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ABSTRACT

It is demonstrated that the velocity shear, intrinsic to the e^+e^- plasma present in the pulsar magnetosphere, can efficiently convert the nonescaping longitudinal Langmuir waves (produced by some kind of a beam or stream instability) into propagating (escaping) electromagnetic waves. It is suggested that this shear induced transformation may be the basic mechanism needed for the eventual generation of the observed pulsar radio emission.

Subject headings: plasmas — pulsars: general — waves

It is generally believed that the radio emission from a pulsar has its origin in the processes occurring in its magnetospheric plasma which has two main constituents: an ultrarelativistic (primary) beam, and a relativistic (secondary) e^+e^- plasma, created via the *pair cascade process* (Sturrock 1971). These processes (dependent, perhaps, on the differential dynamics of these constituents) generate a variety of waves some of which propagate out of the magnetosphere, travel through the interstellar medium and are seen as radio emission by a distant observer (Ginzburg and Zheleznyakov 1970). Over the years, several different models for the pulsar radio emission (Ginzburg and Zheleznyakov 1975; for the most recent and comprehensive review see, e.g. Melrose 1995) have been suggested, and certain aspects of the phenomenon, like the polarization properties of the emission, are rather well understood

(Kazbegi et al. 1991; Kazbegi, Machabeli and Melikidze 1991; Kazbegi et al. 1996). However, there are still many unanswered questions. One of the most significant and puzzling problems is the delineation of a satisfactory mechanism for the conversion of potential waves (like the Langmuir waves), readily generated in the magnetosphere, into escaping radio waves. In this *letter* we propose that the velocity shear inherent in the magnetospheric e^+e^- plasma can provide the desired conversion mechanism; this may lead to a more comprehensive theory for the generation of the observed radio emission.

The first step in this process, perhaps, is the excitation of Langmuir waves by some kind of a beam or two-stream instability (Ruderman and Sutherland 1975; Cheng and Ruderman 1980; Asseo, Pellat, and Rosado 1980; Asseo, Pellat, and Sol 1983). Initially, the instability was attributed to the primary ultrarelativistic electron or positron beam. However, the beam has too low a density and too large a Lorentz factor, so that the characteristic growth time turns out to be a few times more than the time needed for the beam particles to escape the pulsar magnetosphere (Benford and Buschauer 1977; Egorenkov, Lominadze, and Mamradze 1983). In order to overcome this difficulty Usov (1987) (see also Ursov and Usov 1988) suggested the interesting idea of a *nonstationary plasma flow.* According to this model clouds of e^+e^- plasma are injected into the pulsar magnetosphere from time to time (with small enough intervals). Fast particles from the following clump overtake slower ones from the preceding clump creating favorable conditions for the development of a two-stream instability leading to the generation of Langmuir waves propagating along the magnetic field lines. In this model the instability is attributed to the dense and low Lorentz factor e^+e^- plasma, and its growth rate is found to be large enough. Thus it appears that by either Usov's or through an alternative mechanism, it should be possible to produce Langmuir waves of sufficient intensity.

The second crucial step in the development of a model is to pinpoint a mechanism(s) which will convert the energy "accumulated" in the Langmuir waves into the energy of such waves that can escape out of a pulsar magnetosphere.

There seem to be a variety of physical processes which could mediate mode conversion: induced wave scattering (Machabeli 1983), wave-wave interaction (Gedalin and Machabeli 1983; Mamradze, Machabeli, and Melikidze 1980), and *mode couplings due to some kind of a plasma inhomogeneity* (Melrose 1995). In the latter class, however, an extremely important inhomogeneity, i.e., the inhomogeneity of the velocity field (velocity *shear*) has, until recently, attracted very little attention (Scharlemann, Arons, and Fawley 1978; Arons and Scharlemann 1979; Kaladze and Mamradze 1984) in spite of the fact that Arons and Smith (1979) had, long ago, outlined a basic mechanism of an electrostatic instability of a sheared stream of charged particles flowing along a strong magnetic field. They conjectured that the energy may be liberated indirectly "through coupling of electrostatic modes generated by the instability to propagating electromagnetic modes." (Arons and Smith 1979, p. 728).

