallel Processors including Multigrid with ADI as the Smoother

Sachit Malhotra
Scientific Computing Associates
One Century Tower, 265 Church Street
New Haven, CT 06510-7010

Craig C. Douglas
University of Kentucky
Department of Mathematics
715 Patterson Office Tower
Lexington, KY 40506-0027

equation. Then we prove the approximation and smoothing property of the corresponding Hierarchical Bases twogrid method. Numerical experiments shall also be given for the corresponding multigrid method.

Multilevel Methods for Exponential Spline Discretizations of Convection-Diffusion Problems

Gerhard Starke
Institut für Praktische Mathematik, Universität Karlsruhe, 76128 Karlsruhe, Germany.

We present a study of multilevel algorithms for boundary value problems of convection-dominated convection-diffusion equations which are discretized by a Petrov-Galerkin scheme using exponential splines. Our approximation is based on a nested sequence of finite element spaces based on a quasi-uniform sequence of triangulations. It is well-known that the Galerkin method using standard linear finite elements on triangles does not produce physically meaningful approximations, in general, for convection-dominated problems. The use of exponential splines (representing local element-wise solutions of the convection-diffusion equation) as trial functions, however, leads to good approximations on rather coarse grids. In the context of the multilevel solution of the resulting linear system of equations this leads to stable coarse-grid operators naturally associated with the hierarchy of trial and test spaces. We investigate the convergence of multilevel (V-cycle) algorithms associated with different variants of Petrov-Galerkin schemes using exponential splines. This includes a set of computational experiments for some test problems on general triangulations.

Recent Developments in Algebraic Multigrid Methods

John Ruge
1005 Gillaspie Dr., Boulder, CO 80303

There has been a resurgence of interest in Algebraic Multigrid Methods (AMG) over the last several years as computers become more powerful and the problems to be solved become larger and more complex, making most conventional solvers inadequate. AMG is a "black-box" linear solver based on multigrid principles, in which the coarser levels and necessary operators are constructed automatically based on the matrix itself. In this talk, we present the main idea of AMG, present some results on recent applications such as 3D elasticity, and discuss some aspects of parallel implementation of AMG.

Towards a Fully-Parallelizable Algebraic Multigrid

Robert L. Falgout * and Van Emden Henson**
rfalgout@llnl.gov
vhenson@nps.navy.mil
*Center for Applied Scientific Computing, Lawrence Livermore National Laboratory, P.O. Box 808, L-316, Livermore, California 94551
**Naval Postgraduate School, Department of Mathematics, Code Ma/Hv 1411 Cunningham Road rm 341, Monterey, Ca. 93943-5216

Algebraic multigrid (AMG) is currently undergoing something of a renaissance. Developed in the nineteen seventies, interest in AMG has increased

arbitrary number of dielectric layers or blocks can be defined within the rectangular domain. There is also no restriction in the number and dimensions of the metal conductors. The idea of the new multigrid solver is to overcome the drawback of a bad overall convergence of traditional smoothers like the Gauss-Seidel or Jacobi method. In /1/ the SOR method is already used to characterize about 14 coplanar elements for the RF circuit design in a frequency range from about 100 MHz to 90 GHz. However, the simulation time increases drastically with the complexity of 3-dimensional structures, which is inconvenient for design tools. An other aim is to achieve more flexibility in finding and defining new structures for high frequency circuits. Future fields of applications are multilayer boards for communication devices (e.g. frontends). The problem now is, to find a discretisation from a very coarse to a fine level. The basic idea is, to place grid lines at each edge point of the structure. The next finer grid will be generated by dividing the distance of two coarser grid points in half, to get a new grid line. The advantage is, that the structure is correctly described in all levels, while the disadvantages are a strong non-equidistant grid and a concentration of gridlines, which are not necessarily on the physical right place. The solution is a minimum distance of 2 grid points in x- and y-direction. This makes the grid more and more homogeneous, while the level increases.

The implementation of the present algorithm in C++ starts with the code suggested in /2/, while some basic ideas where taken from /3/. The multigrid program is able to determine the static potential of an above described structure with a lot of variations of the solver. The smoother is the Gauss-Seidel method with the enhancement of successive overrelaxation or underrealxation. Four options for the ordering of the grid point are available. This can be combined with a multigrid or full-multigrid formulation with a free number of pre and post smoothing steps and cycles. Since the restriction operator has a strong influence on the convergence (especially on non-equidistant rectangular grids), injection, 5-point- and 9-point-restriction can be chosen. Results from the program as well as some convergence considerations will be given on the convergence. In addition a demonstration of the software may be possible on a LINUX-PC.

The result from the multigrid simulation is the static potential distribution in the non-metallization areas. The boundary conditions are 1V on the signal line and 0V on the ground electrodes and all other lines. The capacitance between the signal line and ground will be calculated by an integration of the electrical flux around this line. The result C^Ò in [F/m] is the first element of the equivalent circuit. In the case of N lines C^Ò will be a matrix C^Ò with NxN capacity coefficients. From the same geometry but filled with air instead of dielectric blocks or layers we determine C0^Ò in exactly the same way. With the assumption, that the structure is a TEM-waveguide, which means that no z-component of the electrical field in the propagation direction of the wave exists, a definite relationship between the electrical and magnetical fields is given. Then, the inductivity L^Ò will be calculated from the capacitance C0^Ò. The last element of the equivalent circuit describes the losses of the line. They can be divided into DC-losses and a frequency dependent part, which is depending on the skin effect, where the AC-current flow is concentrated in the surface of the conductor. To calculate R^Ò(freq), the electrical field components within the structure are needed. Ex and Ey are the derivations of the potential in x- and y-direction

for the solution of the system of equations. Adaptive procedures are also considered by defining automatic strategies based on error estimators and multigrid estrategies. All procedures were implemented in C++. Prof. Dr. Marco Lúcio Bittencourt e-mail: mlb@dpm.fem.unicamp.br Dept. of Mechanical Design - DPM/FEM Phone: +55 (19) 239-8384/8475 State University of Campinas Fax: +55 (19) 239-3722

Multilevel Fictitious Space Preconditioner for Nonconforming Approximations on Unstructured Regular Triangulations

Serguei Maliassov
Institute for Mathematics and Its Applications
University of Minnesota
514 Vincent Hall
206 Church Street S.E.
Minneapolis, MN 55455

The paper describes an application of a fictitious space technique to constructing a preconditioner for stiffness matrices generated by nonconforming finite element method on unstructured regular triangulations. Given an unstructured mesh, a structured hierarchical mesh is constructed "approximating" in certain sense the original one. Introducing special interpolation operators we reduce the problem of constructing a preconditioner in a space associated with unstructured grid to the problem of constructing one in a fictitious space corresponding to the structured grid. Within the fictitious space we apply multilevel preconditioner with local refinement.

Iterative Schemes for Nonlinear Heat Transfer in Systems of Multi-materials

Wenlong Dai
University of Minnesota, 116 Church Street SE, Minneapolis, MN 55455

Heat transfer is concerned with physical processes underlying the transport of thermal energy due to a temperature gradient. Our effort to develop numerical schemes for heat transfer is motivated for radiative hydrodynamics and laser fusion. Therefore, the scheme we need should have the following features: (1) stable for any size of time-steps, (2) second-order accurate in both space and time, (3) numerical errors being strongly damped in the limit of large time steps, (4) iterative and fast in convergence, (5) suitable for nonlinear problems, (6) capable to solve a system composed of more than one kinds of material, (7) conserving.

Numerical schemes for heat transfer problems may be divided into explicit and implicit methods. An explicit scheme, i.e., forward Euler scheme, is simple, and is first order accurate, but the size of a time step is limited by a local stability condition which is normally much smaller than the required temporal accuracy. Two typical implicit schemes are backward Euler scheme and Crank-Nicolson scheme. The backward Euler scheme is first order accurate, and is useful for steady states. For the problems in radiative heat transfer and laser fusion, the temporal accuracy is important. Although Crank-Nicolson scheme is second order accurate, numerical errors do not damp in Crank-Nicolson scheme in the limit of large time steps. The

application to 2D and 3D PDEs of 2nd order with general coefficient functions, discuss the adaptive refinement approach and describe how we can deal with general complicated domains. Furthermore, we present the prewavelet multilevel preconditioner in detail.

