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Population Dynamics of Minimally Cognitive Individuals

Part II: Dynamics of Time-Dependent Knowledge

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POPULATION DYNAMICS OF MINIMALLY COGNITIVE INDIVIDUALS

Part II: Dynamics of Time-Dependent Knowledge

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ABSTRACT

The dynamical principle for a population of interacting individuals with mutual pairwise knowledge, presented by the author in a previous paper for the case of constant knowledge, is extended to include the possibility that the knowledge is time-dependent. Several mechanisms are presented by which the mutual knowledge, represented by a matrix **K**, can be altered, leading to dynamical equations for **K**(t). We present various examples of the transient and long time asymptotic behavior of **K**(t) for populations of relatively isolated individuals interacting infrequently in local binary collisions. Among the effects observed in the numerical experiments are knowledge diffusion, learning transients, and fluctuating equilibria. Evidence of metastable states and intermittant switching leads us to envision a spectroscopy associated with such transitions that is independent of the specific physical individuals and the population. Such spectra may serve as good lumped descriptors of the collective emergent behavior of large classes of populations in which mutual knowledge is an important part of the dynamics.

INTRODUCTION

In a previous paper [Schmieder, 1994, hereafter referred to as Paper I], the author developed an approach to simulating the emergent collective dynamics of populations of objects that have minimal cognitive ability. It is assumed that the individuals have the ability to store and process knowledge about other individuals, and that their individual actions depend on that knowledge. This approach will be most appropriate to small populations of complex individuals such as simple animals, robots, computer networks, agent-mediated traffic, simple ecosystems, and games.

To quantify this idea, a simple dynamical principle based on reasonable physical arguments was proposed: First, we define a set of dynamical equations describing the time evolution of the state vector **X** of the system. Next, we define a matrix **K** whose elements are a normalized measure of the amount of pairwise knowledge linking every pair of individuals. Finally, the dynamical equations for **X** are modified by insertion of **K** to modify (generally, weaken) the pairwise interactions. The combined {**X**,**K**} dynamical equations therefore describe the behavior of the population *modified by the cognitive ability of the individuals.*

In Paper I, it was assumed that the matrix **K** is constant in time. This enabled us to demonstrate a variety of interesting behaviors associated with partial and incorrect knowledge. For instance, we found that a set of point vortices with "completely correct" mutual knowledge would circulate forever in a smooth centric flow, whereas if they were given some "incorrect" mutual knowledge, they would move chaotically, switching intermittantly between several quasi-stable configurations and complete chaos.

In this paper, we examine some consequences of allowing the knowledge **K** to evolve in time. After some preliminary remarks about time-dependent knowledge, we identify some physically reasonable mechanisms by which **K**(t) can change. Next we examine some transient effects in small populations, and some phenomena that appear as asymptotic equilibria at long times. As expected, we find processes like diffusion, learning transients, and fluctuating equilibria. In addition, we envision a spectroscopy derived from transitions between metastable configurations. Such spectra may be useful descriptors of the emergent collective behavior of populations in which knowledge plays a significant role, independent of the specific individuals or population.

Definition of the Knowledge

We reiterate here the definition of **K** presented in Paper I. K represents the *amount of knowledge*, normalized to (-1,1), not the knowledge itself. It is derived from physically defined probabilities as follows:

We focus attention on two individuals in the population, {i} and {j}. Assume that {j} can be in any one of G possible states, and that the probability that {i} is able to correctly identify the state of {j} is p_{ij}. Then the (mutual) knowledge K_{ij} is defined as

$$K_{ij} = \begin{cases} \dfrac{G p_{ij} - 1}{G - 1} & 1/G \le p_{ij} \le 1 \\ G p_{ij} - 1 & 0 \le p_{ij} \le 1/G \end{cases}$$

The meanings of these relations for various values of p_{ij} are shown in Table 1.

For a 1-bit state, G=2, and both formulas above reduce to $K_{ij}=2\,p_{ij} - 1$, or $p_{ij} = (1+K_{ij})/2$. In the limit $G \to \infty$, $K_{ij} \to p_{ij}$.

We emphasize that p_{ij} is presumed to be determined by the complex internal structure of the individual, and therefore is traceable through physics (or perhaps biology!). It can be measured empirically by asking {i} to identify the state of {j}, and tabulating the answers from many repeated trials. Therefore, the knowledge **K** is also traceable through physics, and we may assume it is a well-defined physical quantity.

Note that **K** refers to whatever state variables of the individuals we wish. It could refer to position, size, age, color, sex, state of motion, internal state, or any other properties of interest. We could incorporate all properties into a single state variable, or we could separate them and define several matrices **K**, **K'**, **K''** ... , each with its own associated probabilities **p**, **p'**, **p''** ... .

Dynamical character of the knowledge

It is tempting to actually define the knowledge **K** as a dynamical variable, since we could simply specify additional numerical quantities (the matrix **K**(t)) that enter the physical equations of motion. We could modify the state of the system described by the configuration **X** to a system described by {**X**, **K**}, and both **X** and **K** would then evolve according to well-defined dynamical equations. In this context, **K** would act more like properties of the individuals, i.e., dynamical variables. In this case, the population is essentially expanded to include two species, those described by **X** and those described by **K**.

However, **K** really describes pairwise properties of individuals. Thus, it has the character of an interaction between individuals rather than a property of a single individual. In this context, **K** acts more like *constraints* on the system, similar to forces that constrain particles in a rigid body. We would expect that as **K**—>0, the forces of constraint would vanish, and the system would be just a collection of independent individuals.

In principle, both of these viewpoints are correct. An individual can have two kinds of knowledge: knowledge that *does not* represent other individuals, and knowledge that *does* represent other individuals. Both kinds of knowledge can affect the behavior of the individual, and both can be altered by interactions between individuals. The former are more akin to dynamical variables, while the latter are more akin to constraints. Since we are more interested in the pairwise knowledge that affects pairwise interactions, we will generally think of **K** more as a constraint on the system rather than a dynamical variable.

General behavior of knowledgeable populations

We would expect to see certain general behavior in all systems in which knowledge is part of the dynamics. One of these is knowledge *diffusion*. Suppose a quantity of knowledge is given to one individual in the population. During interactions, this knowledge is shared with other individuals, and in the absence of losses the total amount of knowledge in the population grows. If losses do occur, the total amount of knowledge in the population will grow to an *equilibrium*. Thus, the population will exhibit a *learning transient*. In a finite population, the equilibrium knowledge will exhibit *fluctuations* around its equilibrium value. We will find all these phenomena in the examples presented in this paper.

MECHANISMS OF KNOWLEDGE ALTERATION

In this section, we examine several mechanisms by which the knowledge matrix elements K_{ij} can change in time. This list is not exhaustive; rather, it is meant simply to provide several mechanisms with which we can exhibit the basic phenomena associated with variable knowledge. Included here are three individual mechanisms (creation, information, destruction), and two pairwise mechanisms (incretion, decretion). These processes are roughly comparable to inspiration, learning, forgetting, sharing, and eroding. Figure 1 shows schematics of the 5 mechanisms. In each frame, some measure of the knowledge **K** is plotted as a function of time. It is clear that the several mechanisms can compete, possibly producing equilibria at long time, and we will find this to be the case.

Our interest lies mainly in the pairwise mechanisms, since they lead to more interesting behavior. Hence we will first mention and then generally neglect the individual mechanisms.

A possibly important mechanism of knowledge alteration in populations of complex objects is group action: a group of critical size may take some action that produces new knowledge for the individuals, while a smaller group takes no action. Similarly, there might be a specific group size, or a maximum group size that can effect such actions. There will be a rich variety of such mechanisms, and the behavior of populations correspondingly complex and interesting. However, since they can be generated by a simple extension of the pairwise mechanisms, we omit explicit development of these here.

Creation

Perhaps the simplest mechanism for increasing knowledge in the population is to create it *mirabile Dei gratia*. For this purpose, we can use the Heaviside unit step function:

$$K_{ij}(t) = K_{ij}(0) + \Delta K_{ij} H(t)$$

and simply define ΔK_{ij} arbitrarily. Obviously the same mechanism can be invoked to suddenly delete the knowledge from the population.

Information

A second means of increasing knowledge is by receiving information, or *learning*. An individual {i} receives some information $\mathbf{I}_{ij}$ about another individual {j}, and stores it internally as knowledge $\mathbf{K}_{ij}$. We therefore need to relate the incoming information **I** to the stored knowledge **K**.

In the classical definition of information (Brillouin, 1962), we are presented with a system having R possible configurations. Initially, we have no knowledge of the configuration of the system. If we receive an amount of information $I = c \ln(R/R')$, where c is a units constant, we have enough knowledge to reduce the number of possible configurations to R'.

To apply this convention to the present case, let g_{ij} represent the number of states that {i} would infer that {j} has available to it. If {i} initially infers $g_{ij}(0)$, then receives information $I_{ij}(t)$, and thereby infers $g_{ij}(t)$, the classical relation is

$$I_{ij}(t) = c \ \log\left(\frac{g_{ij}(0)}{g_{ij}(t)}\right)$$

Since {i} makes guesses about the state of {j} with probability p_{ij} of being correct, {i} would infer that {j} has

$$g_{ij} \equiv \frac{1}{p_{ij}} = \begin{cases} G\dfrac{1}{1+(G-1)K_{ij}} & 0 \le K_{ij} \le 1 \\[2ex] G\dfrac{1}{1+K_{ij}} & -1 \le K_{ij} \le 0 \end{cases}$$

states available to it. Combining these relations and inverting to obtain $K_{ij}(t)$, we obtain

$$K_{ij}(t) = \begin{cases} \left[\dfrac{1}{G-1} + K_{ij}(0)\right] e^{I_{ij}(t)/c} - \dfrac{1}{G-1} & 0 \le K_{ij} \le 1 \\[2ex] \left[1 + K_{ij}(0)\right] e^{I_{ij}(t)/c} - 1 & -1 \le K_{ij} \le 0 \end{cases}$$

which gives the increase in knowledge K_{ij} due to receipt of information I_{ij}.

If we simply use the definition $p_{ij}=1/g_{ij}$ in the formula for I_{ij}, we obtain

$$p_{ij}(t) = p_{ij}(0)\ e^{I_{ij}(t)/c}$$

This relation shows that the receipt of information causes a simple exponential increase in the probability of correctly identifying the state of the transmitter.

Note that I_{ij} can be negative: this represents incorrect information that produces incorrect knowledge in {i}. It makes {i} a poorer guesser than it was before receipt of the information: $p_{ij}(t)<p_{ij}(0)$.

Let us restrict the system to have $G=2^n$ possible states, i.e., it is an n-bit system. If the initial configuration is one of zero pairwise knowledge, then $K_{ij}(0)=0$, and $p_{ij}(0)=1/G=2^{-n}$. If {i} receives exactly $I_{ij}(t)=n$ bits, it would be able to identify the state of {j} with certainty, i.e., $p_{ij}(t)=1$. This gives

$$1=(1/2^n)\exp(n/c) \Rightarrow 1/c=\ln(2)$$

From this we find the knowledge attained from receiving $I_{ij}(t)$ bits of information:

$$K_{ij}(t) = \begin{cases} \dfrac{\left[2^{I_{ij}(t)}-1\right]}{\left[2^n-1\right]} & 1 \le I_{ij} \le n \\[2ex] \left[2^{I_{ij}(t)}-1\right] & -\infty \le I_{ij} \le 1 \end{cases}$$

For a 1-bit state, n=1, and $K_{ij}(t)=2^{I_{ij}(t)}-1$, valid for the full domain $-\infty \le I_{ij}(t) \le 1$.