In this *letter* we intend to prove that this hypothesis of Arons and Smith is highly plausible. In particular, we shall demonstrate that the velocity shear of the relativistic e^+e^- plasma flow can mediate an efficient conversion of the longitudinal, nonescaping waves (Langmuir waves) into the desired electromagnetic waves which can propagate outwards. Notice that shear induced mode conversion and energy exchange is known to be an efficient and widespread phenomenon (see, e.g. Chagelishvili, Rogava and Tsiklauri 1996, Chagelishvili and Chkhetiani 1995; Rogava and Mahajan 1996; Rogava, Mahajan, and Berezhiani 1996).

We consider a collisionless, viscosity-free and *cold* e^+e^- plasma. Following Arons and Smith (1979), we neglect the plasma pressure, and model the flow by the following, relativistic two-fluid equations:

$$\partial_t n^{\pm} + \nabla \cdot \left(n^{\pm}\mathbf{V}^{\pm}\right) = 0, \tag{1}$$

$$\left[\partial_t + (\mathbf{V}^{\pm}, \nabla)\right]\mathbf{P}^{\pm} = \pm e\left(\mathbf{E} + \mathbf{V}^{\pm} \times \mathbf{B}\right), \tag{2}$$

$$\nabla \cdot \mathbf{E} = 4\pi e[n^+ - n^-], \tag{3}$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \tag{4}$$

$$\nabla \times \mathbf{B} = 4\pi e[n^+\mathbf{V}^+ - n^-\mathbf{V}^-] + \partial_t \mathbf{E}. \tag{5}$$

where the notation is standard with the speed of light taken to be unity. The equilibrium velocity of electrons and positrons in the sheared stream will be modelled by $\mathbf{V}_0^{\pm} \equiv \mathbf{V}_0 = \{U_0 + Ay,\ 0,\ 0\}$, where A measures the strength of the shear. It will be assumed that the stream is weekly sheared in the sense that Ay is much smaller than the average part U_0. The resulting momentum becomes $P_{x0}(y) \simeq P_0 + ay$, where $a \equiv mA\gamma_0^3$, $P_0 \equiv m\gamma_0 U_0$ and $\gamma_0 \equiv (1-U_0^2)^{-1/2}$ is the average Lorentz factor. In the first part of our model, the equilibrium velocities of electrons and positrons are equally sheared ($A^+ = A^- = A$) and there is no mutual streaming of the two species ($\gamma_0^+ = \gamma_0^-$).

In order to delineate the basic features of shear-induced mode conversion, we make the following simplifying assumptions; 1) the plasma is quasineutral ($n_0^{\pm} \equiv n_0$) with an equilibrium (one fluid) mass density $\rho_0 \equiv 2mn_0$, 2) the magnetic field $\mathbf{B}_0 = \mathrm{const} = B_{x0}$ tends to infinity restricting the e^+e^- plasma motion to x axis (a quasi-one-dimensional system). Thus the perpendicular dynamics will be altogether neglected, and 3) the perturbation wavevectors lie in the X-Y plane (defined by $\mathbf{B}_0$ ($\mathbf{U}_0$) and by the direction of the velocity shear). We consider, from now, the *lt waves* for which the electric field vector $\mathbf{E}$ lies in the X-Y plane, and the magnetic field perturbation $\mathbf{B} = \{0\ ,0\ ,B_z\}$ is along the z-axis.

With these simplifications, the magnetospheric plasma can be described by the following set of linearized equations:

$$D_t\rho_q + \partial_x J_x = 0, \tag{6}$$

$$D_t J_x = (\omega_p^2/4\pi\gamma_0^3)E_x, \tag{7}$$

$$\partial_x E_x + \partial_y E_y = 4\pi\rho_q, \tag{8}$$

$$\partial_t B_z = \partial_y E_x - \partial_x E_y, \tag{9}$$

$$\partial_t E_y = -\partial_x B_z, \tag{10}$$

where $D_t \equiv \partial_t + (U_0 + Ay)\partial_x$, $\omega_p^2 \equiv 8\pi e^2 n_0/m$, and we have used the *one fluid* variables: $\rho_q \equiv e(n^+ - n^-)$, the perturbed charge density, and $\mathbf{J} \equiv en_0(\mathbf{u}^+ - \mathbf{u}^-)$, the perturbed current density.

