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# Alternative Methods of Marginal Abatement Cost Estimation: Non-parametric Distance Functions

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This paper implements a economic methodology to measure the marginal abatement costs (MAC) of pollution by measuring the lost revenue implied by an incremental reduction in pollution. It utilizes observed performance, or "best practice", of facilities to infer the MAC rather than engineering cost analysis. The empirical results are based on data from an earlier published study on productivity trends and pollution in electric utilities [yaisawarang, 1994] to test this usefulness of this approach and to provide insights on its implementation to issues of cost-benefit analysis studies needed by the DOE.

[Färe, 1993] shows that the output distance function, which is a mathematical representation of the underlying production technology, is dual to the revenue function. This duality may be exploited to infer the relative and absolute shadow prices for pollution outputs. These shadow prices are the lost revenue from electricity sales, evaluated at the observable electric price, and may be interpreted as the marginal abatement costs of pollution.

The paper extends the results of others [coggins, 1996] in several ways. The first is the use of non-parametric methods to obtain the shadow prices. The second is the application of the directional output distance function. The third is extending the size of the data set to include a wider range of power plants. The non-parametric method does not impose any restrictions on the functional form or sign of the MAC. This allows for the possibility economic benefits, i.e. cost reduction, corresponding to emission reduction. The non-parametric approach is based on a linear programming (LP) problem which computes the value of the directional distance function. The shadow prices of electricity and $SO_2$ are obtained from sensitivity analysis of the LP and are used to derive the MAC for $SO_2$ for 62 baseload coal fired power plants.

To assess the future applicability of this methodology for DOE, it is desirable to qualitatively compare these results from 'similar' analysis using traditional engineering cost studies. This comparison finds the two methods in reasonable agreement, suggesting that this method, which can be easily extended to include multiple pollutants, can be a useful source of information for policy studies.

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This project implements a economic methodology to measure the marginal abatement costs of pollution by measuring the lost revenue implied by an incremental reduction in pollution. It utilizes observed performance, or "best practice", of facilities to infer the marginal abatement cost. The initial stage of the project is to use data from an earlier published study on productivity trends and pollution in electric utilities (Yaisawarng and Klein 1994) to test this approach and to provide insights on its implementation to issues of cost-benefit analysis studies needed by the Department of Energy.

The basis for this marginal abatement cost estimation is a relationship between the outputs and the inputs of a firm or plant. Given a fixed set of input resources, including quasi-fixed inputs like plant and equipment and variable inputs like labor and fuel, a firm is able to produce a mix of outputs. Some of these outputs are joint products and tend to be produced simultaneously. Examples include refineries that produce gasoline and residual fuel oil, meat packing plant which produce beef and cow hide, and power plants that produce electricity and air pollution emissions. In the refinery example both products are desirable outputs and the firm optimizes the mix of production based on a unrestricted production technology and the market price of the products. In the meat packing plant example, there may be a positive value for cow hide as an intermediate product, e.g. inputs to a leather producer, or the firm may have to divert resources to dispose of the by-product. In this case the hides are not freely disposable and how they are treated depends on their value, positive or negative, to the firm. The power plant could freely dispose of air pollution emissions, except that they are regulated and input resources are required by law to reduce, or 'dispose' of the emissions. It is the regulations that place restrictions on the production choices available to the firm. This paper uses this theoretical view of the joint production process to implement a methodology and obtain empirical estimates of marginal abatement costs. These estimates are compared to engineering estimates. Futher areas of research are identified.

## Methodology

The cost methodology presented here is based on the analysis of the observable production choices available to the electric power industry. Graphically figure 1 illustrates the unrestricted production set, i.e. the combinations of electricity, 'goods', and air emissions, 'bads', that a plant could produced with some vector of inputs, X. The boundary of this set is ABC, which represents the maximum feasible production level of both goods and bads. This might be similar to the production set of a refinery or any other firm that produces several outputs. Regulation of emissions set limits on the disposability of these bad outputs, so the regulated production set is the subset of the unrestricted production set. The boundary of the regulated production set is OBD. The essential feature of the regulated production set is that to reduce emissions, given a fixed set of resources, some loss in desirable production occurs.