Destruction

Spontaneous loss of knowledge, i.e., *forgetting*, is an obvious mechanism for knaoeldge change. If the process is quasi-continuous, we can invoke a relation like

$$K_{ij}(t) = K_{ij}(0) - K_{ij}(0)\ t/t$$

If the interval t is sufficiently small, a large number of steps will approximate an exponential decay, with characteristic time constant τ.

Pairwise interactions

Populations normally consist of well-defined, relatively autonomous individuals that move (literally or figuratively) relative to one another. Infrequently, pairs of individuals interact ("collide"), at which time something interesting happens. Figure 2 shows schematically one possible sequence of successive pairwise interactions of 4 individuals. We assume that the collisions are relatively well-isolated from other individuals, and relatively abrupt in time.

If the individuals are cognitive, their individual knowledge can be altered by the interaction, and such changes are likely to be complex. However, two very simple processes appear to capture much of the dynamics of such pairwise interactions. We can formulate these as follows:

(1) Interacting individuals each copy a fraction of the total knowledge carried by the other individual;
(2) All other individuals lose a fraction of the knowledge they have of the interacting partners.

The first mechanism seems obvious and reasonable, and while it is superficially similar to diffusion, it has no known direct analog in physics. We refer to this process as *knowledge incretion.*

The second is not so intuitively obvious, and while it bears some similarity to spontaneous decay, there is again no direct analog in physics. This process will be called *knowledge decretion*.. We are familiar with this mechanism in a social context; with the passage of time, we tend to know less and less about a lost friend (so long as that friend is interacting with others). After a year of no contact with the friend, we may not know if he or she has the same address, phone number, or job. After several years, we may be unsure whether he or she is married, healthy, or still working. After 20 years we may not know whether he or she is still alive. Note that we have not forgotton; it is the external interactions of the individuals with other individuals that leads to our loss of knowledge about them. This erosion of knowedge is easily compensated—a single brief interaction will regenerate all this knowledge, and much more.

We now elaborate linear versions of these two mechanisms in detail, and subsequently combine them to obtain a dynamical equation for the knowledge **K**.

Linear incretion

The first pairwise mechanism, knowledge increase by copying, seems intuitively obvious. Assume that individuals {A,B,C} are initially completely independent of each other (they have no knowledge of each other). For a while, all individuals proceed according to their independent dynamics. After an {A,B} interaction, individuals {A} and {B} know about each other, but neither knows anything about {C}. Next, a {B,C} interaction occurs. Now individuals {B} and {C} know about each other, and therefore, {C} knows something about {A}, although {A} knows nothing yet about {C}. This process leads to an expansion and general increase of knowledge within the population.

We can formulate this in a linear limit as follows: Before the {i,j} interaction the knowledge held by {i} and {j} can be represented symbolically as

$$K_i=\{K_{i1}, K_{i2} \dots K_{iN}\}$$
$$K_j=\{K_{j1}, K_{j2} \dots K_{jN}\}$$

where $0 \le K_{iq}, K_{jq} \le 1$ and $K_{ii}=K_{jj}=1$, and N is the number of individuals in the population. We now assume that during the interaction, {i} gains a fraction (a) of all the knowledge held by {j}. After the interaction, {i}'s knowledge is

$$\begin{aligned} K_i' &= K_i + a\,K_j \\ &= \{K_{i1}, K_{i2} \dots K_{iN}\} + a\,\{K_{j1}, K_{j2} \dots K_{jN}\} \\ &= \{K_{i1}+a\,K_{j1}, K_{i2}+a\,K_{j2} \dots K_{iN}+a\,K_{jN}\} \\ &\equiv \{K'_{i1}, K'_{i2} \dots K'_{iN}\} \end{aligned}$$

The same relation, with i<—>j, is obtained for {j}'s post-interaction knowledge.

Written as a recursion relation, this transformation is

$$K_{iq}(n+1) = K_{iq}(n) + a\,K_{jq}(n) \qquad q=1 \dots N$$
$$K_{jq}(n+1) = K_{jq}(n) + a\,K_{iq}(n) \qquad q=1 \dots N$$

with the proviso that any matrix element $K_{ij}>1$ is automatically truncated to $K_{ij}=1$, and also $K_{ii}=K_{jj}=1$ always.

Linear decretion

The second mechanism, knowledge decrease by erosion, is not intuitively obvious. The following argument demonstrates the reality of this process: In any interaction, both partners are inevitably and permanently altered (this must be so, or else no interaction took place). We can indicate this by {A,B}—>{A',B'}. Next, {B'} and {C} interact: {B',C}—>{B",C'}. However, since {A'} has no means to *know* how {B'} was altered by {C} during this event, it knows less about {B"} than it did about {B'}. With the passage of more and more time, {A} has less and less knowledge of {B} because {A} has no knowledge of {B}'s *other* interactions. This process clearly leads to a contraction and general decrease in knowledge.

As above, we formulate this in a linear limit: Before the {i,j} interaction, the set of individuals that has knowledge of {i} can be represented by the i^{th} vector of the transpose matrix $\mathbf{K}^T$, and similarly for {j}:

$$K^T_i=\{K_{1i}, K_{2i}, \dots K_{ji} \dots, K_{Ni}\}$$
$$K^T_j=\{K_{1j}, K_{2j}, \dots, K_{ij} \dots, K_{Nj}\}$$

We now assume that during the interaction, every individual {p≠j} that has knowledge K_{pi} of {i} loses a fraction (b) of that knowledge. Then after the interaction,

$$\begin{aligned} K^T_i{}' &= K^T_i - b\,(1-\delta_{ij})\,K^T_i \\ &= \{K_{1i}, K_{2i} \dots K_{ji} \dots K_{Ni}\} - b\,\{K_{1i}, K_{2i} \dots 0 \dots K_{Ni}\} \\ &= \{(1-b)K_{1i}, (1-b)K_{2i} \dots K_{ji} \dots (1-b)K_{Ni}\} \\ &\equiv \{K'_{1i}, K'_{2i} \dots K'_{ji} \dots K'_{Ni}\} \end{aligned}$$

Similarly, every individual {p≠i} that has knowledge K_{pj} of {j} loses a fraction (b) of that knowledge, and we have the same relation with i<—>j.

Written as a recursion relation, this transformation is

$$K_{pi}(n+1) = K_{pi}(n) - b\,K_{pi}(n) \qquad p=1\dots\neq j\dots N$$
$$K_{pj}(n+1) = K_{pj}(n) - b\,K_{pj}(n) \qquad p=1\dots\neq j\dots N$$

with the same provisae as in the previous section.

Combining incretion and decretion

The two binary interaction mechanisms naturally (perhaps unavoidably) appear together in the dynamics. They are conveniently visualized together as a matrix transformation **K**—{i,j}—>**K**', or **K**{i,j}**K**'. For example, if we have N=7 individuals, the matrix **K**' after an {i,j}={3,5} transformation is:

i\j	1	2	3	4	5	6	7
1	1	K_{12}	$(1-b)K_{13}$	K_{14}	$(1-b)K_{15}$	K_{16}	K_{17}
2	K_{21}	1	$(1-b)K_{23}$	K_{24}	$(1-b)K_{25}$	K_{26}	K_{27}
3	$K_{31}+aK_{51}$	$K_{32}+aK_{52}$	1	$K_{34}+aK_{54}$	$K_{35}+a$	$K_{36}+aK_{56}$	$K_{37}+aK_{57}$
4	K_{41}	K_{42}	$(1-b)K_{43}$	1	$(1-b)K_{45}$	K_{46}	K_{47}
5	$K_{51}+aK_{31}$	$K_{52}+aK_{32}$	$K_{53}+a$	$K_{54}+aK_{34}$	1	$K_{56}+aK_{36}$	$K_{57}+aK_{37}$
6	K_{61}	K_{62}	$(1-b)K_{63}$	K_{64}	$(1-b)K_{65}$	1	K_{67}
7	K_{71}	K_{72}	$(1-b)K_{73}$	K_{74}	$(1-b)K_{75}$	K_{76}	1

It is readily seen that knowledge incretion has the general form of a transverse extension,

$$x' = x + a\,y$$
$$y' = y + a\,x$$

while knowledge decretion has the general form of a longitudinal contraction

$$x' = x - b\,x$$
$$y' = y - b\,y$$

The simple vector diagrams shown in Figure 3 capture these relations. They strongly suggest that circumstances can easily be found in which incretion and decretion compensate, producing a population of asymptotically constant, but fluctuating, total knowledge.

Successive interactions

Multiple successive binary interactions will be represented by multiple matrix transformations **K**{i,j}**K**'{i',j'}**K**"... The result of successive transformations clearly depends on the sequence of interacting partners, which in turn may depend on the dynamics of the individuals themselves. We illustrate this for four individuals {A,B,C,D} interacting with incretion and decretion. Assume that initially the individuals have no knowledge of any individuals other than themselves. Then **K** takes the following forms sequentially for two different collision sequences:

$$\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix}$$

{B,C}

$$\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & a & 0 \\ 0 & a & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix}$$

{A,B}

$$\begin{Vmatrix} 1 & a & a^2 & 0 \\ a & 1 & a & 0 \\ 0 & (1-b)a & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix}$$

{C,D}

$$\begin{Vmatrix} 1 & a & (1-b)a^2 & 0 \\ a & 1 & (1-b)a & 0 \\ 0 & (1-b)a & 1 & a \\ 0 & (1-b)a^2 & a & 1 \end{Vmatrix}$$

{A,B}

$$\begin{Vmatrix} 1 & (1-b)a & (1-b)^2a^2 & 0 \\ a & 1 & (1-b)a+a & a^2 \\ a^2 & (1-b)a+a & 1 & a \\ 0 & (1-b)^2a^2 & (1-b)a & 1 \end{Vmatrix}$$

$$\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix}$$

{A,C}

$$\begin{Vmatrix} 1 & 0 & a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix}$$

{A,D}

$$\begin{Vmatrix} 1 & 0 & a & a \\ 0 & 1 & 0 & 0 \\ (1-b)a & 0 & 1 & 0 \\ a & 0 & a^2 & 1 \end{Vmatrix}$$

{B,D}

$$\begin{Vmatrix} 1 & 0 & a & (1-b)a \\ a^2 & 1 & a^3 & a \\ (1-b)a & 0 & 1 & 0 \\ a & a & a^2 & 1 \end{Vmatrix}$$

{A,C}

$$\begin{Vmatrix} 1 & a & 2a & (1-b)a+a \\ (1-b)a^2 & 1 & (1-b)a^3 & a \\ (1-b)a+a & a & 1+a^2 & (1-b)^2a \\ a & a & (1-b)a^2 & 1 \end{Vmatrix}$$

State-independent pairings

Normally, we might expect that the selection of collision partners {i,j} would depend on the individual state variables $\{\mathbf{X}_i(t),\mathbf{X}_j(t)\}$. For instance, individuals would experience a collision when their spatial positions coinicide, or when their internal structures are in resonance. The complete population behavior would emerge from the simultaneous solution of the coupled $\{\mathbf{X}(t),\mathbf{K}(t)\}$ dynamical equations.

If, however, we assume that the selection of interacting pairs {i,j} is independent of $\{\mathbf{X}_i(t),\mathbf{X}_j(t)\}$, we can investigate the dynamics of $\mathbf{K}(t)$ *independent of the individuals comprising the population*. This will be a good approximation, say, if the fraction of the individual used to store the knowledge is small. In such populations, every individual looks like every other individual. An example is a computer network. With protocals such as TCP/IP, the exchange of data between any pair of computers is independent of the details of the computers.

We conclude from this that, with respect to the movement of knowledge within a population, *so long as the individuals are "big and stupid," it does not matter what they are*. We will use this approximation to study several exemplary systems.

