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Ductile Damage Model with Void Coalescence

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A general model for ductile damage in metals will be presented. It includes damage induced by shear stress as well as damage caused by volumetric tension. Spallation is included as a special case. Strain induced damage is also treated. Void nucleation and growth are included, and give rise to strain rate effects. Strain rate effects also arise in the model through elastic release wave propagation between damage centers. The underlying physics of the model is the nucleation, growth, and coalescence of voids in a plastically flowing solid. The implementation of the model in hydrocodes will be discussed.

1. Introduction

On the microscopic scale, high strain rate ductile fracture is due to the nucleation, growth, and link up of voids. We present a general 3D model of this process. The model consists of analytic expressions that approximate the behavior. Earlier 2D work [1-2] has suggested the general structure of the model.

The general phenomenology is as follows. At high strain rates, a disordered initial void configuration gives rise to spatially disordered breaking, where voids have little time to communicate with each other. In other words, when voids link up into a cluster, there is not time for the enhanced stress and strain fields, which further extend the cluster's void link up range, to form at cluster boundaries. Thus, the cluster size effect on linking is drastically curtailed. The sample breaks due to the general build up of wide spread damage. At low strain rates, the ductile damage consists of flat, disk-like clusters or cracks. The sample breaks with little general damage, when the biggest crack rapidly outstrips is neighbors. It can do so because there is time for its size to greatly enhances further linking. Consequently, the strain to fracture in the low strain rate case is significantly less than in the high strain rate case. These two different kinds of damage behavior, at low and high strain rates, are illustrated in 2D in previous work. [1-2] Inertia effects of void growth are not included but they have been shown to be negligible for typical void sizes and driving strain rates.[3] Inertia is included in the retardation of void linking due to release wave propagation.

Two analytical models have been formulated to explain the point of fracture. At high strain rates, the point of fracture is explained with random percolation theory. The organized clusters at low strain rates are explained using a probabilistic theory for cluster growth

2. General Physical Modeling and Formulas

In order to form a continuous internal surface that separates the sample, the growing voids must coalesce or link up. In dynamic situations, this will occur when the intervoid ligament undergoes a mechanical localized instability that rapidly thins it out and causes an elastic unloading in the surroundings.[4-6] Once the applied stress is great enough to establish the local instability, some local straining is then necessary to thin out the intervoid ligament.

The amount of average, external strain to add a new void to a cluster, i. e. thin out the separating intervoid ligament, depends on the cluster size. It is appreciable for small clusters but can become very small for large clusters because the external strain is greatly amplified at the periphery of a large cluster.

To model void linking, the stress conditions triggering the local instability must be known. Work by Thomason [4] has shown that the intervoid distance of linking depends on cluster geometry and the applied stresses. Thomason has modeled the local instability for periodic void arrays for the quasistatic situation with slip line fields and instability theory in both two and three dimensions.[4-6]. For example, the range of a spherical void under uniaxial tension is roughly the size of its diameter.[4]

We have generalized this work to treat single voids linking to an already formed void cluster. It is apparent that a large linked void group constitutes a big void with an enhanced linking range. The enhancement comes from the enhanced stress and strain fields at the periphery of the cluster. This enhancement occurs in uniaxial stress experiments, for example, done on sheets with drilled-in holes[7-8]. We have also generalized the void linking laws of Thomason to include both shear and tensile stresses and linking strain effects.

A major task in the model development was to extend earlier work in 2D [1-2] to 3D. The method used to model 3D damage was to include only the most probable path of damage evolution. This involves, to a certain approximation, disks of linked voids perpendicular to the direction of greatest principal stress. This orientation intercepts the greatest applied stress and causes the greatest cluster size enhancement of the applied stress at the cluster periphery. The disks are assumed to be one void thick. A greater thickness is much less likely since it would require a juxtaposition of initial void locations. Also, a greater disk thickness will not give a greater cross section to the greatest principal stress and, so, will not afford much greater peripheral stress enhancement. The shape perpendicular to the greatest principal stress is likely to be a circle because this shape will give the most stress enhancement per intercepted perpendicular surface area. Hence, we consider a population of disks of various diameters oriented perpendicular to the greatest principal stress.

The disk populations will be grown from the basic voids and will not interact. This should be accurate for the early and middle regions of damage growth.

We will need to include the possibility that the other principal stresses are almost as strong as the dominant one. This will be done by growing three populations of disks, each oriented perpendicular to one of the directions of principal stress.

To capture changes in direction of the axes of principal stress, averaged effective principal axes will be used to define the orientations of the three disk populations and their corresponding effective strains. The averaging is with respect to the total inelastic strain, i.e. both plastic and volumetric. Strain increments are included only when the local instability is active.

The above scheme should approximate well a stress history that has a dominant stress direction, but will be less accurate for many strong changes in the directions of principal stress.