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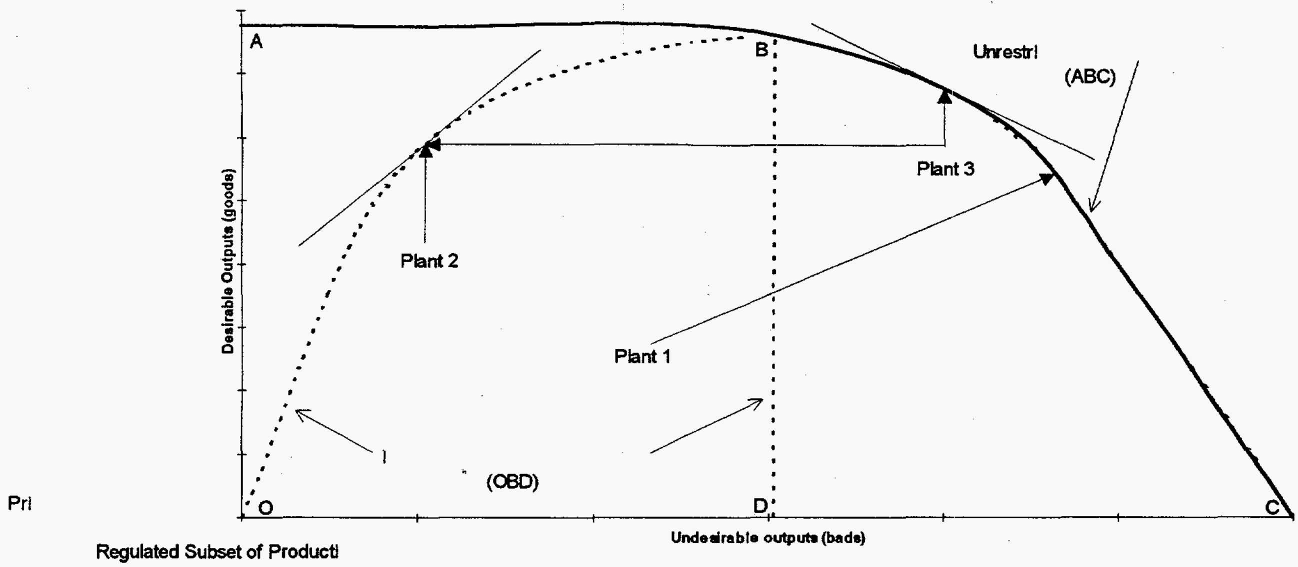


**Figure 1 Distance Function Approach to Marginal Abatement Cost Methodology**

Mathematically the production set may be described by the output distance function, introduced by (Shepard 1970). The function is defined as

$$D_O(x,u) = \inf\{f : (u/f) \in P(x)\} \qquad (1)$$

where $x$ and $u$ are input and output vectors, respectively, and $P(x)$ is the production set. $P(x)$ can be either a restricted or unrestricted production set. Graphically the output distance function can be see as the amount by which outputs, u, can be expanded and still be in the production set, as shown by plant 1 in figure 1.

(Färe et al. 1993) show that, since the revenue function is dual to the output distance function, the gradient vector for the distance function gives information regarding the shadow (revenue) prices of the desirable and undesirable outputs. If the shadow price of the desirable output equals the market price, then the shadow price for undesirable outputs may be computed as

$$r_b = r_g \cdot \frac{\partial D_o(x,u) / \partial u_b}{\partial D_o(x,u) / \partial u_g} \qquad (2)$$

where

$r_b$ is the shadow price of the bad output and
$r_g$ is the shadow (and market) price of the good output.

One approach is to use observable data to estimate a differentiable functional form of the output distance function and apply (2). An alternative approach is to use non-parametric methods to construct a piecewise linear version of $P(x)$, derive the shadow prices from a non-parametric estimate of $\partial D_o(x,u) / \partial u_*$. This is the initial approach used in this study.