Note that the change of knowledge during the {i,j} interaction may very well depend on $\mathbf{K}$ itself. For instance, we could easily define a process in which knowledge is copied *only* if it exceeds a threshold, or has some other property. Thus, the dynamical equation for $\mathbf{K}(t)$, even without coupling to the equations for $\mathbf{X}(t)$, may very well be nonlinear.

Random pairings

Another approximation we will find useful is to assume that the interacting pairs {i,j} are selected *at random*. This assumption imposes "full mixing" on the population, and converts it from a deterministic system to a stochastic one. It therefore has the useful potential for exhibiting average behavior, such as the learning transient and fluctuations, that otherwise would be masked by systemic behavior.

Thus, we will find it useful to examine the behavior of populations with *random, state-independent pairing,* since they will exhibit properties of the knowledge dynamics that cannot be seen in more complex populations.

DYNAMICAL EQUATIONS FOR K

Discrete-time

For {i}-{j} pairwise interactions, we define an interaction matrix $\Gamma\{i,j\}$ by

$$\Gamma\{i,j\}_{pq} = \delta_{pi}\delta_{qi}+\delta_{pj}\delta_{qj}$$

and a corresponding noninteraction matrix $\Lambda\{i,j\}$ by

$$\Lambda\{i,j\}_{pq} = \delta_{pi}\delta_{qj}+ \delta_{pj}\delta_{qi}$$

For example, for N=4, we have

$$\Gamma\{2,3\} = \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{Vmatrix} \qquad \Lambda\{2,3\}= \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Vmatrix}$$

Thus, for pairwise interactions, Γ has 1's on the diagonal only for the interacting partners and zeros elsewhere, while Λ has 1's for the off-diagonal elements of the partners and zeros elsewhere. A straightforward extension allows us to include collisions of more than 2 partners.

Let us identify the interacting pair {i,j} with the generation (n). With these definitions, we can write the dynamical equation for **K**, including only incretion and decretion, as

$$\mathbf{K}(n+1) = \mathbf{K}(n) + a\,\Gamma(n)\bullet\mathbf{K}(n) - b\,\mathbf{K}(n)\bullet\Lambda(n)$$

In order to complete the dynamical model of the population, we must specify how the matrices $\Gamma(n)$ and $\Lambda(n)$ and the state variables $\mathbf{X}(t)$ evolve in time. A typical behavior we might expect is that $\Gamma(n)$ and $\Lambda(n)$ remain constant while $\mathbf{X}(t)$ evolves continuously in time. Infrequently, a binary collision occurs, the partners being determined by $\mathbf{X}(t)$. This allows us to specify $\Gamma(n)$ and $\Lambda(n)$ and update $\mathbf{K}(n)$ to $\mathbf{K}(n+1)$. After that, $\mathbf{X}(t)$ continues to evolve until the next collision.

Continuous time

In the limit of continuous time, we consider the matrices Γ and Λ to be continuous functions of t. That is, every element in $\Gamma(t)$ and $\Lambda(t)$ is a function of t. In a sense, every individual is interacting continuously with every other individual, although some of these interactions may be defined to be zero. With this assumption, the recursion relation goes over into a differential equation

$$\frac{d\mathbf{K}}{dt} = a\ \Gamma \bullet \mathbf{K} - b\ \mathbf{K} \bullet \Lambda$$

where $a\Gamma(t)$ and $b\Lambda(t)$ play the role of time-dependent frequencies. The simplest assumption we could make is that Γ and Λ are constants. This allows us to write the formal solution of the previous equation as

$$\mathbf{K}(t) = e^{at\Gamma}\mathbf{K}(0)e^{-bt\Lambda}$$

If in addition, $a\Gamma = b\Lambda = \Omega$, we have

$$\mathbf{K}(t) = e^{\Omega t}\mathbf{K}(0)e^{-\Omega t}$$

This is a similarity transformation, and it is interesting because under such transformations, eigenvalues and matrix norms are preserved. There is obviously complete formal identity between the last expression and the propagation of matrix operators forward in time, familiar from quantum mechanics. For instance, if Ω and **K** commute, **K** is constant in time. This is, of course, not a coincidence: earlier we remarked on the dualism of regarding **K** as a dynamical variable *versus* a constraint. Here we see that dualism: **K** plays the role here of an observable physical quantity; its evolution in time is given by the well-known relation for observables in the Heisenberg picture.

The circumstance that $a\Gamma \neq b\Lambda$ ruins this neat picture; we must admit that the evolution of knowledge **K** in general is more complex. This is traceable to the fact that the individuals in these populations are not simple objects that have well-defined symmetry and obey Newton's Third Law. The emergent behavior of our systems will, of course, be correspondingly richer.

NUMERICAL EXPERIMENTS

In this section, we present several numerical experiments that illustrate the transient behavior of knowledge. We will assume a small population of individuals experience infrequent isolated binary collisions (numbered by n) that change their mutual knowledge **K**(n) only by the pairwise processes of incretion and decretion. Furthermore, we will assume the pairings are selected randomly, and independent of any state variable **X**. The dynamical equation for **K**(n) is that given above for discrete time, in which the matrices $\Gamma(n)$ and $\Lambda(n)$ are randomly reset after every collision.

Knowledge Diffusion

We have remarked that the spread of knowledge by incretion and decretion is similar to diffusion. Figure 4 shows this clearly. Initially, a population of 30 individuals with no knowledge of each other was established; the matrix K is the unity matrix, $K_{ii}(0)=\delta_{ij,}$ indicated by the black squares on the diagonal. At successive generations, the incretion/decretion mechanisms (with rates a=0.5, b=0.5) were applied, choosing nearest neighbors randomly for interacting partners. This causes off-diagonal elements $K_{ij}(n)$ to be incremented, indicated by the gray squares. After 30 generations, the diagonal has diffused into a band; each individual knows about several others within its immediate neighborhood, but knows little or nothing of more distant individuals. After 300 generations the matrix has equilibrated, and never diffuses beyond the ragged diagonal band.

Several interesting structural entities emerge in these matrices:

(1) *Experts:* These individuals *know about* significantly more other individuals than the average individual does.
(2) *Celebrities:* These individuals *are known by* significantly more other individuals than the average individual.
(3) *Isolates:* Distant individuals know more about one individual than do close ones.

The last of these is most surprising. Apparently during the diffusion process, knowledge can be transferred through some individuals to others (which then lose it), resulting in "islands" of knowledgeable individuals.

Individual knowledge

The knowledge K_i held by individual {i} is the sum of its knowledge of all other individuals and of itself:

$$K_i(n) = \sum_{j=1}^{N} K_{ij}(n)$$

Figure 5 show $K_i(n)$ for 3 populations of 9 individuals interacting as described above, for 100 generations. The initial population in each case was set with random mutual knowledge ($K_{i \neq j}=0$).

For Fig. 5(a), the incretion/decretion rates were a=1, b=0.1. This population learns quickly and its knowledge erodes relatively slowly, leading to relatively high individual knowledge (near the maximum $K_i=\Sigma(1)=9$). Irregularly, the individual K_i fluctuate downward, although there is no directly compensating upward fluctuation in other individuals.

For Fig. 5(b), the incretion/decretion rates were a=0.5, b=0.5. In this population, knowledge grows and erodes are roughly the same intermediate rate. The individual K_i undergo upward rises, followed by sudden drops, producing a sawtooth pattern.

For Fig. 5(c), the incretion/decretion rates were a=0.1, b=0.1. This population learns slowly, but its knowledge also erodes slowly. The result is that the rise and fall of any individual's knowledge is more symmetrical than the sawtooth pattern, and the fluctuations are smaller over the same time scale.

A striking result of these simulations are the rather long swings, either upward or downward. One might have expected more randomness, considering that the collision partners were randomly chosen at each generation. We interpret this behavior as an expression of the coupling between all the individuals: they are not independent, since they have some knowledge of each other. Hence, complete chaotic behavior would not be expected. But they are also not totally dependent, so some stochasticity is reasonable. We conclude that we are seeing behavior that is characteristic of the peculiar processes of incretion and decretion. We would expect other mechanisms of knowledge change to introduce their own characteristic behaviors.

Total population knowledge

The total knowledge K of the population is the sum of all individuals' knowledge:

$$K(n) = \sum_{i=1}^{N} K_i(n) = \sum_{i=1}^{N} \sum_{j=1}^{N} K_{ij}(n)$$

Figure 6 shows K(n) for the same three cases of the population of 9 individuals. These figures show that the total knowledge K(n) is not constant, although it appears to stay within bounds.

Learning Transient

If we start with no pairwise knowledge $K_{ij}(0)=\delta_{ij}$, the matrix elements gradually increase by the process of incretion. Eventually, this increase will be countered by decretion, so that K(n) will approach some equilibrium value at long times. This process can be described as a *learning transient*. Figure 7 shows this clearly for the population of 9 individuals as above with incretion/decretion rates a=1, b=0.1. Initially the total knowledge was K(0)=9. As the individuals interacted, their pairwise knowledge increased, then approached an equilibrium value slightly below the maximum possible $\Sigma_{ij}\ (1) = 81$.

The number of generations in the transient (the "rise time") will be of the order of a few times N/a, as seen from the following argument: If we selected interaction partners systematically, never choosing any individual more than once, it would take exactly N/2 generations to mix the N individuals. But when collision partners are randomly selected, it becomes increasingly likely that the next selection will include previously selected individuals, hence not contribute to the mixing. Therefore, several times N/2 generations will be required to select, say, half the individuals. Since the fractional mixing at each collision is a, by definition of the incretion process, it should take several times (N/2)(1/a) generations to produce half full mixing, i.e., the FWHM of the learning transient. The numerical experiments confirm these ideas.

It may be noted that if the population starts with *more* than its equilibrium knowledge $K(n \to \infty)$, K(n) drops asymptotically. Figure 8 shows results of numerical experiments on two populations of 20 individuals with incretion, decretion rates a=1, b=1. Within the fluctuations, the knowledge in the two cases approaches the same asymptote.

Asymptotic equilibrium knowledge

Whatever the dynamics of the knowledge matrix **K**, it will typically evolve from an initial matrix to an equilibrium at long times. The general behavior of the equilibrium knowledge K(n) for n->∞ as a function of the incretion,decretion rates a,b therefore is of interest. Figure 9 shows numerical experiments on 3 populations of 4 individuals. Each population has a fixed value of the decretion rate b (0.1, 0.5, and 1.0). Each population is carried through 1000 generations, during which the interacting pairs are selected randomly. In Fig. 9, we plot the normalized total population knowledge

$$k_m(n) = \frac{K(n) - N}{N(N-1)}$$

at the n=1000th generation, as a function of the incretion rate a. This quantity is a reasonable approximation to the asymptotic value $k_m(\infty)$. It approaches 1 as every individual approaches full knowledge of every other individual.

The plots indicate that increasing a increases the total knowledge, while increasing b reduces the total knowledge, as expected. An interesting general result is that for b=a, $k_m(\infty) \approx 0.5$. That is, *if the incretion and decretion rates a,b are comparable, the asymptotic total population knowledge is about half its maximum possible value*.

Another interesting observation is that for small b there is a substantial range of a within which *the asymptotic knowledge is independent of a*. Apparently, the asymptotic equilibrium knowledge is controlled more by decretion than by incretion. The incretion, however, determines the duration of the transient to reach the asymptotic equilibrium.

We have empirically found that $k_m(\infty)$ can be approximated quite well by the remarkably simple function

$$f(a,b) = 1 - \exp[-(a/b)^{3/2}]$$

Figure 10 shows plots of this function. The curves of Fig. 9 can be identified as sections of this plot. We have no understanding of the significance of this formula.