On the Efficient Implementation of Adaptive Multigrid Methods

Ulrich Rüde
Institut für Mathematik, Universität Augsburg, D-86135 Augsburg, Germany

Practical PDE problems often show increased activity in subdomains and thus an adaptive mesh generation can drastically reduce the number of unknowns in the discretized system. Modern adaptive multilevel methods are thus often based on hierarchically nested, unstructured finite element meshes. Unfortunately, the data structures required to implement these algorithms lead to a high overhead on many advanced computer architectures. The two main sources of this performance degradation are the effects of non-uniform data access combined with indirect addressing that prohibits the use of instruction-level parallelism essential in many modern CPUs. The second, and possibly more fundamental question arises from hierarchical memory architectures with several layers of caches that require programs with data access locality. These problems are addressed in the patch-adaptive multigrid method (PAM), a recently developed experimental software system implementing an adaptive multigrid method based on a locally uniform mesh data structure. The advantages of the approach are shown by performance comparisons with conventional unstructured mesh multigrid systems.

Multigrid for Locally Refined Meshes

Yair Shapira
Los Alamos National Laboratory
Mail Stop B-256
Los Alamos, NM 87545

A two-level method for the solution of finite element schemes on locally refined meshes is introduced. An upper bound for the condition number is derived and, in some cases, implies mesh-independent convergence. This is also verified numerically for a diffusion problem with discontinuous coefficients. The discontinuity curves are not necessarily aligned with the coarse mesh; indeed, numerical applications with ten levels of local refinement yield a fast convergence for the corresponding ten-level multigrid V-cycle, even when the discontinuities are not visible on most of the coarse meshes.

Object oriented programming techniques and FAC method in numerical reservoir simulation

Mazen SAAD and Huilong ZHANG
MAB, ERS CNRS 123, Université de Bordeaux I, 351 Cours de la Libération, 33405 Talence, FRANCE

Flexible gridding has the potential of increasing the accuracy in numerical simulation of flow through hydrocarbon reservoirs within limitations in

methods is that every linearization step a matrix of Jacobians must be evaluated and stored. Since the basis of our method is nonlinear multigrid, Jacobians are only evaluated locally (point- or linewise) in the smoother on every grid level. The nonlinear Krylov acceleration is performed on the finest grid level, and can be seen as an outer iteration for the multigrid preconditioner. Therefore it is very easy to implement it in already existing codes with a nonlinear multigrid variant as a solver, like Navier-Stokes or Euler codes. Another advantage of the nonlinear acceleration scheme presented is the cheapness in computational cost of our method, with respect to the multigrid preconditioner. The (nonlinear) search directions are constructed from available intermediate solution vectors. Jacobians are approximated by the residual vectors for the intermediate solutions, so that they are not recomputed explicitly in our Krylov acceleration technique. In this sense the method suits very well to the nonlinear multigrid method.

Numerical results with this approach are presented for nonlinear elliptic scalar PDEs, like for the Bratu problem, where the convergence difficulty to obtain the second solution for certain parameters with FAS is nicely improved by the Krylov acceleration technique. Also results are presented for systems of the incompressible Navier-Stokes and Euler equations, where the discretization is based on the primitive variables.

Multilevel Evaluation of Point Forces

Witsch
Angewandte Mathematik Universitaetsstr. 1 40225 Duesseldorf Germany

The fast evaluation of point forces as e. g. Coulomb forces is an essential requirement for molecular dynamics. We investigate a multilevel type algorithm. It does not rely on potential forces or the precise form of the forces. We describe the algorithm, report on first numerical results and discuss the complexity in different situations.

Stranger than Fiction: On smoothing symmetric nine-point operators.

Irad Yavneh
Department of Computer Science Technion - Israel Institute of Technology
Haifa, 32000 Israel

Analytical formulae are derived for the smoothing factors yielded by damped Jacobi relaxation and by red-black relaxation applied to symmetric nine-point stencil discretizations of elliptic partial differential operators in 2D. The results include point and line relaxation and full and partial coarsening. Several unusual results are implied by the formulae. Numerical results of multigrid cycles match the analytical predictions well.

systems arise, for example, in certain discretizations of Navier-Stokes equations. The main results of the talk are for an inexact Uzawa algorithm. The classical Uzawa algorithm requires the action of the inverse of the operator A . The inexact Uzawa methods replace the action of the exact inverse of A with an ``incomplete" or ``approximate" evaluation of its action. In practice, this is provided by a preconditioner for the symmetric part of A such as one multigrid V-cycle. A convergence result for the inexact algorithm will be reported which shows that the iterative algorithm is a contraction in an appropriate norm. This norm convergence is achieved without the assumption of a sufficiently accurate approximation to the inverse of A . Applications of the inexact Uzawa method to the numerical solution of steady state Navier-Stokes equations will also be discussed.

Overlapping Non-matching Grids Mortar Element Methods for Elliptic Problems

Marcus Sarkis
Department of Computer Science University of Colorado Campus Box 430
Boulder, CO 80309-0430

We introduce some mortar finite element methods for solving elliptic problems discretized on overlapping non-matching grids. We focus on the two-subdomain cases, and provide an error analysis for the discretization. Numerical examples are presented to support the theory. This is a joint work with Xiao-Chuan Cai, Maksymilian Dryja.

A Comparison of Multilevel Methods for Total Variation Regularization

Panayot Vassilevksi and Gordon Wade
Department of Mathematics and Statistics
Bowling Green State University
Bowling Green, OH, 43403-0221

We consider numerical methods for solving problems involving total variation (TV) regularization for semidefinite quadratic minimization problems for image recontruction. The objective functional (before regularization) is the norm squared of (Lu-z), where u is the reconstructed image, L is a compact linear operator, and z is data containing inexact or partial information about the image. TV regularization entails adding to the objective functional a term which penalizes the total variation of u; this term formally appears as (a scalar times) the L1 norm of the gradient of u. The Euler equation for the regularized objective functional is a quasilinear elliptic equation of the form

$$[\ L^*L + A(u)\ ]\ u = -L^*z$$

in two space dimensions. Here, A(u) is a standard self-adjoint second order elliptic operator in which the coefficient a depends on u, by

$$[a(u)](x) = c/\mathrm{sqrt}(|\mathrm{grad}\ u(x)|^2 + b^2)$$

where b and c are small constants. A common and effective strategy for solving the Euler equation is the fixed point method.

Total variation regularization has enjoyed significant success in image denoising and deblurring, laser interferometry, electrical tomography, and

A fully coupled method for the resolution of the incompressible Navier-Stokes equations is presented. This method previously used by Deng et al. (1991) employs a cell-centered colocated grid, standard linearization of convection terms, central difference discretization for both convective and diffusive terms and a pressure Poisson equation approach, leading to deduce from the incompressibility constraint an equation for the pressure variable. The originality of this present work is to introduce auxiliary variables --the so-called pseudo-velocities-- to make easier the flux reconstruction step. The resulting structure of the nodal unknowns matrix consists in seven or nineteen bands of sparse blocks. Direct solvers have been used to solve coupled systems but their use for three-dimensional applications is penalized by strong storage limitations. In order to improve the global efficiency of the algorithm by seeking grid independent convergence rates, a non-linear multigrid approach is chosen by implementing a FMG-FAS (Full Multigrid-Full Approximation Scheme) procedure for the resolution of the coupled system. Numerical treatments and implementation aspects of this non-linear procedure are detailed.