The usual method to estimate the output distance function non-parametrically is to solve the linear programming problem

$$
\begin{gathered}
\mathit{Max}\ \phi\ \ \mathit{subject\ to} \\
z \cdot G \geq \phi \cdot g^0 \\
z \cdot B = \phi \cdot b^0 \\
z \cdot X \leq x^0 \\
\\
z \geq 0 \\
\phi \geq 0
\end{gathered}
$$

where:

| | | |
|---|---|---|
| $x^0$ | = | Observed inputs for a single plant, |
| $g^0$ | = | Observed "good" outputs for a single plant, |
| $b^0$ | = | Observed "bad" outputs for a single plant, |
| X | = | Matrices of inputs for the entire sample, |
| G, B | = | Matrices of good and bad outputs for the entire sample, and |
| z | = | An activity vector. |

This linear programming problem is based on activity analysis and constructs a convex hull of the observed data to create the unrestricted production set, *P(x)*. The dual values of the output constraint are estimates of the derivative required in (2). Graphically we see that the duals are described by the slope of the line tangent to the production set boundary, shown in figure 1.

The practical problem with this approach is that this method is applicable to the unrestricted production set, P(x), and that the shadow values one may obtain are both negative, as shown for plant 1. If regulations are binding on all plants, then the dual value for undesirable outputs will be positive, while the dual for desirable outputs is positive. We wish to alter this approach so as to obtain the 'correct' shadow values. To do this we follow (Turner 1994), who develops a sub-vector output distance function.

$$D_{SV,o}(x, g, b) = \inf\{\phi : (g / \phi, b) \in P(x)\} \qquad (3)$$

Note that the output vector u is now partitioned into desirable and undesireable components and that the optimization is only over the desirable portion of the output vector. Turner proves a similar form of duality for the sub-vector output distance function as in equation (2). A similar linear programming problem can then be solved:

$$
\begin{gathered}
\mathit{Max}\ \phi\ \ \mathit{subject\ to} \\
z \cdot G \geq \phi \cdot g^0 \\
z \cdot B = b^0 \\
z \cdot X \leq x^0 \\
\\
z \geq 0 \\
\phi \geq 0
\end{gathered}
$$

Note that the inequality constraint on undesirable outputs has been also replaced by an equality constraint. This reflects the assumption that bad outputs are not freely disposable (Färe, Grosskopf, and Lovell 1985).

As a final consideration in the specification of the distance function, we also adopt the approach used by (Yaisawarng and Klein 1994) on the input side. Inputs as well as outputs can be specified as weakly disposable. The sulfur content of the fuel can be viewed as a weakly disposable input. This changes the inequality constraint in the above LP from

$$z \cdot X \leq x^0$$

to

$$z \cdot X_d \leq x_d^{\;0}$$

$$z \cdot X_s = x_s^{\;0}$$

where the subscript *d* represent the 'desirable' inputs of labor, fuel, and capital, and *s* represents the undesirable percentage of sulfur in the fuel.

Based on the data from (Yaisawarng and Klein 1994) this LP was solved for each of the 61 observations for the year 1989. This reveals that only 13 out of 61 plants were operating in the restricted portion of the output set and have negative shadow prices. This result is consistent with those obtained by Turner, who obtained 344 out of 962 negative shadow prices[1], although she obtains negative shadow prices for a larger subset of her data. The range of shadow prices are from \$74 to over \$12,000 per ton. Ignoring the outliers on both ends, the average is \$1120 per ton and the median is \$876.

The large fraction of non-negative shadow prices may be illustrated by comparing figure 1 and an alternative restricted production function in figure 2. In this figure we see that many more plants are operating in the unrestricted area of production, given the plant inputs of labor, capital, etc. The implications of these results is that for the observed plant resources, much lower emissions are feasible, as shown by the horizontal arrow, rather than the vertical arrows.