The behavior of the knowledge for small a,b is quite peculiar. For b=0, knowledge is never lost, so $k_m(\infty)$ will grow inevitably to its maximum possible value: $k_m(\infty)_{a,b=0}=1$. But for a=0, there is no increase of knowledge, so any initial knowledge will be lost: $k_m(\infty)_{a=0,b}=0$. Thus, there is a discontinuity at a=b=0, and the long-time evolution of the population is extremely sensitive to the *relative* values of a and b. In this region, $k_m(\infty)\approx 0$ if $a > b$, $k_m(\infty)\approx 1$ if $a < b$. Thus, *in the limit that a population increases and decreases its knowledge very slowly, all the individuals will eventually attain either complete knowledge or zero knowledge.* Because organisms in Nature are very complex, the rate at which they alter their knowledge is numerically very small (the fractional change in any interaction is very small). Therefore, we might conclude that to the extent that the mechanisms of incretion and decretion are significant in Nature, real populations will be inherently unstable: eventually either all individuals will share the same complete knowledge of each other, or eventually all individuals will have no knowledge whatever of other individuals.

Fluctuations

At equilibrium, the total knowledge K(n) fluctuates around its equilibrium value. The magnitude of these fluctuations is smaller in a population with a larger number N of individuals, and depends on the values of a and b. We have found that in some cases these fluctuations can be minimized.

Figure 11 shows results from a set of experiments on a population of 10 individuals interacting by knowledge incretion and decretion with a=1 and b=0.1, 0.2, 0.3, 0.5, and 0.8. Collision partners were selected randomly, and the population was first evolved for 10,000 generations, enough to be fairly certain that it had reached equilibrium. Then it was evolved for another 1000 generations. The histograms show the number of times the value k_m (bin width 0.01) occurred in the 1000 generations, i.e., it is the trace of the fluctuating population knowledge. Surprisingly, the fluctuations have a minimum at b=0.3.

The magnitude of the fluctuations is associated partially with the stochasticity of selecting collision partners at random. But we might also expect larger fluctuations if the individuals were somehow nonlinearly coupled by their knowledge; the total population knowledge K(n) would undergo larger swings because of theis coherence. Therefore, we conjecture that the minimum fluctuations are produced by randomly selecting partners, and additional fluctuations are produced by coherent effects of the mutual knowledge itself.

Eigenvectors and eigenvalues

Eigenvectors of the matrix **K** represent linear combinations of the individuals constructed in such a way that the eigen-individuals have no knowledge of each other. Thus

$$\mathbf{K} \bullet \mathbf{S}_i = \kappa_i \mathbf{S}_i$$

defines the eigen-individual $\mathbf{S}_i$. The eigenvalues κ_i represent the total knowledge held by the eigen-individuals.

Eigenvalues and eigenvectors are important to this work, since they are another measure of the collective properties of the knowledge, in the same sense as the total population knowledge K(n). We therefore expect to see the processes of diffusion, learning transients, asymptotic equilibrium, and fluctuations in the eigenvalues.

Figure 12 shows values of κ_i vs generation for two populations of 10 individuals, one with a=1, b=.3 and one with a=.3, b=.05. These numerical experiments suggest that one eigen-individual usually dominates, containing almost all of the knowledge, while the others fluctuate around roughly the same (lower) value. The learning transient is clearly seen in the eigenvalues, as expected.

Figure 13 shows another series of numerical experiments, all constrained to have a=b. Together with Fig. 12, these results suggest that the dominance of one eigen-individual is associated with small b. In the limit b—>0, the singular eigen-individual has knowledge κ=N, and all other eigen-individuals have knowledge κ=0. A low rate of decretion leads to dominance by one eigen-individual, while a high rate of decretion leads to democratization among several eigen-individuals. The dominant eigen-individual is a single assembly of all the individuals in the population. Another way of describing this is as follows: Mutual knowledge provides links between individuals, enabling the assembly of a coherent structure. If those links are eroded quickly, the structure cannot persist.

Figure 14 shows that the population of 10 individuals has not yet reached equilibrium, even at 10,000 generations. This is consistent with the idea that each individual needs perhaps 10/a interactions to be significantly modified, and for N individuals to be modified by sequential pairwise interactions we must therefore have (N/2)10/a=(5)10/0.01=50,000 generations.

All of this is understandable in the following way: Initially, the population is comprised of N independent individuals (rows in **K**). Successive generations mix these individuals (by adding a fraction of each row to other rows), so that the rows (i.e., individuals) are no longer independent. If there is no mixing of columns (b=0), the rows become more and more mixed with time, asymptotically approaching complete linear dependence. If there is some mixing of columns (b≠0), this process of mixing is hindered, so the rows retain some measure of independence. The independence is manifested as lower equilibrium population knowledge K(n->∞), larger fluctuations of K(n->∞), and reduced asymmetry of the matrix K.

Note that the value of b is related to the probability that the individual changes its state during the interaction: smaller b means lower probabliity of change, or more stable individuals. We therefore have the implication that populations that are very stable spontaneously assume a configuration with one eigen-individual that has complete knowledge of all other individuals, while all other eigen-individuals are independent. On the other hand, when b is large, the individuals have high probability of changing state upon interaction, and the population does not separate as cleanly into one dominant eigen-individual.

There are numerous physical analogies that can help visualize this situation. For instance, very slow solidification at low temperature results in crystalline solids with high order, while fast solidification at high temperature results in amorphous solids with low order. We can think of the crystal atoms as comprising one large eigen-individual; every atom has complete knowledge of at least one other atom; the entire solid is thereby linked.

Symmetry

In general, the matrix **K** is asymmetric for finite values of (a,b). We interpret this to mean that {i} may know more (or less) about individual {j} than {j} knows about {i}. This concept is quite natural: a circling hawk knows some things about a mouse on the ground below, but the mouse knows little of the hawk. For (a,b) sufficiently small, **K** will be nearly symmetric, i.e., individual pairs will be approximately symmetric in their pairwise knowledge.

It was noted previously that asymmetry in **K** is indicated by the appearance of imaginary parts of the eigenvalues. Numerical experiments suggest that *the symmetry of* ***K*** *at equilibrium is greater than the symmetry of a random matrix*. That is, if we initialize a population with random pairwise knowledge, after equilibration by many interactions of randomly selected partners, the pairwise knowledge will be more symmetric. As an example, consider the following matrix describing a population of 10 individuals, in which the off-diagonal elements in this matrix were generated randomly:

1	0.18	0.37	0.66	0.42	0.42	0.3	0.96	0.99	0.4
0.81	1	0.7	0.74	0.77	0.74	0.007	0.79	0.024	0.48
0.45	0.18	1	0.28	1.	0.76	0.18	0.34	0.97	0.28
0.85	0.46	0.49	1	0.21	0.88	0.75	0.71	0.037	0.92
0.69	0.87	0.046	0.31	1	0.14	0.38	0.93	0.25	0.22
0.98	0.56	0.88	0.027	0.078	1	0.21	0.94	0.5	0.98
0.61	0.041	0.62	0.46	0.99	0.39	1	0.18	0.28	0.63
0.98	0.29	0.3	0.21	0.084	0.96	0.8	1	0.12	0.65
0.1	0.35	0.21	0.88	0.57	0.12	0.9	0.73	1	0.033
0.4	0.76	0.44	0.4	0.29	0.46	0.37	0.58	0.081	1

The eigenvalues, which sum to 10.00000 (=the trace of the matrix), are:

5.48365
0.993922 + 0.563914 I
0.993922 - 0.563914 I
0.796437 + 0.375661 I
0.796437 - 0.375661 I
0.343349 + 0.791205 I
0.343349 - 0.791205 I
0.337418
-0.04424 + 0.120764 I
-0.04424 - 0.120764 I

After 1000 generations using a=1,b=.1, the matrix and its eigenvalues became

1	0.25	0.12	1	0.12	0.12	0.5	0.12	0.063	0.045
0.094	1	0.14	0.85	0.18	0.085	0.78	0.027	0.11	0.03
0.35	0.25	1	0.7	1	0.21	0.78	0.16	0.12	1
0.45	0.25	0.11	1	0.12	0.12	0.48	0.029	0.063	0.036
0.25	0.19	0.76	0.51	1	0.15	0.57	0.13	0.11	0.7
1	0.43	0.41	1	0.32	1	1	0.85	0.3	0.46
0.28	0.25	0.22	0.61	0.15	0.38	1	0.061	0.12	0.3
1	0.43	0.4	1	0.33	1	1	1	0.34	0.36
1	0.25	0.38	1	0.36	0.26	1	0.5	1	0.13
0.39	0.25	0.5	0.75	0.21	0.27	0.78	0.14	0.12	1

4.09448
1.55883
1.21346
0.822087 + 0.0908688 I
0.822087 - 0.0908688 I
0.689982
0.403431
0.247912
0.0840975
0.0636397

where again the eigenvalue sum is 10.00000. The second matrix is fully conditioned; it represents an equilibrated population. Its elements fluctuate wildly from generation to generation, which will produce wildly fluctuating behavior of the individuals, but the population as a whole exhibits relatively stable behavior.

Metastability and spectroscopy

Figure 11 shows the evolution of the small eigenvalues of a population of 10 individuals interactin by incretion , decretion with rates a=b=0.01. The only difference between the two experiments was the exact sequence of interacting partners at each generation (they were randomly chosen in each case). It is evident that the population sometimes had a relatively constant eigenvalue, and that sometimes one of those eigenvalues experienced a sudden change. This suggests the interesting implication that certain configurations of the knowledge matrix **K** are more stable than others, and some may be *very* stable (i.e., metastable). Metastable configurations can switch spontaneously to other configurations, and there could be a characteristic time for this switching. The jump in eigenvalue in such a switch is analogous to the energy difference between bound states of a quantum system. This difference can be expressed as a spectral line. In observing this line we are observing transitions between two discrete configurations of the population knowledge. Thus, we have the suggestion of a rich and very complex spectroscopy associated with populations based entirely on the dynamics of their pairwise knowledge.

It is emphasized that this is independent of any specific physical dynamical model, definition of the individual, population size, etc.

RELATION TO OTHER WORK

System that can be described as a population of cognitive individuals are almost always described making use of the verb "to know." Regardless of whether the objects exhibit intelligence in human terms, we say that the object "knows" about other objects" and "knows what to do." Examples of these systems include globally coupled relaxation oscillators [Christiansen and Levinsen, 1993; Hansel, Mato, and Munier,1993], metapopulations [Gilpin and Hanski, 1991], cellular automata [Gutowitz, 1991], simulated fish schools [Huth and Wissel, 1992], flocking birds [Kshatriya and Blake, 1992], artificial life [Langton, 1989, 1992], and ant swarms [Milonas, 1992], to name but examples. It is, however, possible to incorporate the present language in the larger body of theory and modeling of individual-based population models [DeAngelis and Gross, 1992; Lomnicki, 1988], and through that, with a much broader spectrum of models and theory [Goel, Maitra, and Montroll,1971; Hoppensteadt 1982; Kampis, 1991; Murray, 1993; May, 1975].

CONCLUSIONS

We have noted that the knowledge matrix **K** can be treated either as a dynamical variables or as constraints. We found that many properties of **K**(t) can be found without reference to any specific dynamical system. General considerations predict that in any dynamical system, K will exhibit diffusion, a learning transient, and a fluctuating asymptotic equilibrium. We proposed a variety of mechanisms by which the knowledge can be altered: creation, information, destruction, incretion and decretion.

The dynamical equations for **K**(t) were derived for discrete and continuous time. In a series of numerical experiments, we found configurations of K corresponding to experts, celebrities, and isolates. The knowledge held by an individual usually varies, as does the total population knowledge. The properties predicted above are observed in these experiments. The FWHM of the learning transient is a few times N/2a. We found that if **K**(t) changes very slowly, the population eventually relaxes to either complete mutual knowledge, or zero mutual knowledge. The fluctuations at equilibrium can be minimum for certain values of the incretion, decretion rates a,b.