More detailed modeling of history effects, here and elsewhere, is avoided because it would quickly introduce far too much complexity and too many mesh variables to be accommodated in a production hydrocode. We have tried to include the most important and most probable history effects in simple ways that still capture most of the physics

A disk is "grown" in a stepwise fashion by adding rings of linked voids to a previously existing disk. This damage growth path of adding rings is the most probable one because it enlarges a disk more effectively than other paths. The voids in the ring must link to their neighbors and to the voids on the periphery of the disk. The ring voids and the disk periphery voids are given the same model linking range which is enhanced by the disk size.

Thus, we need to know the linking range of two voids which link under the influence of the stress or strain field at the disk periphery whose field is enhanced by the size of the disk. Since the linking of the ring voids into a ring will involve, loosely speaking, a ring segment adding an additional void, the stress/strain condition at the place of linking is roughly two dimensional. We can then use some 2D linking results from Thomason[4], who gave the local stress necessary for the local instability of linking. We combine his result with a relation between the external stress and the stress available at the intervoid neck to give the following formula for r_σ, the center to center stress linking range of two voids in the ring:

$$r_\sigma = D\,[1 + (\sigma_I/\sigma_y)\sqrt{2R/D}]\,, \qquad (1)$$

where D is the void diameter, R is the disk radius, σ_I is the effective stress at the disk surface, σ_y is the plastic yield stress. The square root factor approximates the effect of the disk size to enhance the local stress/strain at the periphery of the disk. The square root can be approximately justified by 2D plane strain slip line fields. [9-10] The effective stress σ_I is equal to $\sqrt{(\sigma_\perp)^2 + (\sigma_{parallel})^2}$, where the two stress components act, respectively, perpendicular and parallel to the disk surface. The use of σ_I can be justified by the work of Green [11] on the yielding of joints between two blocks of material.

r_σ is the intervoid separation at which the local instability occurs between two voids. External strain is still required to thin down the intervoid ligament. Accordingly, we will include the strain to thin intervoid ligaments in the modeling, since it can be appreciable for small growing clusters. In general, we will use an effective integrated principal strain (inelastic) ε_I, for each of the disk populations with effective principal stress direction I. ε_I is the appropriate projection of a total, accumulated inelastic strain tensor, which is incremented only when the local instability is active. ε_I includes strains both tensile and shear with respect to the disk orientation.

We will let r_ε be the strain intervoid linking distance for two voids on the periphery of a disk. For voids closer than this distance, the intervoid ligament is actually gone, so that the two voids have become one larger void. We will use a 2D derivation for r_ε for the reasons outlined above. We will first assume that the effective relative displacement of material above and below the intervoid axis necessary to thin the intervoid ligament is (d/2)[1+(d/3D)], where d is the edge to edge separation of the voids. The value d/2 was given by Thomason[4] for the necessary displacement for small void separations. The effective relative displacement is defined as the square root of the sum of the squares of the displacements normal and tangential to the disk surface. The above displacement can be combined with a strain field enhancement factor at the disk periphery of $\sqrt{2R/D}$, and a geometric factor relating the macroscopic strain to the local ligament strain, to obtain the following expression for r_ε: $r_\varepsilon = D[1 + \eta\varepsilon_I\sqrt{R/D}]$, where η is $6\sqrt{2}$.

The relative size of r_σ and r_ε determines, for a given time step, whether or not an effective external principal strain increment $d\varepsilon_I$ contributes to the effective strain for void linking. If $r_\sigma < r_\varepsilon$, no new voids are experiencing the local instability and the effective strain increment should not be added to ε_I since it is not causing any new ligament thinning.

The use of a time integrated effective principal strain ε_I for all linking in disk population I is an approximation to eliminate history variables whose use would require too much calculation time for a production hydrocode.

When a disk grows by adding a ring, the ring diameter is taken to be midway in the link range, which is the most likely, or average position, since this gives results close to summing over possible ring positions.

In the following derivations, the index indicating the disk population will be dropped for convenience.

The probability p(c), of formation for a ring of circumference c, is given approximately by: $p(c) \approx \alpha e^{-c\gamma/fD}$, where f is $1 + \eta\varepsilon_I\sqrt{R/D}$. This equation can be obtained from an integral equation given by Domb[12]. γ is given by the nontrivial solution of the equation: $\beta e^{-\beta} = \gamma e^{-\gamma}$, where β is $\alpha' f^3 \rho D^3$, α' is a constant of about π, and ρ is the number density of void centers. The quantity γ is transcendental must be approximated by $\beta - \ln(\beta)$ for small β and $\beta e^{-\beta}$ for large β. The prefactor α is $(\beta-\gamma)/[\beta(1-\gamma)]$.

Using the above formula for p(c), we can finally find the existence probability P(R) for disks as a function of the disk radius, R, given an initial void at R=0. We add rings having the sequence of radii R_i, where $R_{i+1}=R_i+r_{\varepsilon,i}/2$ and $r_{\varepsilon,i}$ is obtained from the equation for r_ε with R set equal to R_i. R_1 is 0. The probability of formation for each ring i is given by p(c) with $2\pi R_i$ inserted for c. Thus, the probability of existence, $P(R_i)$, for a disk of radius R_i is given by:

$$P(R_i) = \prod_{k=1,i} \alpha_k exp(-\gamma_k 2\pi R_k/(Df_k)) . \qquad (1)$$

This product can be exponentiated and approximated by an integral. The results will be given in a longer paper. We note that at low strain rates the solution for large R/D asymptotes to a constant. This means that the disk probability does not die off with size but is finite. This is a feature of a correlated growth process in which bigger clusters link farther. In this case, after a cluster reaches a certain size, it can grow arbitrarily large with almost certain probability. This leads to the result that large systems will surely break.