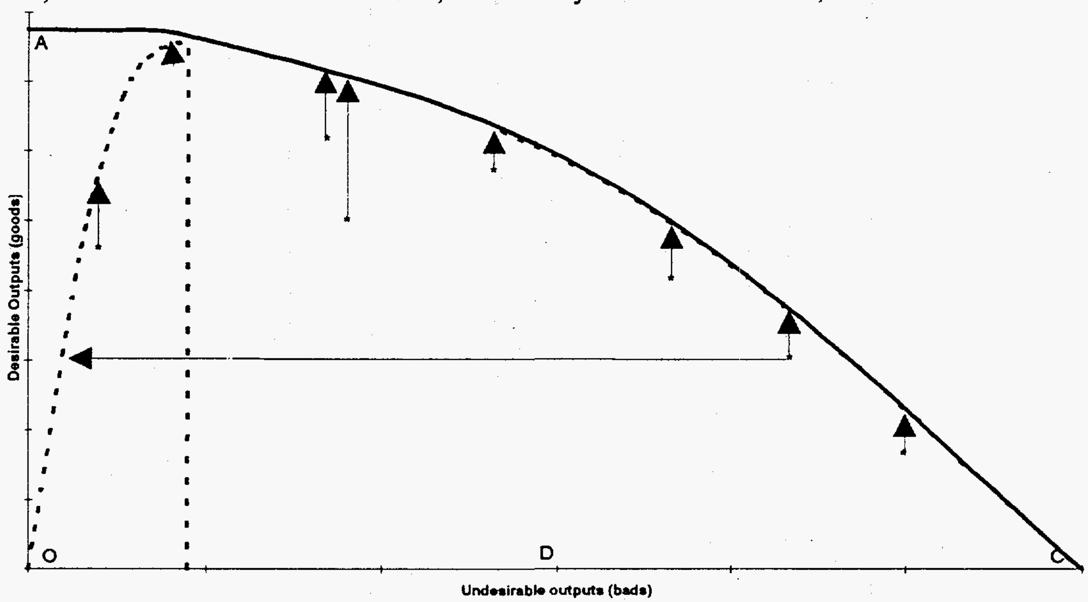


**Figure 2 Alternative Production Output Set**

If we change the formulation of the distance function to represent reductions in undesirable outputs, rather than increasing desirable outputs we expect to only identify segments of the restricted production set, where shadow prices are negative. This type of analysis was performed for the 62 plants in the data set and the results confirm the 'picture' represented in figure 2. When the following optimization problem

1 Data used by Turner was generating unit level, not plant level.

$$
\begin{aligned}
&Min\ \phi\ \ subject\ to\\
&z \cdot G \geq g^0\\
&z \cdot B = \phi \cdot b^0\\
&z \cdot X \leq x^0\\
\\
&z \geq 0\\
&\phi \geq 0
\end{aligned}
$$

was run[2], all resulting shadow prices were negative; the values of $\theta$ were rather low, 72% on average, and magnitude of the shadow prices was very high, median values of $2,668/ton. The simple implication of the low values of $\theta$ are that emissions could be reduced by over one quarter, given current plant resources, i.e. at 'no cost'.

While both of these approaches have some intuitive appeal, each are based on a narrow focus of the production/environment frontier. It would be useful to define the distance function in a more general way. Chambers, Chung, and Fare (forthcoming) refine the theoretical basis for this type of efficiency measurement; the directional distance function. The directional distance function provides a more integrated approach to modeling economic and environmental performance. This formulation of the distance function allows for the type of inefficiency that is typified by Porter and others, who maintain that there is significant opportunity to reduce environmental impacts and increase productivity, simultaneously.[3] Figure 3 illustrates the directional distance function. As in figures 1 and 2 above, the output set is denoted by P(x), the good output by y and the bad by b.

The directional output distance function, as any distance function, is a function representation of the technology. Shephard's distance function applied to the output vector (y,b) places it on the boundary of P(x) at A. The directional output distance function on the other hand takes (y,b) in the "g" direction and places it on the boundary at B. Here the directional distance function increases the good output and decreases the bad. Formally, it is defined as

$$(8) \qquad \vec{D}_o(x,y,b;g) = \sup\{\beta : ((y,b) + \beta g) \in P(x)\}.$$

In Figure 3 this amounts to the ratio of the distances (BC/0g).

One can prove

$$(9) \qquad \vec{D}_o(x,y,b;g) \geq 0 \text{ if and only if } (y,b) \in P(x).$$

Which is required to establish the completeness of the directional distance function as a model of the joint production technology. Based on this relationship one can also prove the revenue function duality required to compute the marginal abatement costs as was done above. It is easy to see that the approach of Turner and the second approach implemented here are simply variations of (9) where g=(1,0) and g=(0,-1), respectively.