For small decretion rates b, the population is formed into one eigen-individual that contains essentially all the knowledge. For high b, the population forms a set of roughly comparable individuals that share the knowledge. Numerical experiments suggest that the population can form various metastable configurations, and that it can switch spontaneously between these configurations. The spectroscopy associated with these transitions may be a useful indicator of the eigenvalue structure.

This paper was concerned primarily with demonstrating certain fundamental behaviors of $\mathbf{K}(t)$ common to all cognitive dynamical systems. We placed most emphasis on the incretion,decretion mechanisms for changing $\mathbf{K}(t)$. With this background we can look forward to applying this formalism to more complicated populations.

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Table 1 - Correspondence between mutual knowledge and probability

{i}'s knowledge of {j}			Probability that{i} correctly identifes the state of {j})
	K_{ij}	p_{ij}	
completely correct	1	1	certainty
partial, correct	$0<K_{ij}<1$	$1/G<p_{ij}<1$	greater than random
none	0	1/G	random
partial, incorrect	$-1<K_{ij}<0.$	$0<p_{ij}<1/G$	less than random
completely wrong	-1	0	zero

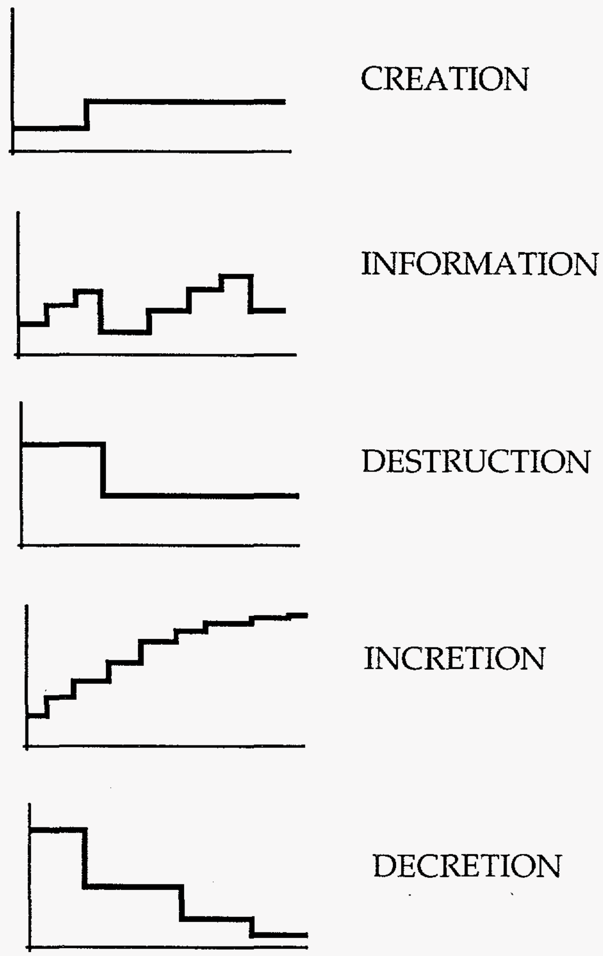


Figure 1 - Five mechanisms for changing the knowledge, together with popular and technical descriptors of the processes. In each frame, some measure of the knowledge is plotted as a function of time.

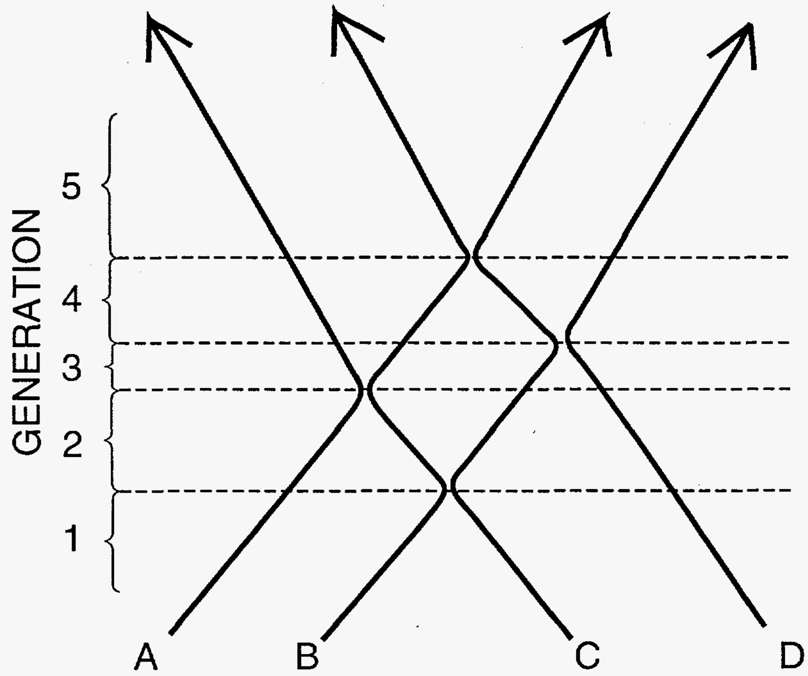


Figure 2 - Schematic of four individuals that change their total knowledge by pairwise interactions. In this Feynman-like diagram, time increases upward.

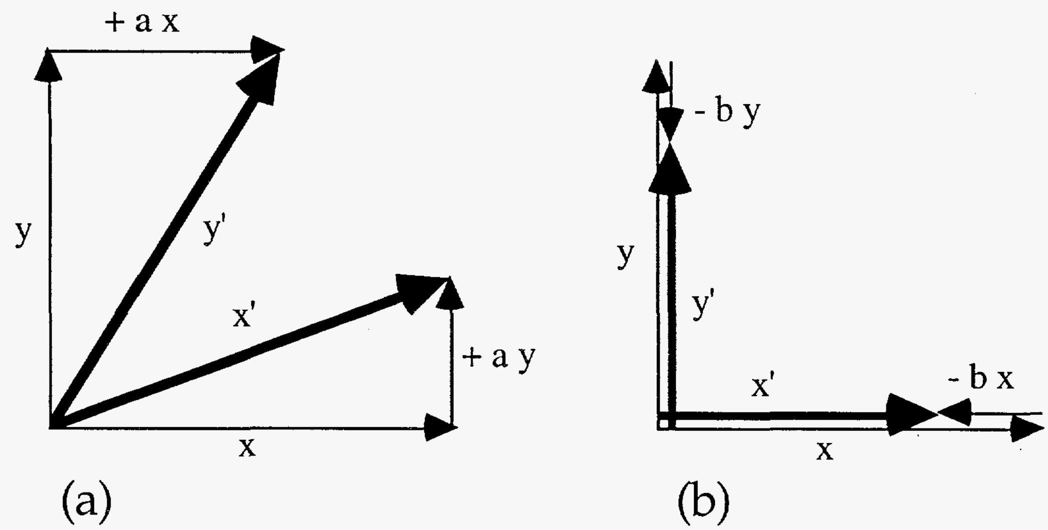


Figure 3 - Vector diagrams representing (a) knowledge incretion and (b) knowledge decretion.

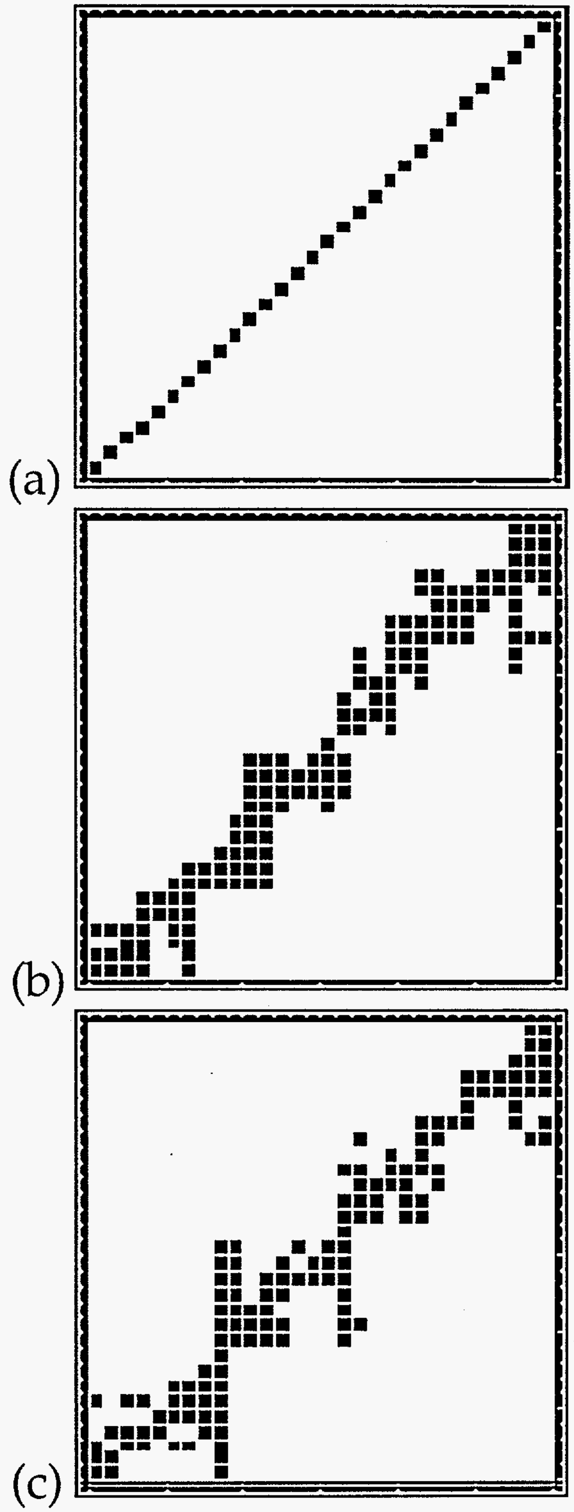


Figure 4 - Diffusion of knowledge into a population. The matrix elements K_{ij} are plotted as black (K_{ij}=1) or gray (K_{ij}<1) squares for: (a) initial population (no mutual knowledge); (b) after 30 generations; (c) after 300 generations.