Note that (9)–(10) contain the usual time derivative, while in (6)–(7) we have the convective derivative D_t. Since it is assumed that $Ay \ll U_0$ we can approximate $\partial_t \simeq D_t - U_0\partial_x$. The advantage of the resulting system is that it may be handled by the standard method of "Kelvin modes" (see, e.g. Marcus and Press 1977, Criminale and Drazin 1990). This method requires the change of variables: $x_1 = x - (U_0 + Ay)t$; $y_1 = y$; $t_1 = t$ that leads to a substantial simplification in the solution of the initial-value problem. The differential operators, appearing in the above equations, are so transformed: $D_t \equiv \partial_t + (U_0 + Ay)\partial_x = \partial_{t_1}$, $\partial_x = \partial_{x_1}$, $\partial_y = \partial_{y_1} - At_1\partial_{x_1}$ that an initial inhomogeneity in space (y) is exchanged for a new inhomogeneity in time. The Fourier transform in the new spatial variables converts (6)–(7) and (9)–(10) to a set of first order, ordinary differential equations (ODE's) for the evolution of the spatial Fourier harmonics (SFH) (see, e.g. Chagelishvili, Rogava, and Segal 1994). The wave vector components may also be written in the original (x, y, t) coordinates: $k_x = k_{x_1}$ and $k_y(t) = k_{y_1} - Atk_{x_1}$. It is of principal importance to note that the velocity shear induces linear drifts of SFH so that initially longitudinal modes can become eventually oblique.

By introducing the dimensionless quantities: $\mathcal{D} \equiv \widehat{\rho}_q/en_0$, $J \equiv \widehat{J}_x/en_0$, $e_{x,y} \equiv (k_{x_1}/en_0)\widehat{E}_{x,y}$, $b_z \equiv (k_{x_1}/en_0)\widehat{B}_z$, $\sigma \equiv \omega_p^2/4\pi k_{x_1}^2$, $\tau \equiv k_{x_1}t_1$, $R \equiv A/k_{x_1}$, $\beta_0 \equiv k_{y_1}/k_{x_1}$, $\beta(\tau) \equiv \beta_0 - R\tau$, we can reduce the original system to the following complete set of dimensionless equations:

$$\partial_\tau \mathcal{D} = -iJ, \tag{11}$$

$$\partial_\tau J = -(2\sigma/\gamma_0^3)[4i\pi\mathcal{D} + \beta(\tau)e_y], \tag{12}$$

$$(\partial_\tau - iU_0)e_y = -ib_z, \tag{13}$$

$$(\partial_\tau - iU_0)b_z = -i[1 + \beta^2(\tau)]e_y + 4\pi\beta(\tau)\mathcal{D}. \tag{14}$$

In this letter we shall investigate the evolution of those modes for which the initial perturbations are purely longitudinal ($|k| = k_x$ and $\beta_0 = 0$). This is indeed the most important case because purely longitudinal Langmuir waves are the easiest to excite in a pulsar magnetosphere. For further analysis, it is convenient and revealing to combine (11)-(14) to obtain two equations for the variables $E(\tau) \equiv e^{-iU_0\tau}e_y$ and $D(\tau) \equiv -4\pi i\mathcal{D}$,

$$\partial_\tau^2 D + W^2 D = -W^2 R\tau e^{iU_0\tau} E, \tag{15}$$

$$\partial_\tau^2 E + (1 + R^2\tau^2)E = -R\tau e^{-iU_0\tau} D, \tag{16}$$

where $W^2 \equiv 8\pi\sigma/\gamma_0^3$.

Equations (15)–(16) clearly reveal that shear ($R \neq 0$) is responsible for the mutual coupling of the purely potential, longitudinal Langmuir oscillations (with phase velocity $\omega/k = W$) and the purely transverse electromagnetic waves (with phase velocity $\omega/k = 1$). The entire time dependence (of the coupling terms as well as of the effective frequency) is due to the nonzero shear, and will be slow or *adiabatic* for $R \ll 1$. For the problem at hand the effective shear parameter indeed turns out to be small; it is typically a few orders of magnitude smaller than unity. The detailed estimates will be given in a later larger paper.

Though, the physical meaning of Eqs. (15)–(16) is transparent enough, it is instructive to look at some representative solutions. In Fig. 1, we plot the functions $e_x(\tau)$, $e_y(\tau)$, $b_z(\tau)$, and $e_y + R\tau e_x$ (the latter function measures in dimensionless notations value of $\nabla \times \mathbf{E}$) obtained by a numerical integration of the defining equations. For this case, the values of parameters are $R = 4 \times 10^{-2}$, $\sigma = 1$, $\gamma_0 = 10$, and the initial perturbation is taken to be a pure longitudinal Langmuir wave ($e_x(0) \neq 0$, while $e_y = b_z = 0$). We see that as time progresses, the fields $e_y(\tau)$ and $b_z(\tau)$ are excited and the wave becomes more and more nonpotential ($e_y + R\tau e_x$ is increasing); the initial perturbation (longitudinal and purely potential Langmuir wave) begins to acquire transverse "features" as it evolves.