Steady laminar lid-driven cavity flows calculations have been performed on three-dimensional geometries to discuss the performances of this approach. The retained test-problems were the three-dimensional versions of the ones investigated by Demirdzic et al. (1992) and Oosterlee et al. (1993) : regular, skewed and L-shaped lid-driven cavities. The non-linear multigrid approach (MGC) is compared with the single grid coupled method (SGC) and decoupled methods based on the sequential PISO algorithm (DC, DC-MG) with different pressure linear solvers respectively Krylov subspace solver and a linear multigrid solver. Computations were performed on cartesian, orthogonal and stretched grids for the regular cubic cavity, cartesian and non-orthogonal grids for the skewed lid-driven cavity (skewness angle : 30 degrees) and finally curvilinear grids for the L-shaped cavity. The chosen Reynolds numbers (Re) are equal to 100 or 400. From the whole numerical results, the main conclusion is that the non-linear multigrid approach seems quite powerful, at least more efficient than decoupled or single grid coupled methods.

Multiscale Methods: 1996 Review

A. Brandt
Department of Applied Math. and Computer Science, Weizmann Institute, Rehovot, 96100, Israel

Highlights of recent research, including: steps toward ``textbook multigrid efficiency" in fluid dynamics; calculations of n eigenfunctions of Schroedinger equation in O(n log n) computer operations; Dirac solvers for advanced models; fast adaptive integral transforms; multigrid/renormalization in mechanical statistics; climbing the scales in molecular mechanics; early vision: multiscale edge detection and picture segmentation; multilevel X-ray tomography.

A Note on Multi-block Relaxation Schemes for Multigrid Solvers

Jim E. Jones
ICASE, Mail Stop 403, NASA Langley Research Center, Hampton, VA 23681-0001
N. Duane Melson
Mail Stop 128, NASA Langley Research Center, Hampton, VA 23681-0001

Efficient and robust multigrid solvers for anisotropic problems typically use either semi-coarsened grids or implicit smoothers: line relaxation in 2D and plane relaxation in 3D. However, both of these may be difficult to implement in codes using multi-block structured grids where there may be no natural definition of a global `line' or `plane'. These multi-block structured grids are often used in fluid dynamic applications to capture complex geometries and/or to facilitate parallel processing. In this paper, we investigate the performance of multigrid algorithms using implicit smoothers within the blocks of a such a grid. By looking at a model problem, the 2-D anisotropic diffusion equation, we show that true multigrid efficiency is achieved only when the block sizes are proportional to the strength of the anisotropy. Further, the blocks must overlap and the size of the overlap must again be proportional to the strength of the anisotropy.

Directional Coarsening and Smoothing Multigrid Strategies for Anisotropic Navier-Stokes Problems.

Dimitri J. Mavriplis
ICASE, Mail Stop 403, NASA Langley Research Center, Hampton, VA 23681-0001

Unstructured multigrid techniques for relieving the stiffness associated with high-Reynolds number viscous flow simulations on extremely stretched grids are investigated. One approach consists of employing a semi-coarsening or directional-coarsening technique, based on the directions of strong coupling within the mesh, in order to construct more optimal coarse grid levels. An alternate approach is developed which employs directional implicit smoothing with regular fully coarsened multigrid levels. The directional implicit smoothing is obtained by constructing implicit lines in the unstructured mesh based on the directions of strong coupling. Both approaches yield large increases in convergence rates over the traditional explicit full-coarsening multigrid algorithm. However, maximum benefits are achieved by combining the two approaches in a coupled manner into a single algorithm. An order of magnitude increase in convergence rate over the traditional explicit full-coarsening algorithm is demonstrated, and convergence rates for high-Reynolds number viscous flows comparable to those achieved for inviscid flows are obtained. Furthermore, convergence rates which are insensitive to the degree of anisotropy in the mesh are demonstrated on meshes with aspect ratios up to 10,000:1.

residual of the momentum equation. To develop a practical method this norm is replaced by a discrete equivalent, as suggested in a recent paper by Bramble and Pasciak. Among the main results are optimal discretization error estimates for conforming finite element approximations, and well-posedness of the discrete algebraic problem.

A FOSLS approach for Three-Dimensional Pure Traction Linear Elasticity

Sang Dong Kim
Visiting Scientist
Department of Applied Mathematics, C.B. 526
The University of Colorado at Boulder
Boulder, CO 80302-0526

In this talk we develop a first-order system least-squares (FOSLS) approach for the solution of the pure traction problem in three-dimensional linear elasticity. This is joint work with Thomas A. Manteuffel and Stephen F. McCormick.

First-Order System Least-Squares: Numerical Experiments

Barry Lee
Department of Applied Mathematics University of Colorado Boulder, CO 80309-0526

We extend some of the existing FOSLS theory for general elliptic equations with mixed boundary conditions to general elliptic equations with the Robin boundary condition and to the complex Helmholtz equation with the Sommerfeld radiation condition. We then present some numerical results that demonstrate the multigrid convergence rate and discretization error given by the theory.

An Analytic Basis for Multigrid Methods for Stabilized Finite Element Methods for the Stokes Problem

Zhiqiang Cai and Jim Douglas, Jr.
Department of Mathematics, Purdue University, 1395 Mathematical Science Building, West Lafayette, IN 47907-1395

In recent years, so-called ``stabilization" techniques have been used extensively to stabilize unstable numerical methods for partial differential equations. While existing results indicate that such methods have great promise, a fast solver for the resulting algebraic equations has been missing for many such methods, possibly because of a too simple treatment of the perturbation term and the lack of symmetry of the schemes. The effect of the former is that the bilinear form is either not elliptic or not continuous with respect to norms separating velocity and pressure. The effect of the latter is that existing iterative methods cannot be applied directly. In this paper, we first describe a new absolutely stabilized finite element method for the Stokes problem, with the method being a modification of the approach of Douglas and Wang. In it, a weighted L^2-inner product is replaced by a discrete H^{-1}-inner product. Our bilinear form is then elliptic and continuous with respect to the H^1-norm for the velocity and the L^2-norm for the pressure, and

Solution of a Convolution Equation with Unbounded Domain via Multigrid

Steven M. McKay
Dept. of Mathematics Brigham Young University 310 TMCB Provo, Utah

The solution of integro-differential equations involving a convolution are of interest to researchers in the material science fields due to the similarity in solution to models such as Allen-Cahn which describe behavior of mixed alloy metals during the quenching process. It is well known that current models for this process produce regions which have a high concentration of one metal. We consider a convolution equation which is a continuous analogue to the discrete models used to study this process. The movement of the alloy from a mixed or homogeneous state to a mixed state appears to occur on two levels. There is a rapid change phase which develops the regions of high concentration, and a slower phase which occurs after the regions are developed. This slow phase is mainly concerned with sharpening of the interface between two different regions. We will apply a multigrid algorithm to the equation in two different ways: First, we will apply multigrid on an irregular grid in order to more efficiently solve the problem. Second, we will use irregular time stepping on the grids to accelerate the convergence of the solution to steady state.

First-Order System Least Squares (FOSLS) for PDEs: Basic Tools

Steve McCormick
Department of Applied Mathematics, C.B. 526
Univ. of Colorado, Boulder, CO 80309-0526

First-order system least squares (FOSLS) is a fairly recent methodology aimed reformulating many PDEs as well-posed functional minimization problems. The basic idea is to begin by introducing new variables, new but consistent equations, and new boundary conditions so that the PDE becomes a well-posed first-order system. A least-squares principle is then applied, usually based on L2-type norms and possibly incorporating preconditioners and careful scaling. The aim is to produce a minimization principle that is highly accurate, easily solved, and robust. A typical successful use of FOSLS leads to a system of loosely coupled Poisson-like equations in each variable. This means that virtually any standard type of finite element discretization and multigrid solution method can obtain optimal accuracy and efficiency. Other benefits accrue as well, including an especially powerful strategy for estimating errors.

The purpose of this talk is to provide a very short introduction to the FOSLS methodology and its basic principles, and to describe some of the algorithmic tools used in typical FOSLS applications.

First-Order System Least Squares (FOSLS) for Transport Problems

Tom Manteuffel
Department of Applied Mathematics, C.B. 526
Univ. of Colorado, Boulder, CO 80309-0526

A systematic solution approach for the neutron transport equation,

computed. These serve as an initial guess for the next stage, which uses a finer version of the coarsened Radon Transform, and which results in a four by four image, which, in turn, is interpolated to provide the initial values for the next stage. 1. A. Brandt and J. Dym, ``Fast Calculation of Multiple Line Integrals", to appear in SIAM Jour. on Sci. Comp.