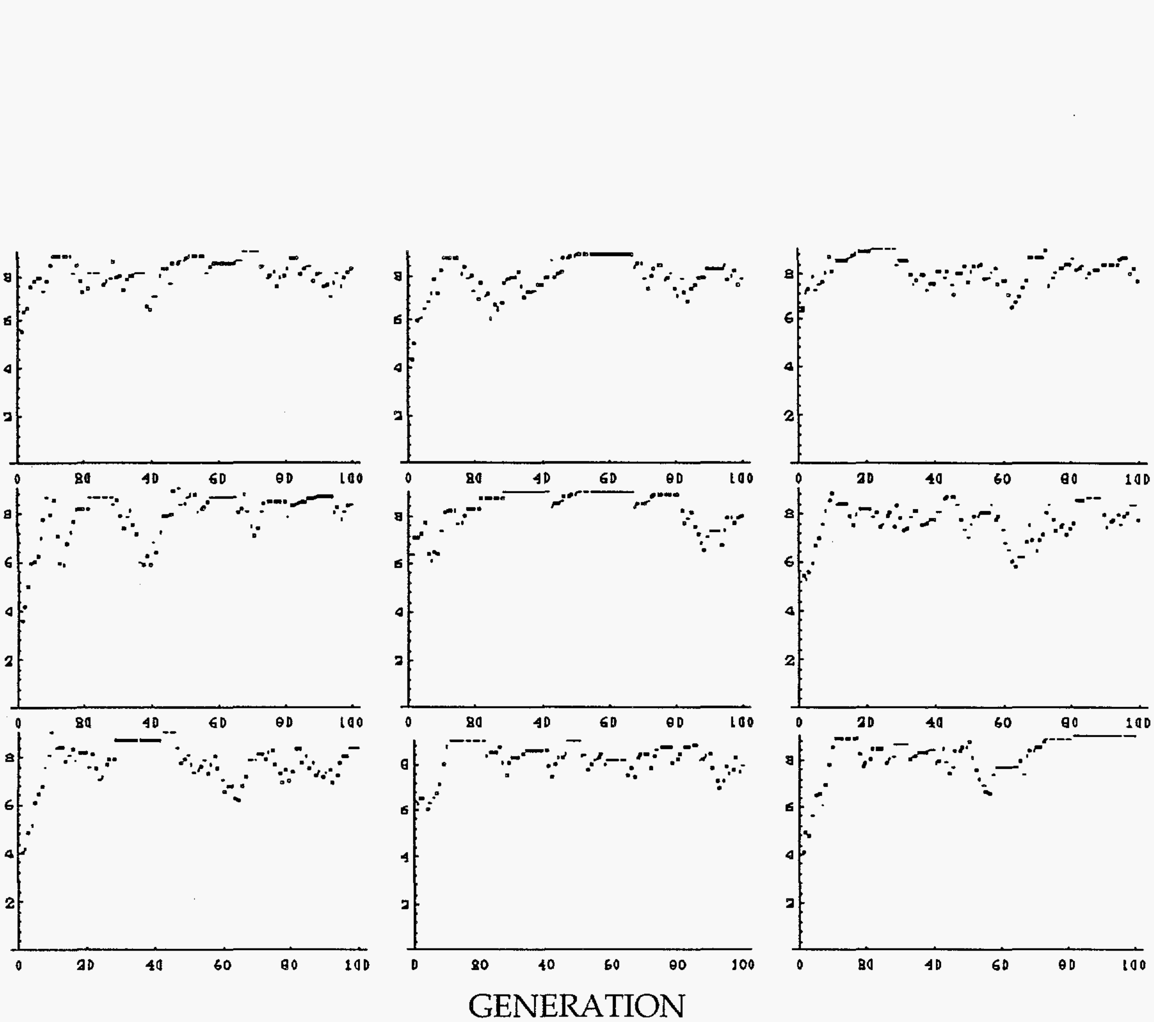


Figure 5(a) - Knowledge held by individuals as a function of time. The plots show $K_i = \Sigma_j K_{ij}$ for each of 9 individuals interacting randomly with incretion,decretion rates a=1.0, b=0.1. The initial mutual knowledge between all individuals was random.

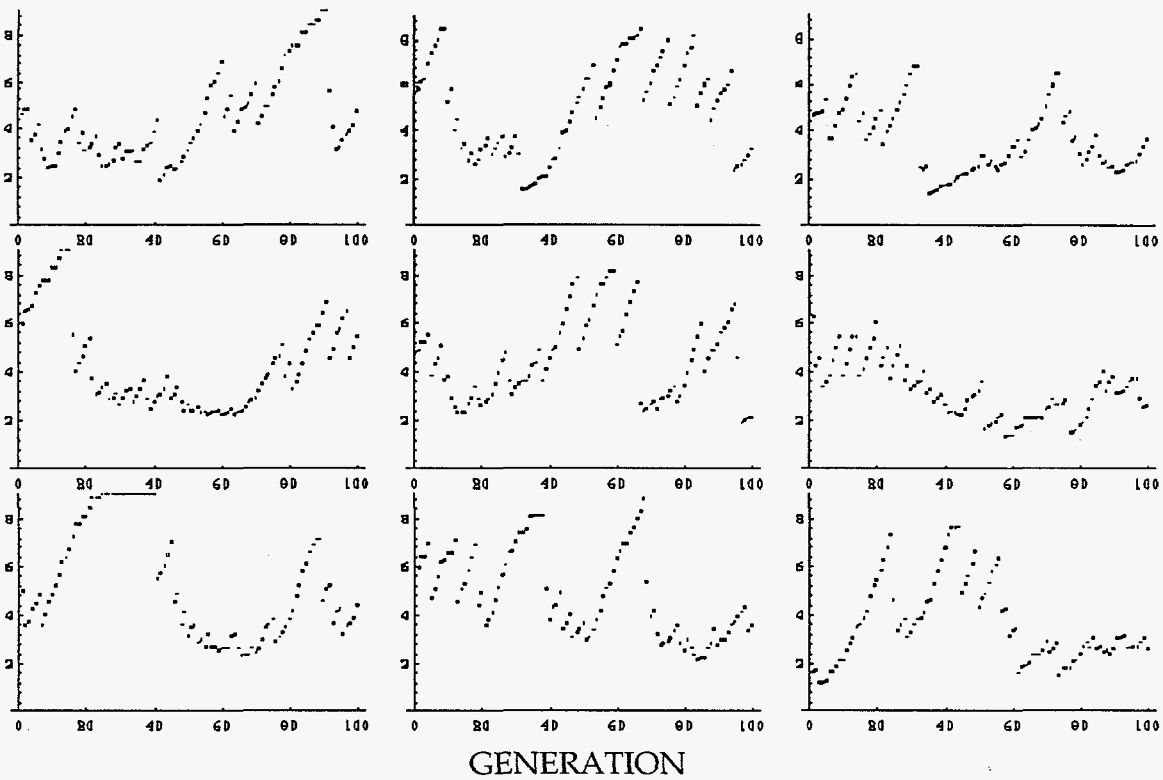


Figure 5(b) - Same as Fig. 5(a), for incretion,decretion rates a=0.5, b=0.5.

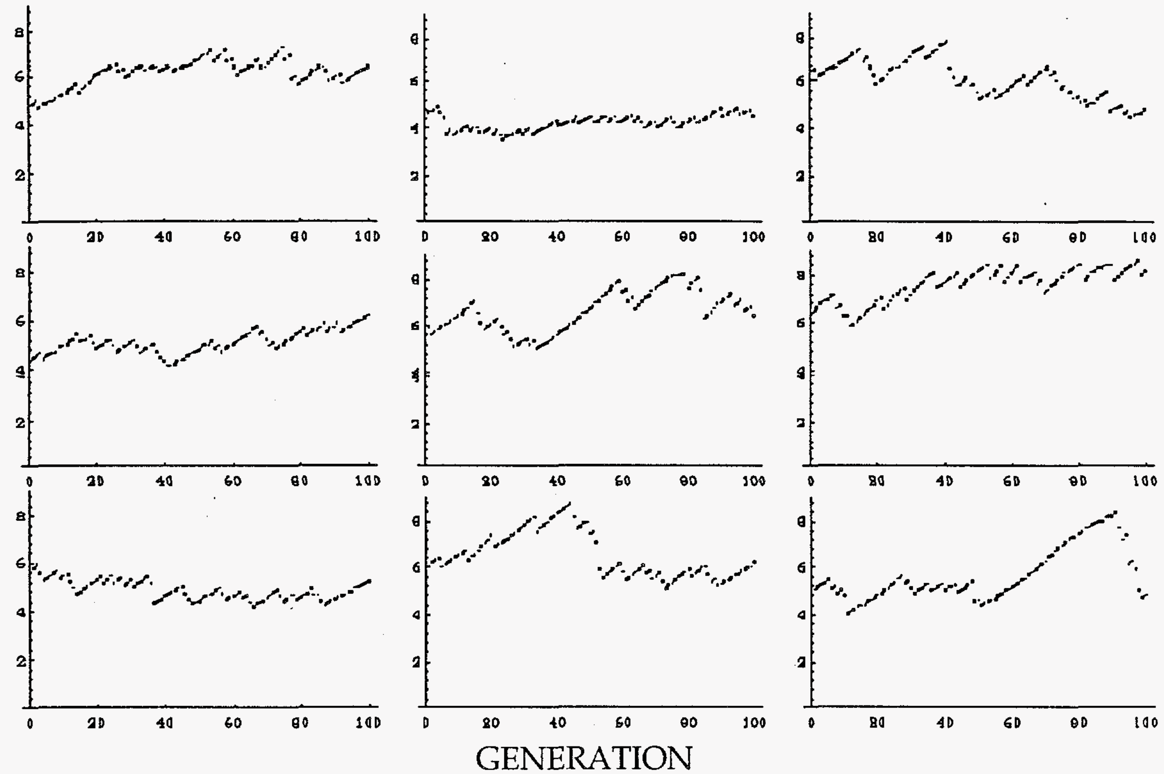


Figure 5(c) - Same as Fig. 5(a), for incretion,decretion rates a=0.1, b=0.01.

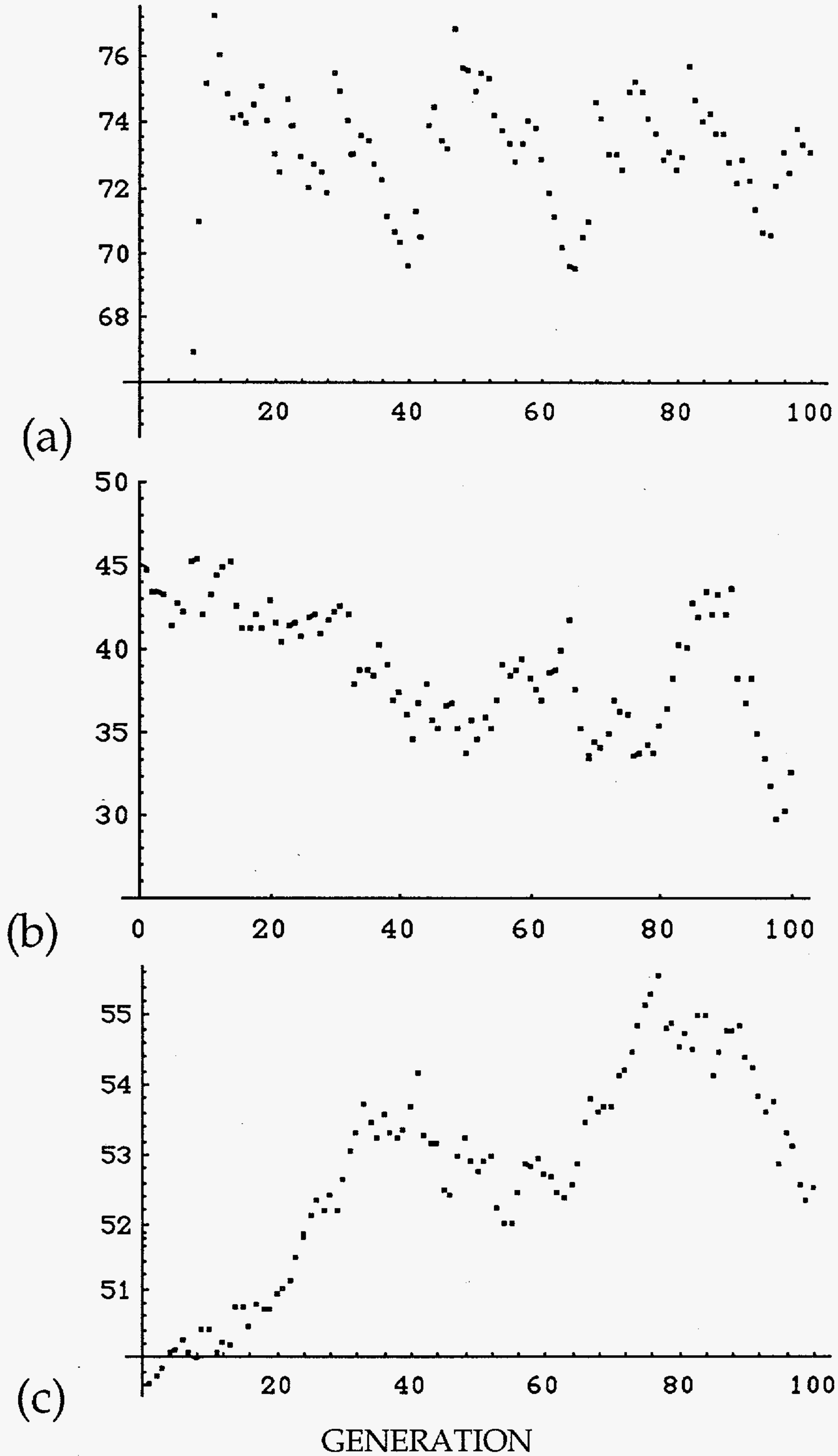


Figure 6 - Total knowledge held by a population as a function of time. The plots show $K=\Sigma_{ij}K_{ij}$ for 3 populations of 9 individuals interacting randomly with incretion,decretion rates (a) a=1.0, b=0.1; (b) a=0.5, b=0.5; (c) a=0.1, b=0.1. The initial mutual knowledge was random.