It would seem that we have now identified both pieces of the puzzle: 1) a reasonable mechanism (some kind of a beam or stream instability) for generating longitudinal, potential Langmuir waves (with $\mathbf{k}(0)\|\mathbf{B}_0$) in the e^+e^- plasma in the pulsar magnetosphere, and 2) an effective shear induced coupling to transform these nonescaping waves into the longitudinal-transversal, nonpotential waves which are perfectly capable of escaping the stellar environment.

We now propose a comprehensive model. We do this by incorporating Usov's (1987) nonstationary injection hypothesis into our model. Let us now consider two streams of e^+e^- plasma with average Lorentz factors γ_1, and γ_2 ($\gamma_2 > \gamma_1$). The resulting equations,

$$\partial_\tau^2 D_1 + W_1^2(D_1 + D_2) = -W_1^2 R\tau e^{iU_1\tau} E, \tag{17}$$

$$(\partial_\tau + i\Delta U)^2 D_2 + W_2^2(D_1 + D_2) = -W_2^2 R\tau e^{iU_1\tau} E, \tag{18}$$

$$\partial_\tau^2 E + (1 + R^2\tau^2)E = -R\tau e^{-iU_1\tau}(D_1 + D_2), \tag{19}$$

where $\Delta U \equiv U_2 - U_1$, $W_1^2 \equiv 8\pi\sigma/\gamma_1^3$, and $W_2^2 \equiv 8\pi\sigma/\gamma_2^3$, explicitly encompass both of the essential processes leading to the pulsar radio emission: The onset and amplification of Langmuir oscillations due to a built-in two-stream instability, and the subsequent conversion of these oscillations into escaping radiation. Corresponding plots are presented on Fig. 2 for two streams with $\gamma_1 = 10$ and $\gamma_2 = 10^2$ ($\sigma = 1$, as above). Figures 2(a) and 2(c) represent the zero shear case ($R = 0$), while for Figs. 2(b) and 2(d), $R = 2 \times 10^{-3}$. In the former case, the two-stream instability is "switched on," and the amplitude $e_x(\tau)$ increases with time. But $e_y(\tau) = 0$ for all times, and the wave remains potential. In the latter case, however, the presence of nonzero shear changes the situation drastically: the wave becomes nonpotential and the electromagnetic component $e_y(\tau)$ is strongly excited.

The transformation of purely longitudinal, non-propagating modes into electromagnetic waves is just one of the many mode transformation processes that can actually happen in the magnetospheric plasma (see, e.g. Rogava, Mahajan, and Berezhiani (1996) for Alfvén modes). A comprehensive paper dealing with shear mediated interactions of various linear waves sustained by an e^+e^- plasma (see, for review, e.g. Volokitin, Krasnoselskikh, and Machabeli 1983; Lominadze et al. 1986; Lyubarsky 1995) is under preparation.

In summary, we have demonstrated in this *letter* that the mode coupling induced by velocity shear could be a vital link in the chain of processes which must be invoked in order to solve the puzzle behind the pulsar radio emission. We must also remember that one of the most severe criterion, imposed on possible pulsar radio emission models is that the *bona fide* mechanism must apply to both the weak-B_0 (millisecond) and the strong-B_0 (young, fast) pulsars (Melrose 1995). In other words this criterion demands that the true mechanism must apply in a range of four to five orders of magnitude in B_0. The velocity-shear based mechanism of mode conversion seems to be tailor-made to satisfy this requirement.