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[2] At this time no formal proof of the duality theorem was undertaken, although it is likely to be very similar to that proven by Turner and follows the same intuitive logic.

[3] Empirical measurement of the extent of these opportunities is an important part the public policy debate and the directional distance function is the appropriate theoretical tool for this modeling. We employ this framework to identify the relevant portion of the production frontier and compute the marginal abatement costs.

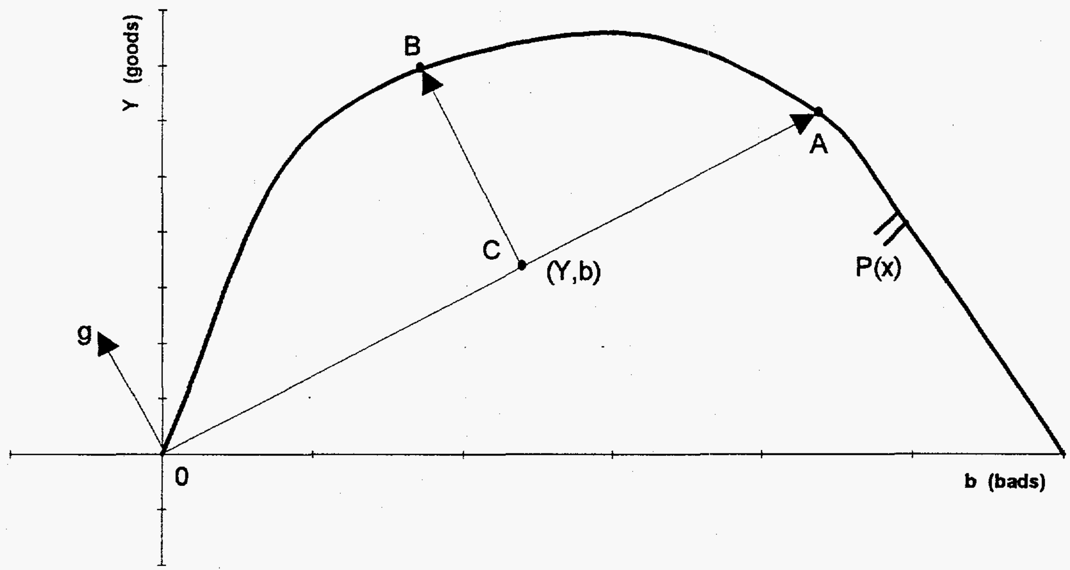


**Figure 3 Directional Output Distance Functions**

The directional distance function, where g=(1,-1) can be estimated from the following LP;

$$
\begin{gathered}
Max\ \beta \ \ subject\ to \\
z \cdot G \geq (1+\beta) \cdot g^0 \\
z \cdot B = (1-\beta) \cdot b^0 \\
z \cdot X \leq x^0 \\
\\
z \geq 0 \\
\beta \geq 0
\end{gathered}
$$

which was done for each plant in the test data set. As one would expect, the results for the shadow prices and efficiency measures are somewhere between the other two cases.[4] Since this approach appears more theoretically encompassing and satisfying, these results are examined in more detail.

Directional Distance Function Results:

The directional distance function places 29 observations, about 1/2 of the data set, on the frontier. This implies that 1/2 of the plants in the test data operate efficiently and could not reduce air pollution and increase output, given the resources available at the plant. The average value for $\beta$ for the remaining observations is 6.7%. This means that those plants not on the frontier could reduce emissions and increase generation by an average of 6.7%. Five plants had values of $\beta$ greater that 10%, one of which was 20%. These results are within normal variation in plant level heat rates, so are not at all unreasonable[5].

[4] One should note that each of the earlier problems as special cases of the directional distance function.

[5] Heatrates, the amount of fuel use per kWh generated can vary due to plant age, design, and operating characteristics. This result does not imply that a plant with a less efficient design could costlessly transform itself into a more efficient

The directional distance function generates negative shadow prices for about 1/2 of the data set[6]. This is in contrast to only 13 observations using the method proposed by Turner. To compute an absolute shadow price for $SO_2$, we must have a price for desirable output. For this analysis a price of 5 cents/kWh was assumed. The average price used by Coggins and Swinton was 5.2 cents/kWh. It would be desirable to have a representative wholesale price for each plant. Such a price would better represent the level of costs particular to that plant. However, such data was not in the test data set.