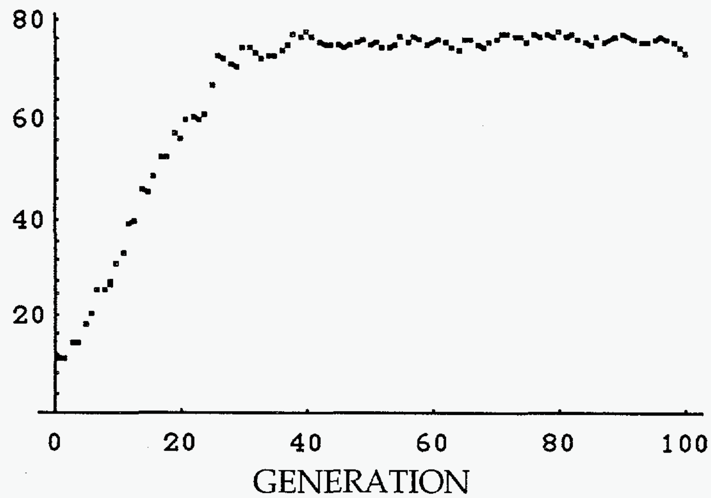


Figure 7 - Learning transient. The plot shows the total knowledge $K=\Sigma_{ij} K_{ij}$ of population of 9 individuals with incretion,decretion rates a=1,b=0.1. The initial mutual knowledge was zero (i.e., there was only self-knowledge $K_i(0)=1$).

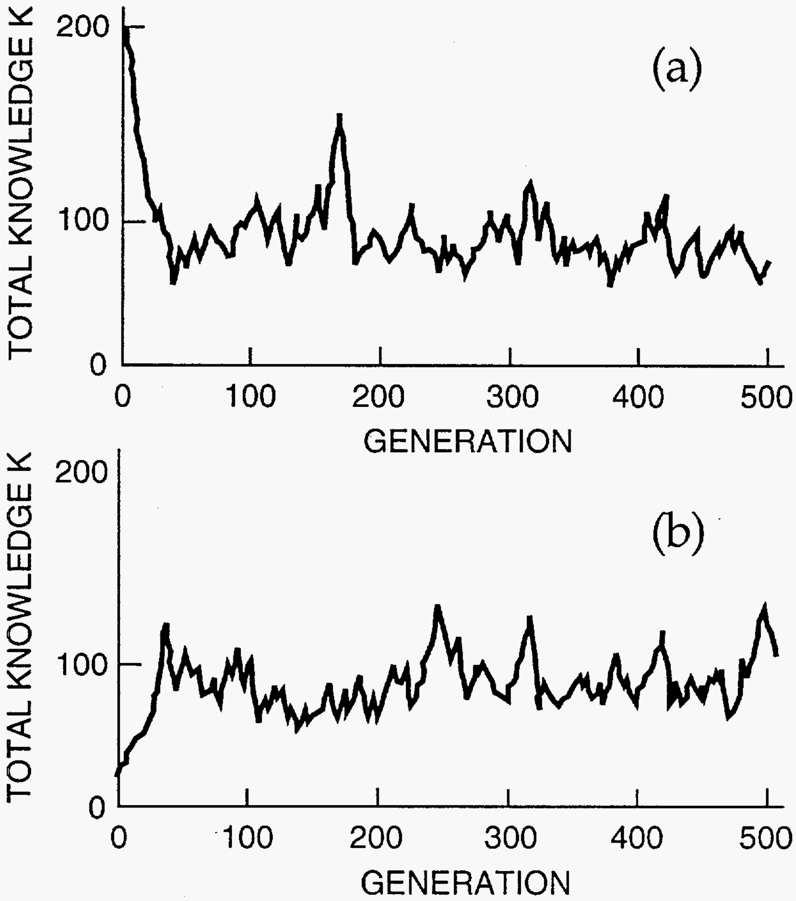


Figure 8 - Learning transient. Same as Fig. 7, but for 20 individuals with incretion,decretion rates a=1,b=1. Initial total knowledge including self-knowledge: (a) 200; (b) 20.

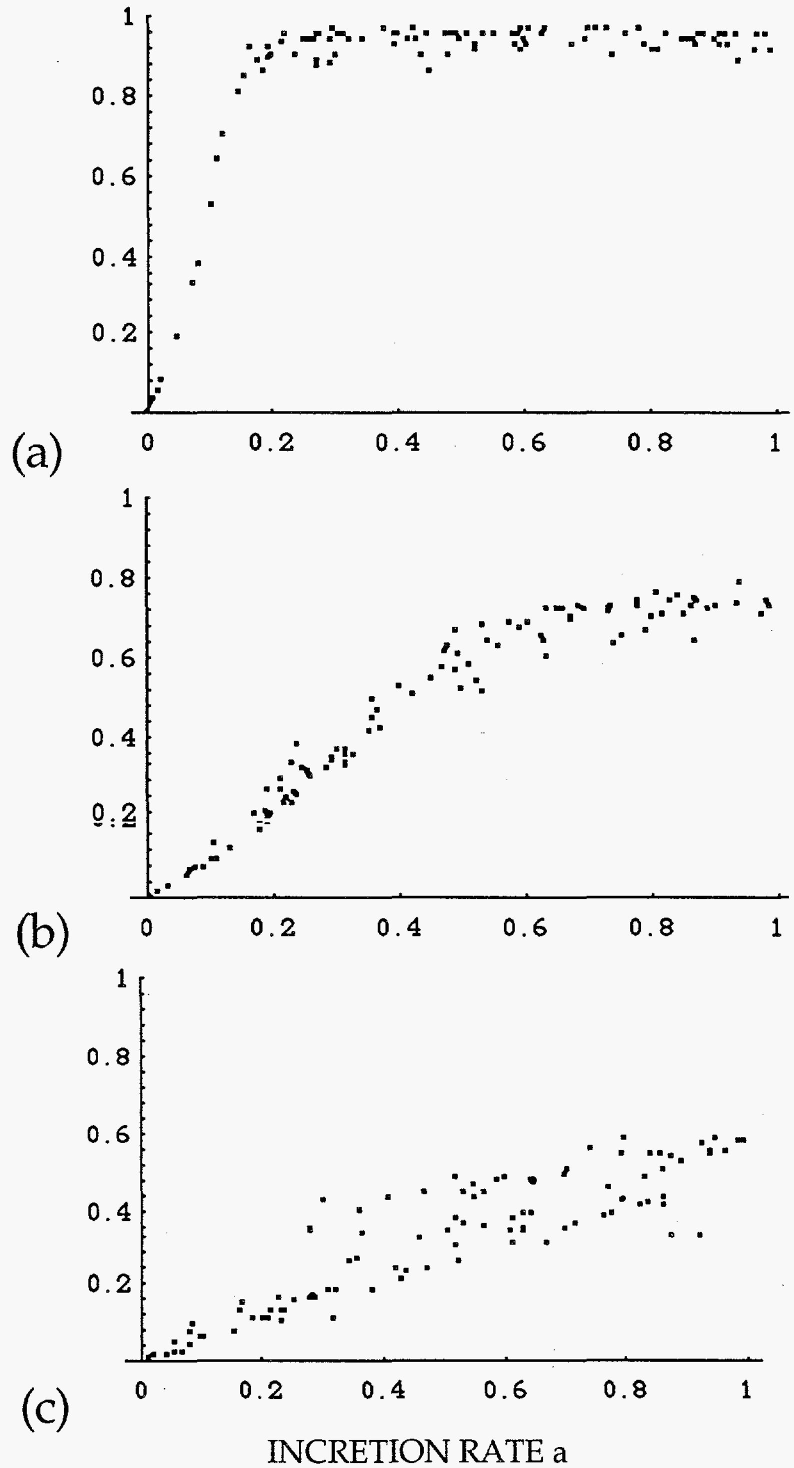
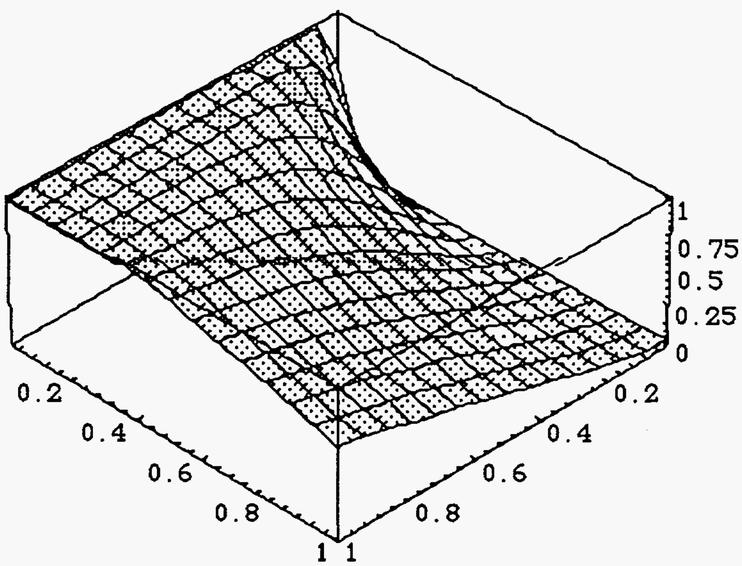
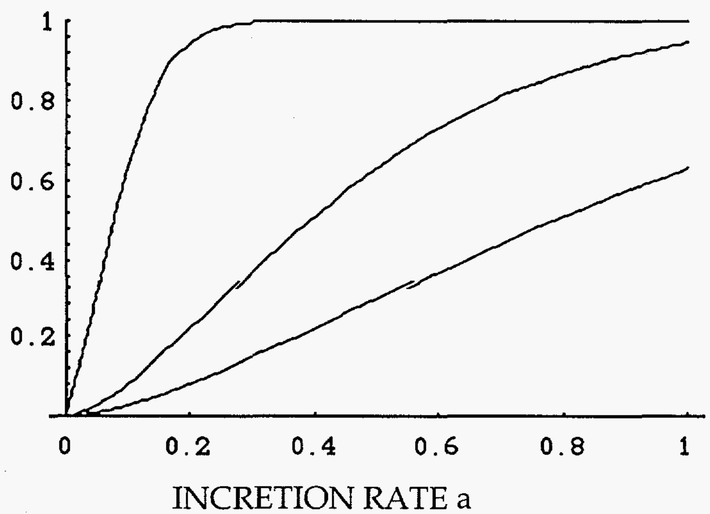


Figure 9 - Equilibrium total knowledge normalized as $[K(n\to\infty) - N]/N(N-1)$ as a function of the incretion rate a for three values of the decretion rate b. (a) b=0.1; (b) b=0.5; (c) b=1.0. The 3 population of 4 individuals were carried through 1000 generations.



(a)



(b)

Figure 10 -Analytic approximation to the data in Fig. 9.
(a) The surface $f(a,b) = 1 - \exp[-(a/b)^{3/2}]$;
(b) Three sections of the surface that correspond to the numerical experiments in Fig. 9.