A Wavelet-based Direct Multilevel Solver

David Gines
University of Colorado at Boulder
Dept. of Electrical and Computer Engineering
Box 425
Boulder, CO 80309-0425

A direct multilevel method based on multiresolution LU factorization is introduced, which, for a large class of elliptic operators, is capable of solving the corresponding linear system of algebraic equations, or computing the inverse operator, in O(N) operations. This method is somewhat similar to algebraic multigrid, in that projections between fine and coarse grids are performed directly on the matrix. We compute these projections using wavelet transforms, since such transforms are orthogonal.

We first introduce our approach within the context of multigrid. We then show how the wavelet transform may be used to construct the direct method, and give numerical results which verify the cost estimates. Finally, we show that our method also applies to integral operators which arise from strictly elliptic problems, since such operators have a sparse representation in the wavelet system of coordinates.

Multigrid Method for H(div) in Three Dimensions

Ralf Hiptmair
Lehrstuhl für Angewandte Mathematik I
Universität Augsburg
Universitätsstraße 14
D-86159 Augsburg
Germany

Stable multilevel decompositions of H(div)-conforming finite element spaces with respect to an inner product (j,v)+r(div j,div v) (r>0) form the core of multilevel approaches to the efficient iterative solution of many problems, ranging from mixed discretizations to first order least squares approaches.

For Raviart-Thomas finite elements in 2D Vassilevski and Wang [2] suggested a multilevel splitting as the basis for an optimal preconditioner. This talk examines the generalization of their ideas to three dimensions, which is highly non-trivial, since the representation of discrete solenoidal vector fields plays a key role: In three dimensions we have to resort to H(curl) -conforming finite element spaces introduced by Nédélec [1] to treat divergence free vector fields. We end up with a degenerate variational problem for the bilinear form (curl.,curl.), for which no multilevel decompositions had been available.

The central result to be presented in the talk is that a plain nodal BPX-type splitting of the nested Nédélec-spaces yields a decomposition that is stable

[6] M. Griebel, P. Oswald, On additive Schwarz preconditioners for sparse grid discretizations, Numer. Math., 66 (1994), pp. 449 ff.
[7] C. Zenger, Sparse grids, in Parallel Algorithms for Partial Differential Equations: Proceedings of the Sixth GAMM-Seminar, Kiel, 1990, Notes on Numerical Fluid Mechanics 31, W. Hackbusch, ed., Vieweg, Braunschweig, 1991.

The Analysis of Intergrid Transfer Operators and Nonconforming Multigrid Methods

Zhangxin Chen
Department of Mathematics, Box 156 Southern Methodist University Dallas, Texas 75275--0156

We analyze intergrid transfer operators for nonconforming finite elements and two approaches for defining multigrid methods for discretizations of second and fourth-order elliptic problems using these elements. The first approach is the usual one, which uses discrete equations on all levels which are defined by the same discretization. For this approach we shall find the lower and upper bounds of the energy norm of the coarse-to-fine grid transfer operators for the nonconforming elements considered. The main result of this approach from the analysis of these operators is the convergence of the $\mathcal W$-cycle with any number of smoothing iterations for some of the nonconforming elements, while for others it may diverge unless the number of smoothing iterations on all levels is sufficiently large. The second is based on the ``Galerkin approach" where the quadratic forms over coarse grids are constructed from the quadratic form on the finest grid and iterated coarse-to-fine grid operators. To analyze the second approach we shall need to bound the energy norm of these iterated intergrid transfer operators. We shall show the convergence of both the $\mathcal V$-cycle and $\mathcal W$-cycle multigrid methods with any number of smoothing steps when the energy norm of the iterated intergrid operators is bounded. The second approach also applies to partial differential problems without regularity assumptions, while the theory developed here for both approaches carries over directly to mixed finite element methods for partial differential problems as well. Numerical results are presented to illustrate the present theory.

Multiresolution Reduction of Elliptic PDE's and Eigenvalue Problems

Nicholas Coult
Department of Applied Mathematics Campus Box 526 University of Colorado Boulder, CO 80309

Coefficients of PDE's are often changing across many spatial or temporal scales, whereas we might be interested in the behavior of the solution only on some relatively coarse scale. The multiresolution strategy for reduction explicitly solves for the fine-scale behavior in terms of the coarse scale, leaving us with a coarse-scale equation. We present a multiresolution strategy for reduction of elliptic operators, and demonstrate the importance of using high order wavelets in the reduction procedure. It is known tha the non-standard form for a wide class of operators has fast off-diagonal decay and

A Multigrid Algorithm for Higher Order Finite Elements on Sparse Grids

Hans-Joachim Bungartz
Institut für Informatik, Technische Universität München, D - 80290 München
Germany

Efficient discretization techniques are of crucial importance for most types of problems in numerical mathematics, starting from tasks like how to define sets of points to approximate, interpolate, or integrate certain classes of functions as good as possible, up to the numerical solution of differential equations. Introduced in 1990 and based on hierarchical tensor product approximation spaces, sparse grids have turned out to be a very efficient approach in order to improve the ratio of the number of invested degrees of freedom or variables on the one side and the achieved accuracy on the other side for many problems in the areas mentioned above.

Concerning the sparse grid finite element discretization of elliptic partial differential equations, the class of problems that can be tackled has been enlarged significantly in the last years. First, the tensor product approach led to the formulation of unidirectional algorithms which are essentially independent of the number d of dimensions. Second, techniques for the treatment of the general linear elliptic differential operator of second order have been developed, which, with the help of domain transformation, enable us to deal with more complicated geometries, too. Finally, the development of hierarchical polynomial bases of piecewise arbitrary degree p has opened the way to a further improvement of the order of approximation.

In this contribution, we present a symmetric and an asymmetric variant of the d-dimensional higher order finite element method on sparse grids, using the hierarchical polynomial bases for both the approximation and the test spaces or for the approximation space only, resp., with standard piecewise multilinear hierarchical test functions. For both algorithms, the storage requirement at a grid point does not depend on the local polynomial degree p, and p and the resulting representations of the basis functions can be handled in an efficient and adaptive way. The latter approach, however, allows the straightforward implementation of a multigrid solver. We discuss the approximation qualities of both variants and the convergence behaviour of the multigrid algorithm, and we give numerical examples.

[1] I. Babuska, M. Suri, The p- and h-p-versions of the finite element method: An overview, Comput. Methods Appl. Mech. Engrg., 80 (1990), pp. 5-26.
[2] H.-J. Bungartz, Concepts for higher order finite elements on sparse grids, in Proceedings of the Third Int. Conf. on Spectral and High Order Methods, Houston, Houston Journal of Mathematics, L. R. Scott and A. V. Ilin, eds., pp. 159-170, 1996.
[3] H.-J. Bungartz, T. Dornseifer, Sparse grids: Recent developments for partial differential equations, to be published.
[4] H.-J. Bungartz, M. Griebel, U. Rüde, Extrapolation, combination, and sparse grid techniques for elliptic boundary value problems, Comput. Methods Appl. Mech. Engrg., 116 (1994), pp. 243-252.
[5] T. Dornseifer, C. Pflaum, Discretization of elliptic differential equations on curvilinear bounded domains with sparse grids, Computing, 56 (1996), pp. 197-214.

the rate of decay is controlled by the number of vanishing moments of the wavelets; we prove that the reduction procedure preserves the rate of decay on all scales, and, therefore, results in sparse matrices for computational purposes. Furthermore, the reduction procedure approximately preserves small eigenvalues of elliptic operators. We introduce a modified reduction procedure which preserves the small eigenvalues with greater accuracy than the standard reduction procedure and obtain estimates for the perturbation of those eigenvalues. Finally, we discuss extension of the reduction procedure to hyperbolic problems and to elliptic operators in higher spatial dimensions.