Time delay effects are added to Eq. (5) by limiting R/D to CT_K/D, where C is a typical release wave velocity and T_K is the total linking strain time. The idea is that in order for the size enhancement of the local stress and strain fields to appear at the disk edges, release waves must sweep from each failing intervoid ligament to completely cover the disk circumference. This approximation has the effect on P of making all the factors in the product in Eq. 1 simple exponentials in R for R greater than CT_K/D. This will produce in P(R) an exponential decay in some power of R for such large R, which will greatly curtail disk growth.

In the formulas above, the void diameter, D, acts as a parameter. We give it a separate growth law in the following equation for $\dot{D}/D$:, $\dot{D}/D = 0.5(\dot{\varepsilon}_o/\sigma_o)\{\Sigma_e - \sigma_y E\}\bar{\zeta}$, where $\bar{\zeta}$ is solved for implicitly from the equation:,

$$(3/2)(\Sigma_m/\Sigma_e) = \bar{\zeta} - (\sigma_y/\Sigma_e)[\bar{\zeta}E + \mathrm{asinh}(\bar{\zeta}\phi) - \mathrm{asinh}(\bar{\zeta})] . \qquad (2)$$

where σ_o and $\dot{\varepsilon}_o$ are parameters describing the matrix strain rate dependence and E is $\sqrt{1+\bar{\zeta}^2\phi^2} - \phi\sqrt{1+\bar{\zeta}^2}$. In the above expressions, ϕ is the total porosity, and Σ_m, Σ_e are the volumetric tension and the von Mises effective deviatoric stress, respectively. This expression combines a yield surface, the strain rate dependence of the plastic flow stress (assuming a linear relationship), and includes both pressure and deviatoric stress. It contains the P-α void growth law as a limiting form. It was derived by generalizing the upper bound plasticity work of Cocks.[13]

A cluster size effect on the growth of voids has not been included to avoid complicated history effects.

Void nucleation will be modeled to occur all at once, at a stress and strain threshold given by Goods and Brown[14], which assumed a critical normal stress threshold at the inclusion boundary. In most dynamic applications in engineering materials, a large volumetric tension is suddenly applied which will tend to open up voids at the inclusions in these materials all at once.

There are two fracture criteria for a computational cell. Both are applied simultaneously. The first is simply that the existence probability of a void disk large enough to span the cell is one:

$P_I(L/2)N = 1$, where N is the number of void centers in the cell (ρV, where V is the cell volume), and L is the size of the computational cell. The I subscript refers to any one of the three void populations. When this criteria is satisfied, a single cluster has grown large enough to span and break the cell. This first fracture criteria will predominate at low strain rates where the cluster size enhancement of void linking comes fully into play. Loosely speaking, the disks have time enough during linking for the stress/strain enhancements to form at their edges and greatly promote further linking.

The second fracture criteria is that the stress linking volumes of the disks sufficiently fill in the computational cell, so that no (1D) path of solid and unlinked material exists completely spanning the cell. In this case, a sheet of stress linked voids spans the cell, the plastic flow localizes, and the cell breaks with little additional external strain. The stress linking criteria is the proper one here, and not the strain linking criteria, because lots of local strain is available for the necessary neck thinning. This criteria is equivalent to a random volume percolation of the stress linking range volumes of each disk.[15-16] This percolation will occur [D. L. Tonks, unpublished] when their overlapped volumes equal 0.92 of the total volume, with an actual voided volume fraction of about 0.30. The linking range volume of a disk is a larger disk enclosing the "bare" disk that is larger in all directions by half of its center to center void linking range. Thus, if the linking range volumes of two disks overlap, then voids in one link to voids in the other via the center to center link criteria given earlier. The second fracture criteria is given by:

$$(0.92V) = N\sum_I \int_0^{L/2} dR\tilde{P}_I(R)\,\pi\,(R + D\tilde{f}_{\sigma,I}/2)^2\, D\,(1 + \tilde{f}_{\sigma,I'''}), \qquad (3)$$

where the sum in I is over the three disk populations. Here, $\tilde{f}_{\sigma,I} = g(\sigma_I/\sigma_y)\sqrt{(2R)/D}$. and $\tilde{f}_{\sigma,I'''}$ is this expression with $0.5(\sigma_{I+1} + \sigma_{I+2})$ in place of σ_I. I+1, I+2 mean the two populations other than the Ith. V is the computational cell volume. In the above equation, the linking to voids outside the disk that are situated perpendicular to the disk face is stress enhanced, with an effective cluster size of D. The average of the two perpendicular principal stresses is used by way of approximation.

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