The range of marginal abatement costs for this group is shown in Table 1. We see that the results vary widely, but are slightly more modest than those obtained using the previous methods. When we look at the weighted average prices (weighted by total plant emissions) we see that the $355/ton estimate is slightly higher than the $293/ton weighted average estimate from (Coggins and Swinton 1996). The large difference between the weighted average and the numerical average abatement cost illustrates a major source of divergence between the 'low' estimates and the 'high' estimates. To better assess the 'reasonableness' of the results it is useful to examine the abatement costs estimates relative the magnitude of the plants emissions.

**Table 1: Summary Statistics For Non-Negative Marginal Abatement Costs Estimates (29 Plants)**

| Minimum | Maximum | Mean | Median | Weighted Average[a] |
|---|---|---|---|---|
| $35 | $12,312 | $1,703 | $787 | $355 |

[a] Weighted by total plant $SO_2$ emissions.

Figure 4 shows the marginal abatement plotted against the cumulative plant level emission. The abatement costs estimates are a log scale. This may be thought of as a crude approximation to the supply of allowances. This is a very crude approximation. To accurately aggregate the allowance supply curve, each plant's entire marginal abatement cost ***curve*** would be summed. Instead, figure 4 simply plots each plant's point estimate of marginal abatement costs against its entire emissions. The appeal to figure 4 as an allowance supply curve in only to present a context.

The figure is further normalized so that the emission are plotted as a percent of the total emissions for the 29 plants, which account for about 42% of the emissions in the test data set. Figure 4 shows that over 60% of the emissions have marginal abatement costs less that $100. The abatement costs as a function of cumulative emissions can be approximated as exponential curve (shown on figure 4). It is quite apparent that marginal costs rise quite steeply for plants that contribute only a small share of overall emissions.

Figure 4 also provides a comparison of the abatement costs estimates derived from the distance function analysis with a more 'traditional' engineering cost analysis. (Molburg 1996) computes the incremental average cost of scrubbing or switching to low-sulfur coal for all coal fired units in the U.S. This data[7] was plotted against the incremental emission reduction for each reduction option at each unit and was normalized to 100%. This comparison suggests that the cost estimates at the 'high end' are in fairly good agreement, i.e. it is very expensive to abate those 'last few' tons of $SO_2$. The principle difference between the two analyses is in the abatement costs below $1,000. The analysis by Molburg suggests that most of the emission reductions fall between $100 and $1,000, while the cost estimates from the distance function lie at both extremes, but with little between. There are two possible reasons for this;

- The test data for the distance function has a small number of plants, while the engineering analysis covers a large number of unit level data.
- The low marginal costs in the distance function analysis are attributed to the entire plant emissions.

A larger data sample might very well generate a 'smoother' set of marginal cost estimates. Theory requires the production set, which is the basis of the distance function approach, to include the origin. When non-parametric methods are used, as was done here, then very sharp differences in the abatement costs estimates can occur (see figure 5 for an example). Larger sample sizes may provide estimates that have fewer large discontinuities.

The other problem is that the marginal abatement costs is obtained from point estimates of two derivatives. Figure 4 plots the results from the point estimates against the entire plant emissions. If the low marginal cost eventually

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plant, but simply that the methodology gives results that are consistent with observed differences in plant engineering and operations

[6] 29 plants had negative shadow prices. This was not the same group of plants that were on the frontier.

[7] Methodology is described in the aforementioned report. Data was provided by the author in a private communication.

rise, as illustrated in figure 5, then the results in figure 4 will be biased downward, since the entire emissions of the plant would not be able to be reduced at that low marginal cost.