Some Applications of Black Box Multigrid to Systems

J. Dendy
Los Alamos National Laboratory MS-B284 Loa Alamos, NM 87545

Time permitting, we will discuss two topics. First, we will discuss a semicoarsening multigrid method for the solution of vertex-centered systems. Second, we will discuss a new method for the solution of the system arising in the Morel diffusion scheme. In this system, unknowns reside at cell centers and cell edges of a quadrilateral mesh. We will also discuss the solution of a new difference scheme on such meshes due to Morel and Shashkov.

Multilevel Inversion of the Radon Transform

J. Dym and A. Brandt
J. Dym Department of Math., DRB 155, USC, 1042 W 36th Pl, Los Angeles, CA, 90089-1113 A. Brandt Department of Applied Math. and Computer Science, Weizmann Institute, Rehovot, 96100, Israel

The Radon Transform measures a property of a two dimensional image along all one dimensional rays in many different angles. This information is then used to reconstruct the two dimensional image (which is not directly accesible). Examples of applications include X-ray tomography, where the interior structure of a two-dimensional slice of a patient (or any other object under study) can be divined by measuring the absorption properties through the slice at many different angles. Another example is in SAR surface imaging. The algorithm we describe here can be used for these applications, but can also be generalized to invert transforms for which no algorithm currently exists. An algorithm which can (among other things) compute the Radon Transform very efficiently (all rays in all directions are computed in O(nlogn) operations, with n the number of pixels in the image) is presented in [1]. This algorithm is at the heart of the inversion algorithm presented here. It performs the computation recursively, first building very short `partial rays' then combining them into longer (double length) pieces until finally the full set of all rays is computed. The inversion algorithm reverses this process. Starting with the complete set of rays, the `partial rays' of half the length are solved for. These, in turn, are used to solved for the `partial rays' of one-quarter length, and so on. The rays of minimal length (2, in our implementation) are used to reconstruct the image. An FMG type algorithm is used to accelerate the reconstruction. The original Radon Transform is `coarsened' to the point where it repesents a Transform of a two by two (coarse) image. This image is then interpolated to a finer (four by four) image, from which the `partial rays' of length two are

in the curl-seminorm independently of the number of levels involved. The proof first establishes an isomorphism between the quotient spaces with respect to the kernel of the curl-operator and certain discrete spaces. Then a duality argument is used to show the stability of the nodal multilevel decomposition. Some regularity assumptions are required for this step. Finally, a discrete extension procedure for Raviart-Thomas spaces finishes the proof for the general setting.

Thus, from algebraic multilevel theory we infer the optimal efficiency of both multilevel preconditioned PCG and multigrid methods. Results from numerical experiments for model problems confirm the rapid convergence of the iterations.

1 J. NÉDÉLEC, Mixed finite elements in R3 , Numer. Math., 35 (1980), pp. 315-341.
2 P. VASSILEVSKI AND J. WANG, Multilevel iterative methods for mixed finite element discretizations of elliptic problems, Numer. Math., 63 (1992), pp. 503-520.

Multigrid Preconditioning of Newton-Krylov Methods

D.A. Knoll
D.A. Knoll and W.J. Rider M.S. K717 Los Alamos National Laboratory Los Alamos, New Mexico 87545

We study the application of classic V-cycle multigrid algorithms with standard smothers as a means of preconditioning matrix-free Newton-Krylov methods. Newton-Krylov methods have been applied to a variety of problems using a "mesh-sequencing" algorithm where the solution on a coarse grid is interpolated up and used as an initial guess on a finner grid. This greatly increases the radius of convergence of Newton's method as well as accelerating the nonlinear convergence on a given grid (i.e. reducing the number of required Newton iterations). In the asymptotic limit only one Newton iteration is required per grid. However, as the grid is refined the performance of any single-grid precondtioner, whose memory requirements scale linearly with grid dimension, will degrade. This results in a significant growth in Krylov iterations per Newton iteration. This growth is of great concern if one is using a Krylov algorithm such as GMRES since the storage scales linearly and the work scales quadratically with linear (Krylov) iteration number. We demonstrate the ability of a multigrid V-cycle preconditioner to limit the growth in Krylov iterations per Newton iteration as the grid is refined. Different strategies for constructing the coarse grid Jacobians are compared. Two of the options considered are: 1) Retaining the last Jacobian from all previous grids in the "mesh sequencing" algorithm; 2) Restricting the current solution down through the coarse grids and forming the coarse grid Jacobians from this solution. We will also consider possible methods for forming the coarse grid Jacobians from the existing fine grid Jacobian. Some of the unique properties/capabilities of a multigrid preconditioned matrix-free Newton-Krylov algorithm for nonlinear boundary value problems will be demonstrated. Specifically, the ability to use a lower-order, linear, discretization to precondition a high-order, nonlinear, discretization, while maintaining Newton-like nonlinear convergence characteristics will be demonstrated.

based on a least-squares functional with a finite-element discretization, is presented. This approach includes the theory for the existence and uniqueness of the analytical as well as of the discrete solution, bounds for the discretization error and guidance for the development of an efficient multigrid solver for the resulting discrete problem.

A boundary functional is developed and included in the functional so that spherical harmonics which do not conform to the boundary conditions can be used to approximate the angular dependence.

To guarantee the accuracy of the discrete solution for diffusive regimes, a scaling transformation is applied to the transport operator prior to the discretization. The key result is the proof of the coercivity and continuity of the scaled least-squares bilinear form with constants that are independent of the total cross section and the absorption cross section. For a variety of least-squares finite-element discretizations this leads to error bounds that remain valid in diffusive regimes. Moreover, for problems in slab geometry, a full multigrid solver is presented with V(1,1)-cycle convergence factors approximately equal to 0.1 independent of the size of the total cross section and the absorption cross section.

Strategies for Adaptive Mesh Refinement for First Order System Least Squares (FOSLS)

Markus Berndt
Department of Applied Mathematics
University of Colorado at Boulder
Campus Box 526
Boulder, CO 80309-0526

The FOSLS approach to partial differential equations of second order is based on the minimization of a functional that in many cases is equivalent to a 'nice' norm (like the H^1 norm). This makes multigrid a suitable method to solve the discrete minimization problem. In many applications it is not feasible to solve the discrete problem on a uniform grid due to limited availability of computing resources time. Hence adaptive methods need to be developed to solve the discrete minimization problem up to a given accuracy.

The FOSLS functional is a natural global error measure, in fact it has the property that it is equal to zero at the minimum. We will show how the FOSLS functional provides us with an a posteriori error estimator, that can be used to generate adaptively refined grids.

Negative norm least-squares for the Velocity-flux Navier-Stokes equations

Pavel Bochev
Department of Mathematics Box 19408 University of Texas at Arlington
Arlington, TX 76019

We develop a least-squares method for the velocity-flux first-order form of the Navier-Stokes equations. This form involves the velocity gradient tensor as a new dependent variable, called velocity flux. The method is based on minimization of a functional involving a negative norm for the

an error estimate of the finite element approximation follows immediately. Then, we introduce a symmetrized form of the method which retains ellipticity and continuity with respect to the same norm. Hence, we can use any effective elliptic preconditioner associated with velocity, including one of multigrid or domain-decomposition type, along with a simple preconditioner associated with pressure, such as one of diagonal matrix type. The condition number of the preconditioned problem is then uniform in the number of unknowns.

Multigrid Algorithm with Conditional Coarsening for the Non-aligned Sonic Flow

Boris Diskin
Department of Applied Mathematics and Computer Science The Weizmann Institute of Science Rehovot 76100, Israel

A multigrid approach using conditional coarsening in constructing solvers for non-elliptic equations on a rectangular grid is presented. Such an approach permits to achieve a full multigrid efficiency even in the case where the equation characteristics do not align with the grid. The 2D sonic-flow equation linearized over a constant velocity field have been chosen as model problem. Efficient FMG solver for the problem is demonstrated.

A Comparison of Multilevel Adaptive Methods for Hurricane Track Prediction

Scott R. Fulton
Department of Mathematics and Computer Science
Clarkson University, Potsdam, NY 13699-5815

Accurate prediction of hurricane tracks may require resolving the flow both within and around the storm. Since the spatial scales in these two regions differ substantially, uniform resolution is inherently inefficient: the grid should be refined only near the storm. An adaptive multigrid barotropic model (MUDBAR) which does this has been described previously. Based on the nondivergent barotropic vorticity equation, this model uses an adaptive multigrid scheme to refine the mesh only around the moving storm.

