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Flow impedance in a uniform magnetically-insulated transmission line

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In two recent publications relativistic electron flow in cylindrical magnetically-insulated transmission lines (MITL) was analyzed and modeled under the assumption of negligible electron pressure.[1,2] Cylindrical MITLs were used because of their common occurrence, and because they are the simplest case of finite width. We show in this note that the models apply equally to MITLs of any cross section.

Assume the axis of the MITL is in the z direction, where the boldface z refers to the unit vector in the z direction. Solve Laplace's equation in the transverse direction (x,y plane)

$$\nabla^2 R = 0 \quad , \quad R_c = 0 \quad , \quad R_a = Z_v$$

where the subscripts a and c refer to values at the anode and cathode electrodes and Z_v is the vacuum impedance of the MITL. Define Q(R) and I(R) by[3]

$$\vec{E} = cQ(R)\,\vec{\nabla}R \quad \textit{and} \quad \vec{B} = \frac{I(R)}{c}\,z\times\vec{\nabla}R \quad .$$

The electric voltage, V, and magnetic voltage,[1] A, are given by

$$V = -\int_c^a \vec{E}\cdot d\vec{s} = -\int_c^a cQ(R)(\vec{\nabla}R)\cdot d\vec{s} = -\int_0^{Z_v} cQ(R)\,dR$$

$$cA = -c\int_c^a (z\times\vec{B})\cdot d\vec{s} = -\int_c^a I(R)\ (\vec{\nabla}R)\cdot d\vec{s} = -\int_0^{Z_v} I(R)\,dR \quad .$$

A is the net magnetic flux per unit length. It is also the axial component of the vector potential in this static case, but this is not always true.[2]

The integral of the normal electric field and the tangential magnetic field around a line of constant R gives the enclosed charge per unit length q, and the enclosed current, i,

$$\epsilon_o\oint(z\times\vec{E})\cdot d\vec{s} = Q(R)\ [\epsilon_o/\mu_o]^{1/2}\oint(z\times\vec{\nabla}R)\cdot d\vec{s} = q$$

$$\frac{1}{\mu_o}\oint\vec{B}\cdot d\vec{s} = I(R)\ [\epsilon_o/\mu_o]^{1/2}\oint(z\times\vec{\nabla}R)\cdot d\vec{s} = i$$

In the vacuum case, when Q and I are independent of R, the capacitance per unit length, C, and the inductance per unit length, L are

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$$C = \frac{q}{V} = \frac{q}{cQZ_v} = \frac{1}{cZ_v}[\epsilon_o/\mu_o]^{1/2}\oint(z\times\vec{\nabla}R)\cdot\vec{ds}$$

$$\frac{1}{L} = \frac{i}{A} = \frac{ci}{Z_v I} = \frac{c}{Z_v}[\epsilon_o/\mu_o]^{1/2}\oint(z\times\vec{\nabla}R)\cdot\vec{ds} \quad .$$

Since $C/L = 1/Z_v^2$, clearly

$$\oint(z\times\vec{\nabla}R)\cdot\vec{ds} = [\mu_o/\epsilon_o]^{1/2}$$

and since the Laplacian of R is zero this is true around any path enclosing the inner electrode. The latter integral does not change when Q is dependent on R, so Q is always the enclosed charge for path R. Likewise I(R) is the enclosed current. In this case, where the cathode has been chosen to be inside the anode, Q and I are negative.

Taking the divergence of the electric field and the curl of the magnetic field

$$\rho = \epsilon_o(cQ(R)\ \vec{\nabla}R) = (\epsilon_o/\mu_o)^{1/2}\ \frac{dQ(R)}{dR}\ (\vec{\nabla}R)^2$$

$$\mu_o\,\vec{j} = \vec{\nabla}\times\left[\frac{I(R)}{c}\ z\times\vec{\nabla}R\right] = \frac{1}{c}\frac{dI(R)}{dR}\ (\vec{\nabla}R)^2\ z \quad .$$

The total electromagnetic force, T, is then given by

$$\vec{T} = \rho\vec{E}+\vec{j}\times\vec{B} = -\frac{\epsilon_o}{2}(\vec{\nabla}R)^2\ \vec{\nabla}R\frac{d}{dR}[I^2-c^2Q^2] \quad .$$

If electron pressure is neglected T must be zero, so $I^2 - c^2 Q^2$ must again be independent of R. This is the pressure balance relationship.[1,2]

The electric and magnetic flow impedances can again be defined as[1]

$$Z_f = \frac{V-Z_v cQ_c}{c(Q_a-Q_c)} = \frac{\int_0^{Z_v} Q(R)dR - Z_v Q_c}{Q_a-Q_c} \quad , \quad V = Z_f\ cQ_a+(Z_v-Z_f)\ cQ_c$$

$$Z_m = \frac{cA-Z_v I_c}{I_a-I_c} = \frac{\int_0^{Z_v} I(R)dR - Z_v I_c}{I_a-I_c} \quad , \quad cA = Z_m\ I_a+(Z_v-Z_m)\ I_c$$

and are again the distances of the charge and current centroids from the anode in the R coordinate system (see Eq. 4b of Ref. 1).

The electron flow can again be modeled as a single thin layer positioned at $R=Z_v - Z_f$. For a single thin layer $Z_m=Z_f$, but as discussed in Refs. 1 and 2, the small difference between them is generally not important for the magnetic voltage, A.

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