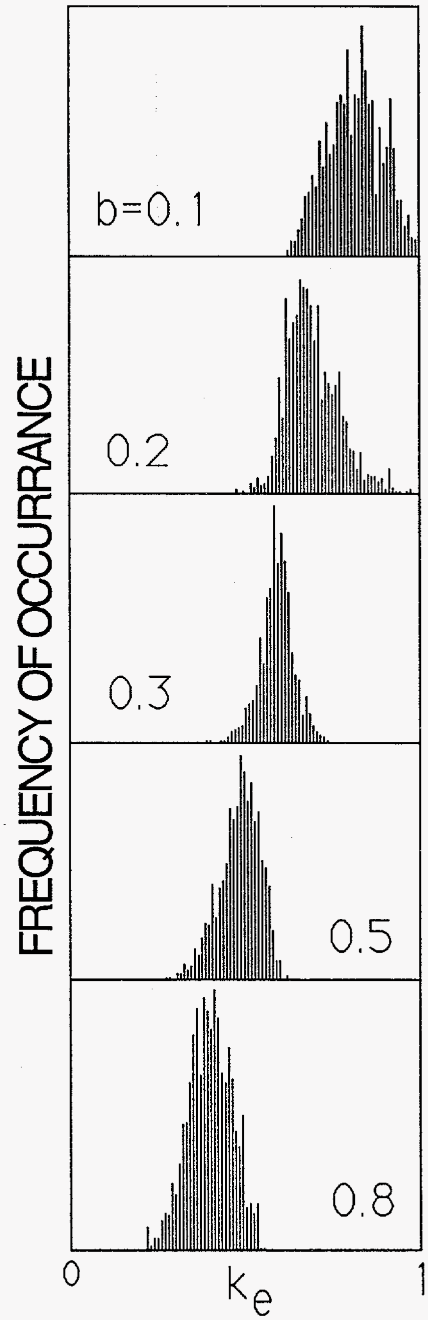


Figure 11 - Obtaining minimum fluctuations of the asymptotic equilibrium knowledge. The population of 10 individuals evolved through 1000 generations with a=1, b=0.1, 0.2, 0.3, 0.5, and 0.8. Minimum fluctuations occur for b=0.3.

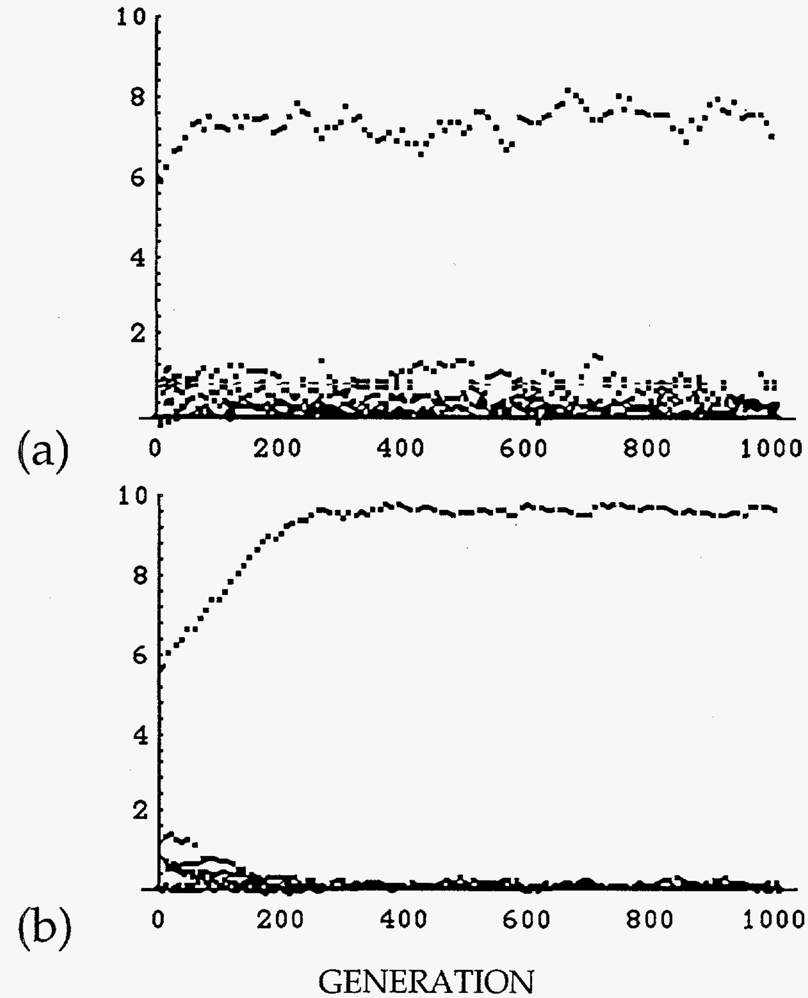


Figure 12 - Eigenvalues as a function of time for a population of 10 randomly interacting individuals. (a) a=1, b=.3; (b) a=.3, b=.05.

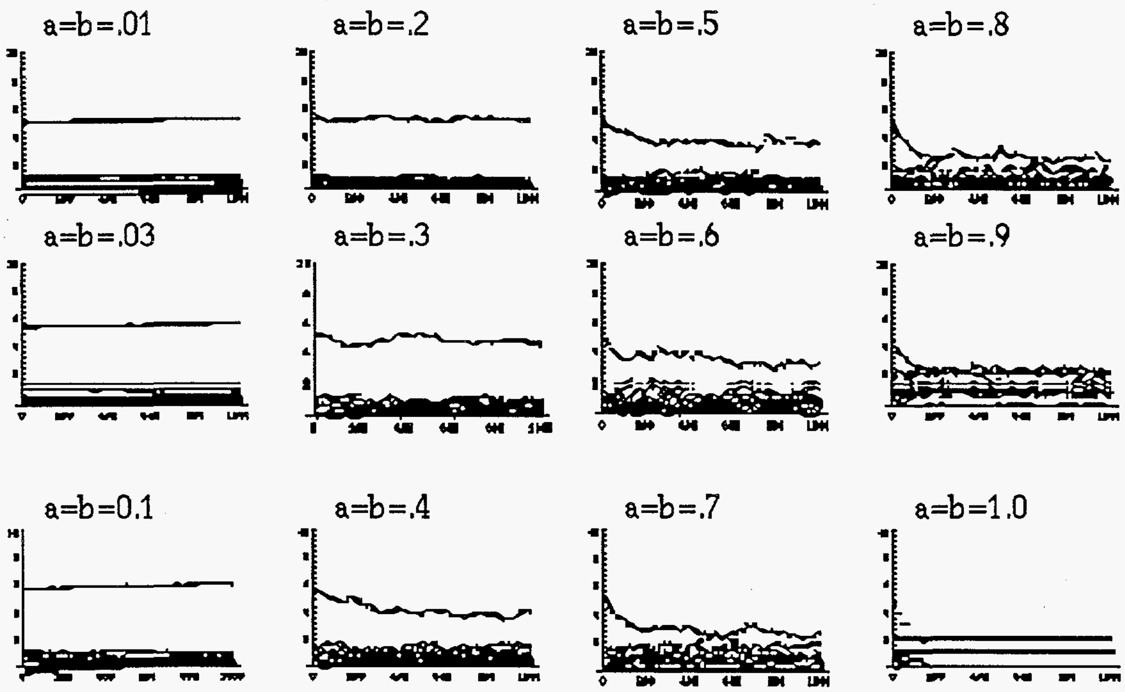


Figure 13 - Eigenvalues as a function of time for a population of 10 randomly interacting individuals. All examples have a=b.

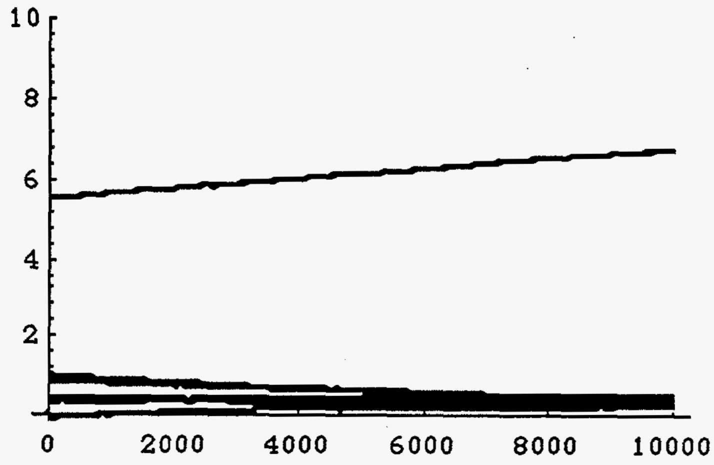


Figure 14 - Approach to equilibrium for very small incretion and decretion, for a population of 10 randomly interacting individuals. a=b=.01

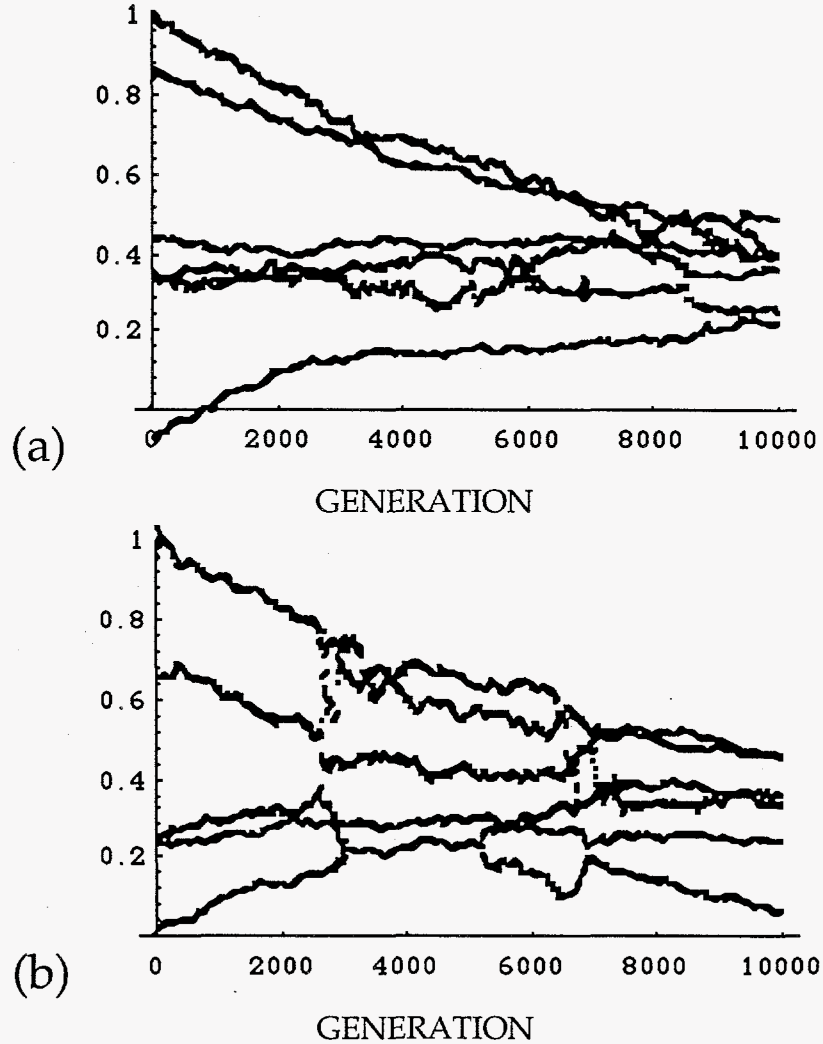


Figure 15 - Switching and spectroscopy. A population of 10 individuals with random initial mutual knowledge was advanced twice through 1000 generations with random incretion, decretion collisions of rates a=b=.01. The obvioius transitions near generations 3000 and 7000 in the second experiment suggest that the spectra assocaited with these transitions is characteristic of the population and its dynamics.

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