This paper describes and evaluates improvements to the MUDBAR model. We concentrate on the solution algorithm, focusing on the method of Berger and Oliger (which uses only local coarse grids) and the method of Bai and Brandt (which uses full coarse grids with FAS processing). We also plan to treat the FAC method. Preliminary results show significant differences in accuracy between these methods; however, these differences are small compared to the savings of simply using nonuniform resolution. For this problem, conservation of energy, vorticity, and enstrophy is crucial; conservation properties of the various methods will be compared. Finally, we investigate the performance of a fully self-adaptive version of the model, using estimates of the local truncation error obtained during multigrid processing to control where to refine or coarsen the grid.

A Two-Level Discretization Method for the MHD Equations

W. J. Layton, A. J. Meir, and P. G. Schmidt
A. J. Meir
Department of Mathematics
218 Parker Hall
Auburn University
Auburn, AL 36849

In this paper we describe and analyze a two-level method for discretizing the equations of stationary, viscous, incompressible magnetohydrodynamics. The MHD equations model the flow of a viscous, incompressible, electrically conducting fluid, interacting with magnetic and electric fields. These arise in plasma physics, nuclear reactor technology, and geophysics.

The suggested algorithm involves solving a small nonlinear problem on a coarse mesh and then one large linear system on a fine mesh.

A multigrid smoother for high Reynolds number flows

Erik Sterner
Dept. of Scientific Computing, P.O. Box 120, S-751 04 Uppsala, Sweden

The linearized Navier-Stokes equations are solved in 2D using a multigrid method where a semi-implicit Runge-Kutta scheme is the smoother. With this smoother the stiffness of the equations due to the disparate scales in the boundary layer is removed and Reynolds number independent convergence is obtained.

A FMG-FAS Procedure for the Fully Coupled Resolution of the Navier-Stokes Equations on Cell-Centered Colocated Grids

G.B. Deng, J. Piquet, X. Vasseur and M. Visonneau
Ecole Centrale de Nantes
LMF UMR CNRS C6598, CFD Group,
1, rue de la Noe, B.P. 92101
F-44321 Nantes cedex 3, FRANCE

These last twenty years, the search of robust and efficient strategies for the numerical resolution of the steady or unsteady incompressible Navier-Stokes equations has been a crucial task giving rise to numerous methods. The major obstacle of these equations lies in the absence of pressure terms in the incompressibility constraint. Several ways have been suggested to overcome this difficulty. The first trend is represented by pressure correction methods like SIMPLE or PISO procedures. The main drawback of such techniques lies in the slowing down of convergence when the number of grid points or the clustering ratios over curvilinear grids increase. A possible cure consists in using non-linear multigrid with sequential pressure correction methods as basis solvers (or smoothers). The second trend is to solve the incompressible Navier-Stokes equations in a locally or fully coupled manner, where momentum and continuity equations are solved simultaneously. Coupled strategies for the resolution of the Navier-Stokes equations in primitive variables allow to develop robust solvers and subsequently to understand the limitations of segregated methods.

MULTIGRID METHODS FOR STRESS INTENSITY FACTORS AND SINGULAR SOLUTIONS

Susanne C. Brenner
Department of Mathematics
University of South Carolina
Columbia, SC 29208

We consider the Poisson equation with homogeneous Dirichlet boundary condition on a bounded two-dimensional polygonal domain with re-entrant angles (and possibly cracks) where the right-hand side is in L2. It is well-known that the unique solution u in the Sobolev space H1 has a singular function representation u=(k1s1+k2s2 +...+kJsJ)+w, where sj is the singular function associated with the jth re-entrant angle, the kj's are the stress intensity factors, and w is the regular part. Because of the singular functions, the solution u is singular in the sense that it is not in H2, and the convergence rates of standard finite element methods are adversely affected.

In this talk we will show that the convergence rates can be improved using standard finite elements on quasi-uniform grids by combining the full multigrid nested iteration technique and the singular function representation. We will present a multigrid method for the computation of the singular solution u and the stress intensity factors using linear finite elements on quasi-uniform grids. The resulting approximate solution to u is a linear combination of the singular functions and a piecewise linear function. The convergence rate of the approximation to u is of order almost 1 in the energy norm, and the convergence rate of the approximate stress intensity factors is of order almost 1 + pi/a, where a is the measure of the largest re-entrant angle. This method can be modified to produce an almost order 2 convergence rate for the stress intensity factors when the right-hand side is in the Sobolev space H1.

We will also discuss the general case where the right-hand side is in the Sobolev space Hm. Using the Lagrange Pm+1 element, the stress intensity factors can be computed with a convergence rate of order almost m+1 by a multigrid method on quasi-uniform grids.

The costs of all the algorithms are proportional to the number of elements in the triangulation.

Inexact Uzawa Algorithms for Nonsymmetric Saddle Point Problems

Joseph Pasciak
Dept. of Mathematics, Texas A&M University, College Station, TX 77840

In this talk I shall consider iterative algorithms of Uzawa type for solving nonsymmetric linear block saddle-point problems of the form

```
| A   B | |U|   |F|
|       | | | = | |
| B^t 0 | |P|   |G|.
```

Specifically, I will consider the case when the upper left block A is nonsymmetric linear operator with positive definite symmetric part. Such

estimation of permeabilities in porus medua flow models. Its main advantage is that it improves the conditioning of the optimization problem while not penalizing discontinuities in the reconstructed image. The main difficulty in its use lies in the fact that the Euler equation is nonlinear with rapidly varying coefficients and can have a rather large number (e.g., 640-squared) of degrees of freedom.

In this paper we present results from numerical experiments in which we use a fixed point approach to solving the the Euler equations, using various multilevel preconditioners. Specifically, we shall explore the performance of the "Hierarchical Basis" method, the "wavelet-modified Hierarchical Basis" method, and a conventional multigrid method.

An Energy-Minimizing Interpolation for Multigrid Methods

Wing-Lok Wan
Department of Mathematics
University of California
Los Angeles, CA 90095-1555

We shall study multigrid methods from energy minimizations and approximations. Through the analysis of a multigrid method in 1D, we introduce the concepts of stability and the approximation property in the classical theory. Based on them, we derive an energy-minimizing interpolation and present a two level analysis for it. Issues on coarsening are also addressed. Finally, we demonstrate the effectiveness of the multigrid method by applying it to unstructured grids computations and discontinuous coefficient problems.

Krylov subspace acceleration method for nonlinear multigrid schemes

Takumi Washio
C\&C Research Laboratories, NEC Europe Ltd., D-53757 Sankt Augustin, Germany
C. W. Oosterlee
GMD, Institute for Algorithms and Scientific Computing, D-53754 Sankt Augustin, Germany

It is well-known that multigrid solution methods are the optimal O(N) solvers, when all components in a method are chosen properly. For difficult problems, like for certain systems of nonlinear equations, however, it is far from trivial to choose these optimal components. The influence on the multigrid convergence of combinations of complicated phenomena, like convection-dominance, anisotropies, nonlinearities or non M-matrix properties is often hard to predict. Problems might then occur with the choice of the best under-relaxation parameter in the smoother, with the choice of the coarse grid correction, or with the transfer operators. In this talk, we are concentrating on nonlinear problems, and aim to construct a nonlinear acceleration equivalence to GMRES for linear problems.

Another (more known) research direction is to construct efficient nonlinear solution methods on the basis of a global Newton linearization. The resulting linear system is then solved with a linear multigrid method, or with Krylov-type methods (Newton-Krylov methods). A disadvantage of these

A multigrid analysis for an anisotropic problem.

Xuejun Zhang
Department of Mathematics
Texas A&M University
College Station, TX77843

In this paper, we provide an analysis for the performance of a standard multigrid method with a line smoother for solving the finite element equation of an anisotropic problem on a rectangular domain. We shall consider the anisotropic equation of the form

$$-[(a(x,y)\, u_x)_x + (\mathrm{epsilon}(x,y)\, u_y)_y] = f,$$

where a(x,y) is of unit size and epsilon(x,y) is possibly small.