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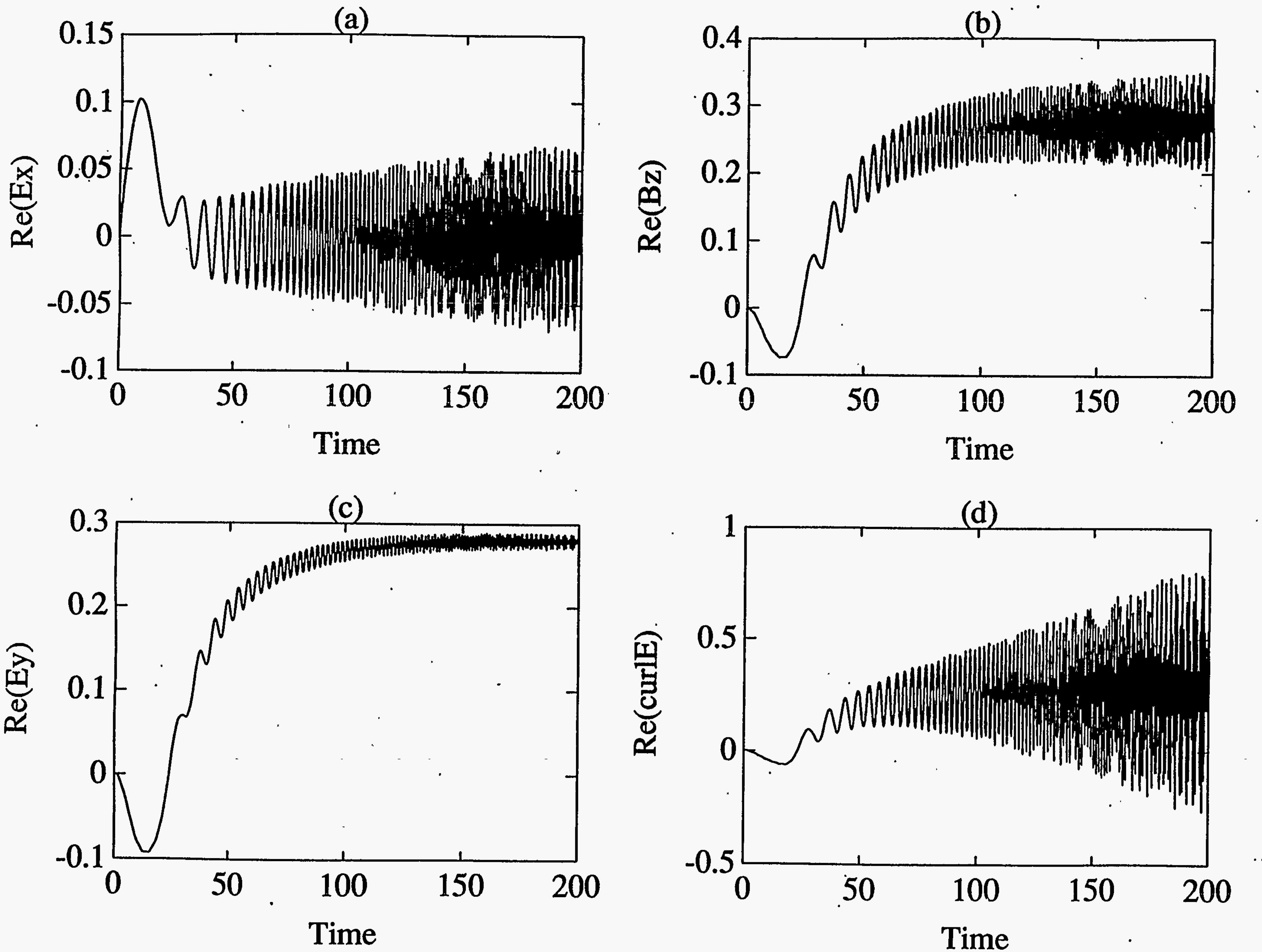
Figure Captions:

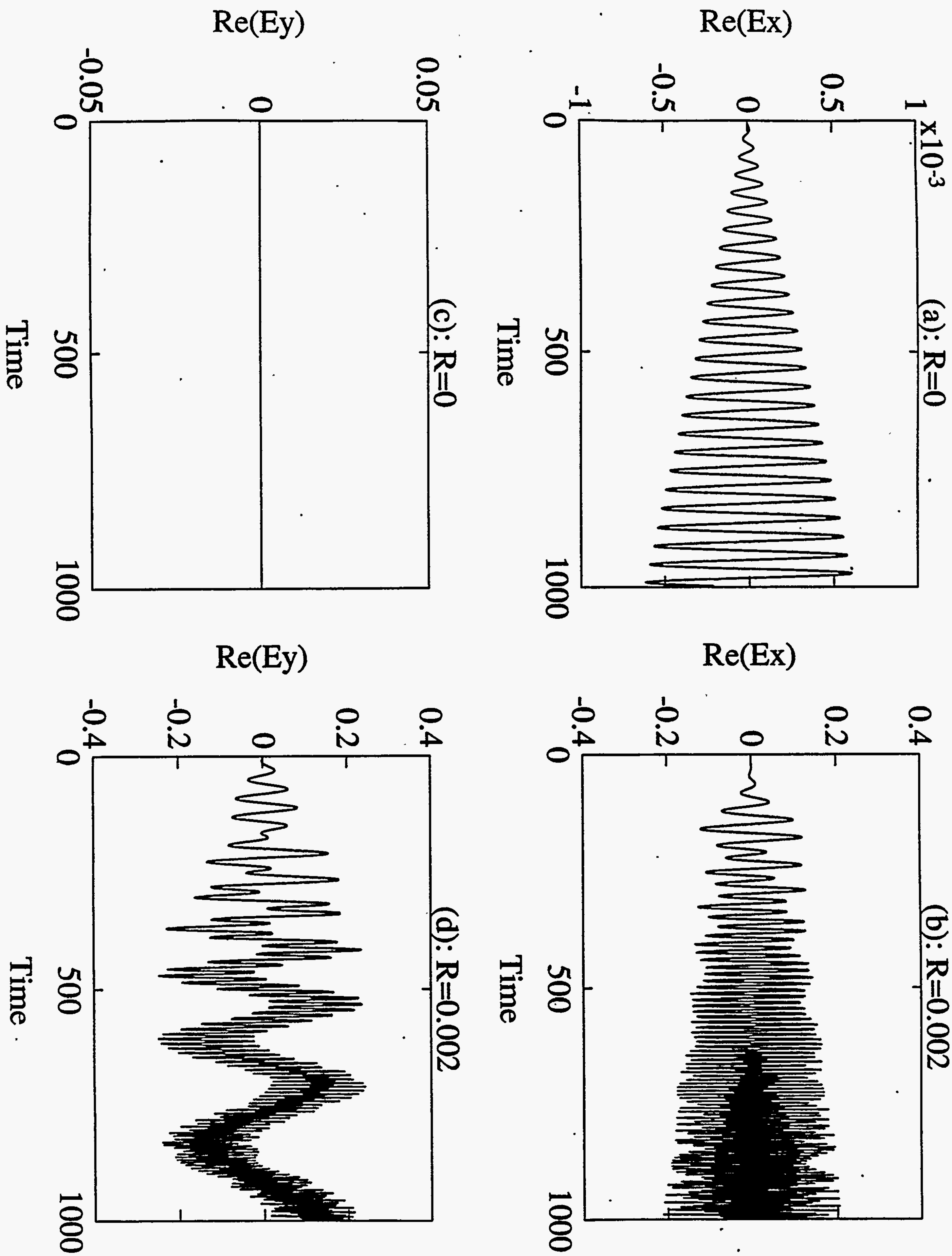
Fig. 1: Evolutionary plots ("one stream case") for real parts of $e_x(\tau)$ [(a)], $b_z(\tau)$ [(b)], $e_y(\tau)$ [(c)], and $e_y(\tau) + R\tau e_x(\tau)$ [(d)]. Time is measured in dimensionless units $\tau \equiv k_{x_1}t$. $R = 4 \times 10^{-2}$, $\sigma = 1$, and $\gamma_0 = 10$.

Fig. 2: Evolutionary plots (real parts of $e_x(\tau)$ and $e_y(\tau)$) for two streams of e^+e^- plasma with average Lorentz factors $\gamma_1 = 10$, and $\gamma_2 = 10^2$ ($\sigma = 1$). Figures 2(a) and 2(c) are plotted for the "zero shear" ($R = 0$) case, while for Figs. 2(b) and 2(d), $R = 2 \times 10^{-3}$.



(a)
Re(Ex)
0.15
0.1
0.05
0
-0.05
-0.1
0
50
100
150
200
Time
(b)
Re(Bz)
0.4
0.3
0.2
0.1
0
-0.1
0
50
100
150
200
Time
(c)
Re(Ey)
0.3
0.2
0.1
0
-0.1
0
50
100
150
200
Time
(d)
Re(curlE)
1
0.5
0
-0.5
0
50
100
150
200
Time

Fig. 1



Re(Ex)
x10^-3
(a): R=0
Time
Re(Ey)
(c): R=0
Time
Re(Ex)
(b): R=0.002
Time
Re(Ey)
(d): R=0.002
Time

Fig. 2