For this anisotropic problem, the standard finite element solution has a ``poor" approximation property and hence the coarse grid solve in the multigrid algorithm is not effective in reducing the smooth components of the errors. It is known that the standard multigrid algorithm with a Jacobi or a Gauss-Seidel smoother does not provide a uniform reduction in the error. The remedy is to use a smoother, such as the line Jacobi smoother, that is effective in reducing components of error in a larger spectrum. For example, numerical experiments and heuristic arguments show that the multigrid method with line Jacobi smoother has a uniform reduction in the error. In this paper, we provide a rigorous justification of this heuristic idea.

This is a joint work with James H. Bramble.

An Adaptive Multilevel Method for Sparse Grid Discretizations of PDEs Based on the Finite Difference Approach

Michael Griebel
Uni Bonn, Institut f. Angewandte Mathematik, Wegelerstr. 6, D 53115 Bonn

For the efficient representation of discrete functions and for the solution of PDEs, the sparse grid technique has been developed in the last years. It is mainly based on the finite element approach using a specific tensor product of 1D hierarchical basis ansatz functions. The resulting linear system can be solved efficiently by multilevel methods. While usual discretization methods require basically O(h-d) grid points, the sparse grid approach needs only O(h-1 log(h-1)d-1) grid points. Here, h denotes the mesh size employed and d denotes the dimension of the problem. The accuracy of the approximation, however, deteriorates pointwise and with respect to the L2- and Lmax-norm only slightly from O(h2) to O(h2 log(h-1)d-1) provided that the function to be represented is sufficiently smooth. In the non-smooth case, adaptive refinement can be applied straightforwardly and helps to maintain the advantage of the sparse grid approach over an adaptive conventional h-version of the finite element method. However, the setup and solution of the linear system is quite complicated, especially in the case of PDEs with non-constant coefficient functions. A sparse grid approach using the finite difference philosophy is in this respect much more simple and gives in some cases even a better preformance, i.e. a O(h2) accuracy without the log-terms. However, the resulting systems are not more symmetric in general. Furthermore, the solver employs the BiCG iteration and a multilevel preconditioner using so called prewavelets. It converges independently of the mesh size of the problem. We report on our method and its

computing time and memory space. This paper is devoted to the description of a computational algorithm for two-phase immiscible, incompressible flows in an adaptive composite grid. An IMPES approach is applied. The pressure equation is discretized on locally refined grids with finite volume scheme. To solve this equation, a FAC method outlined by McCormick is used. For the transport equation, we propose the finite difference schemes : upwind are MUSCL. An object oriented programming language C++ is selected for the implementation. It provides a dynamic memory management and a convenient data structure for evolutive problems. Results for some examples illustrate the efficiency of proposed algorithm.

Multigrid Methods for Standing Wave Equations

Ira Livshits
Weizmann Institute of Science, Rehovot, Israel.

In this work we present a new multigrid solver for the standing wave equations with radiation boundary conditions. The straightforward application of standard multigrid technique cannot provide an efficient solver to these equations, since some special Fourier error components (with frequencies depending on the wave number) have no efficient reduction: They are almost invisible for any relaxation on the fine grids and have no accurate approximation on the coarse grids. Therefore, this type of error needs special treatment. Our approach is based on the fact that each such problematic error can be factorized by representing it as the product of a certain high-frequency Fourier component and a smooth envelope function. The idea is then to reduce this type of error by approximating there smooth envelope functions on the coarse grids. An additional advantage of this approach is that it allows a natural introduction of the radiation boundary conditions.

NON-NESTED MULTIGRID METHODS IN FINITE ELEMENT LINEAR STRUCTURAL ANALYSIS

M.L Bittencour
DPM/FEM/UNICAMP Campinas/SP - Brazil Zip Code: 13083-971
R.A. Feijóo
LNCC/CNPq Rio de Janeiro/RJ - Brazil

The main feature of multigrid methods is a assimptotically linear behavior of the solution cost of systems of equations. In most cases, nested meshes are used, simplifying the restriction and prolongation operators present in multigrid strategies. The geometry complexity in engineering problems, mainly in three-dimensions, makes it difficult to generate nested meshes. For this purpose, algebraic multigrid methods or non-nested approximation spaces are used.

This work considers non-nested multigrid methods applied to two and three-dimensional elastic problems using the finite element method. The generation of non-nested meshes requires the use of automatic mesh generation procedures. The present work uses non-structured and the Delaunay mesh generation techniques. Another important aspect is related to the implementation of the transfering operators based here on quadtree and octree data structures.

The results are compared with direct and iterative conjugated gradient methods. For three-dimensional problems, it was obtained a linear average cost

numerical scheme to be presented in this talk is second-order accurate in both space and time, is stable for any size of time-steps, and numerical errors are strongly damped when a time step is large.

Implicit schemes normally involve solving a large set of algebraic equations at each time step. Exact solvers for the set of equations may not be recommended even for linear problems in two- and three-dimensions. In our scheme, we use the multigrid method to iteratively solve the set of algebraic equations. The implementation of the multigrid method in the scheme dramatically reduces the number of iteration required to reach a required accuracy. Normally, the nonlinearity is a headache in the multigrid method. The coarse grid correction normally doesn't work if the set of nonlinear equations are linearized. Since we iteratively solve the set of nonlinear equations without involving any linearization, the coarse grid correction plays an important role in our scheme. As a result, only a small number of iterations and a small amount of CPU time are needed in our scheme even for nonlinear problems.

Typically, a heat transfer problem may involve more than one kind of material. The ratio between thermal diffusivities of different material may be very large. Therefore, in principle a scheme based on Taylor expansion will not work across interfaces of different materials. Our scheme is based on the physics principle involved in the interfaces, and therefore, it works correctly and efficiently even for the problems involved in laser fusion, in which the ratio of thermal diffusivities for different materials may be as high as 10^9.

Uniform Wellposedness of a Mixed Formulation of Symmetric Problems with Rough Coefficients with Application to Highly Nonhomogeneous Linear Elasticity

Andrew Knyazev
aknyazev@math.cudenver.edu
http://www-math.cudenver.edu/~aknyazev
Department of Mathematics, University of Colorado at Denver
P.O. Box 173364, Campus Box 170, Denver, CO 80217-3364

We prove that a standard mixed formulation of symmetric problems with large jumps in coefficients is uniformly wellposed in a standard norm, independent of the jumps, under some natural assumptions. As an application, we consider a Hellinger-Reissner formulation of nonhomogeneous Lame equations for media with (almost) rigid inclusions. The (almost) incompressible case is covered as well.

A Multigrid-Algorithm Solves the Laplace-Equation in 2D-Electrostatic Structures for the Microwave Circuit Design

Kulke
IMST: Institute of Mobile and Satellite Communication Techniques, Carl-Friedrich-Gauss-Str. 2, Germany, D-47475 Kamp-Lintfort

A multigrid solver was developed to determine the static potential distribution of 2-dimensional microwave quasi TEM-guides. The boundary conditions of these rectangular structures can be Dirichlet or Neumann walls, which are electrical or magnetical walls in the case of waveguides. An

(grad-operator). The resulting equivalent circuit, which consists of a serial C' and L' and a parallel R' can be taken as an interface to any RF- or digital circuit design tool.

/1/ P. Pogatzki, R. Kulke, T. Sporkmann, D. Köther, R. Tempel, I. Wolff: "A Comprehensive Evaluation of Quasi-Static 3D-FD Calculations for more than 14 CPW Structures - Lines, Discontinuities and Lumped Elements", IEEE MTT-S, Volume 2, pp. 1289-1292, San Diego, May 1994
/2/ W. Hackbusch: "Multi-Grid Methods and Applications", Springer Verlag New York Tokyo, 1985
/3/ M. Schuster: "Lösung dreidimensionaler elektrostatischer Randwertprobleme mit dem Multi-Grid-Verfahren", Diplomarbeit an der Uni-Duisburg, Fachgebiet Allgemeine und Theoretische Elektrotechnik, März 1993

Multigrid Methods for the Solution of Singularly Perturbed Differential Equations.

Paul A. Farrell
Department of Mathematics and Computer Science, Kent State University, Kent, OH 44242.

Singularly perturbed equations, in which a small parameter multiplies the highest derivative term, are model equations for many processes such as convection dominated flows. A number of methods in the literature seek to satisfy a criteria known as uniform convergence, in which the error constants are independent of the small parameter as well as the mesh size. One class of such methods involves the use of a refined mesh in the boundary layer(s). We consider the use of multigrid methods for the solution of the difference schemes arising from such discretizations. In the case where the difference scheme uses upwinded differences, the resulting matrix can be highly non-symmetric.

Asymptotic Stability of a 9-Point Multigrid Algorithm for the Convection-Diffusion Equations

Jules Kouatchou
Department of Mathematics
The George Washington University
Washington, DC 20052

We consider the solution of the convection-diffusion equation in two dimension by a 9-point discretization formula combined with multigrid algorithm. We analytically prove the epsilon-asymptotic stability of the coarse-grid operators. Two strategies are examined. A method to compute the asymptotic convergence is described and applied to the multigrid algorithm.

Exponentially fitted hierarchical bases multigrid for the convection-diffusion equation

Marc Küther
Institute for Applied Mathematics, University of Freiburg, Hermann-Herder Str. 10, 79104 Freiburg, Germany

In this paper we construct hierarchical bases for an exponentially fitted finite element discretisation of the one-dimensional stationary convection-diffusion

greatly in recent times, largely because of the apparent applicability of AMG to large, sparse problems discretized on unstructured grids. AMG offers the potential of true multigrid performance on such problems, which are extremely difficult to solve efficiently using other methods.

AMG works by using the algebraic properties of the operator matrix to select some of the equations as "coarse-grid" equations. As in geometric multigrid, the error is smoothed on the full (fine-grid) problem by relaxation, the residual is then transferred to the coarse equations, where the error can be computed at greatly reduced cost. The error is then transferred back to the fine grid, where it is used to correct the original approximation. Recursive application of this technique leads to a multigrid algorithm.

AMG thus consists of two distinct phases. First, a "setup" phase occurs, during which the selection of coarse- and fine-grid equations is made. The intergrid transfer operators are constructed in this phase. Next, the "solution" phase occurs, in which the multigrid cycling takes place.

Many of the problems of current interest are extremely large, with systems having millions of equations and unknowns. Efficient solution of such problems, therefore, will demand the application of parallel procesing technology. Multigrid lends itself well to parallelization, and as a result, the solution phase is easily parallelized, using the same techniques as are used in geometric multigrid. However, the fundamental algorithms of the setup phase are inherently serial in nature, and efforts to increase the efficiency through parallelization have not been terribly successful.

This paper describes new algorithms for parallel implementation of the setup phase. Much of the setup phase is essentially a graph coloring problem, and the algorithms are thus closely related to the question of parallel implementation of graph coloring algorithms. Discussion of the algorithms centers around the mapping of the original equations to the processors, the simultaneous coloring of the individual graphs within processors, and the need for communication to resolve conflicting assignments.

Cache Based Multigrid Algorithms

Craig C. Douglas
University of Kentucky
Department of Mathematics
715 Patterson Office Tower
Lexington, KY 40506-0027

Multigrid methods combine a number of standard sparse matrix techniques. Usual implementations separate the individual components (e.g., an iterative methods, residual computation, and interpolation between grids) into nicely structured routines. However, many computers today employ quite sophisticated and potentially large caches whose correct use are instrumental in gaining much of the peak performance of the processors. This is true independent of how many processors are used in a computation.

We investigate when it makes sense to combine several of the multigrid components into one, using block oriented algorithms. We determine how

Martin H. Schultz
Yale University
Department of Computer Science
P.O. Box 208285
New Haven, CT 06520-8285

Alternating Direction Implicit (ADI) methods have been in use for a long time for the solution of both parabolic and elliptic partial differential equations. In the case where good estimates of the eigenvalues of the operator are available, the convergence of these methods can be dramatically accelerated.

However, in the case of computation on parallel computers, the solution of the tridiagonal system can impose an unreasonable overhead. We discuss methods to lower the overhead imposed by the solution of the corresponding tridiagonal systems. The proposed method has the same convergence properties as a standard ADI method, but all of the solves run in approximately the same time as the "fast" direction. Hence, this acts like a "transpose-free" method while still maintaining the smoothing properties of ADI.

Algorithms are derived and convergence theory is provided. Numerical examples on serial, parallel, and clusters of processors are provided showing how much of a speed up can be gained by the new method.

A Parallel Adaptive Multilevel Method Using the Full Domain Partition

William F. Mitchell
Applied and Computational Mathematics Division
Bldg 820 Rm 365
NIST
Gaithersburg, MD 20899-0001

Adaptive multilevel methods combine multigrid iteration with adaptive refinement to produce fast O(N) solutions on sequential computers. The efficient use of these methods on parallel computers is currently a research topic. Recently the Full Domain Partition (FUDOP) was introduced to distribute adaptive grids over the processors of a distributed memory architecture such that multigrid maintains optimal convergence rates with only two communication steps per cycle. With FUDOP each processor receives a partition of the grid plus the minimum number of extra grid elements to cover the full domain. When considered level by level, FUDOP can be interpreted as domain decomposition with overlapping subdomains on each level. The Extended Full Domain Partition, FUDOP(k), adds additional grid elements to guarantee k levels of overlap on each multigrid level. In this talk the Extended Full Domain Partition will be defined, a multigrid algorithm using FUDOP(k) will be described, and experimental results will be presented.

as coarser grids for a multi-level method (as suggested already in [1]).

Tests have been performed using such a method as preconditioner for a regular cube and a crankshaft segment, which were modeled by tetrahedrons and hexagonal elements. In both cases about 15 cg-iterations are necessary to reduce the residual by 6 orders of magnitude. For the crankshaft segment each iteration involves more smoothing steps than in the other case. In both cases the new solver needs substantial less memory compared to the fastest solver provided by MSC/NASTRAN. Preliminary performance comparison on small test cases (about 10.000 degrees of freedom) indicate, that the multi-level approach is at least as fast as the currently available fastest MSC/NASTRAN solver. Therefore substantial performance improvements are expected for full-size industrial problems.

[1] Axelsson, O.; Gustafsson, I.: Preconditioning and two-level multigrid methods of arbitrary degree of approximation. Math. Comp., 40, pp. 219-242, 1983.

Parallel U-Cycle Multigrid Method

Dexuan Xie
Courant Institute of Mathematical Sciences, New York University,
251 Mercer Street, New York, NY 10012

L. Ridgway Scott
Department of Mathematics, University of Houston, Houston, TX 77204

A simple way to avoid idle processors in implementing the multigrid method based on a sequence of nested grids, \Omega_j for j = 1, 2,..., l with \Omega_1 being the coarsest grid, on a parallel computer is to select a proper grid \Omega_j with 1 < j < l as the new coarsest grid. The aim of this paper is to show that such an approach is an efficient way to implement the multigrid method on a large scale, multiprocessor computer. In particular, the variant of the V-cycle generated by this approach, which is called the U-cycle, is studied in this paper. It is shown that the convergence rate \rho(j) of the U-cycle with grid \Omega_j being the coarsest grid is a decreasing function of j, and the coarsest grid equations of the U-cycle can be solved approximately without increasing the total number of U-cycle iterations. Then, a parallel U-cycle is defined by using domain partitioning techniques, which can be implemented on a MIMD multiprocessor computer without any idle processors. An analysis of the time complexity of the parallel U-cycle shows that the parallel U-cycle is fully scalable, and can have super-linear speedup in comparison to the original V-cycle. Further, the performance of the parallel U-cycle in the memory-constrained case is discussed. Numerical results are presented showing that the U-cycle can have a better performance than the V-cycle on a sequential computer while the parallel U-cycle has a high efficiency on a large scale, MIMD multiprocessor computer. Experiments are presented for both the Intel Paragon and the IBM SP2.