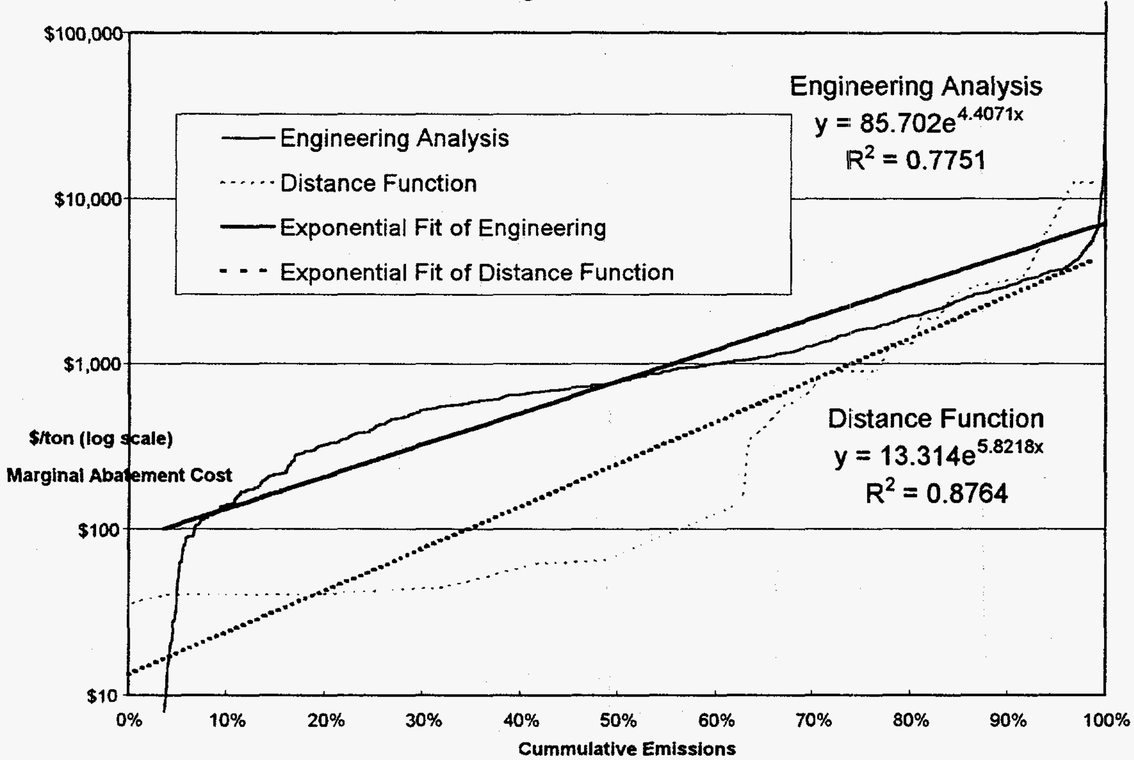


**Figure 4 Comparison of the Directional Distance Function Estimates of the non-negative Marginal Abatement Costs with Engineering Analysis**

This is only half of the results obtained from the directional distance function. The other half of the plants in the test data set had positive shadow prices, in apparent conflict with economic theory. There are several empirical reasons why marginal abatement costs might by negative (i.e. have positive shadow prices for $SO_2$). Among these are:

- Inconsistency in the regulations between plants compared in the data set.
- Inefficiency or lack of optimization in fuel costs and fuel choice due to the economic regulation of power generation.

Inconsistency in the regulations between plants compared in the data set would cause the problems in implementing this methodology. The methodology is based on the notion of a single production set that is restricted by environmental regulation. The range of different environmental regulations that the power plants in the test data operate under in the year 1989 may be too diverse upon which to base a single restricted production function analysis. The large estimates of 'free' emission reductions based on the second version of the shadow price estimation approach is good evidence of this.

The emission rates for $SO_2$ in the test data set ranged from .5 to 6.7 lbs $SO_2$/Mbtu. The average was 2.3 lbs/mBtu. Clearly many plants in the data set were operating in a virtually unregulated fashion in 1989. This is no longer the case, since high emission plants have already come under Phase I of the new CAAA. For this method to be more accurate, the plants in the data must come under a more common regulatory structure. To some extent, this is the strength of the Coggins and Swinton (1995) study. They only examined coal fired plants without scrubbers, hence had a smaller amount of technical variation in control options. They also claim a common regulatory structure, although emission rates range from 0.7 to 3.5 lbs $SO_2$/MBtu. However, one cannot say what the value of this common set of plants is on the *sign of the shadow prices,* since the methods used by Coggins and Swinton *restricts shadow prices to be non-negative.*

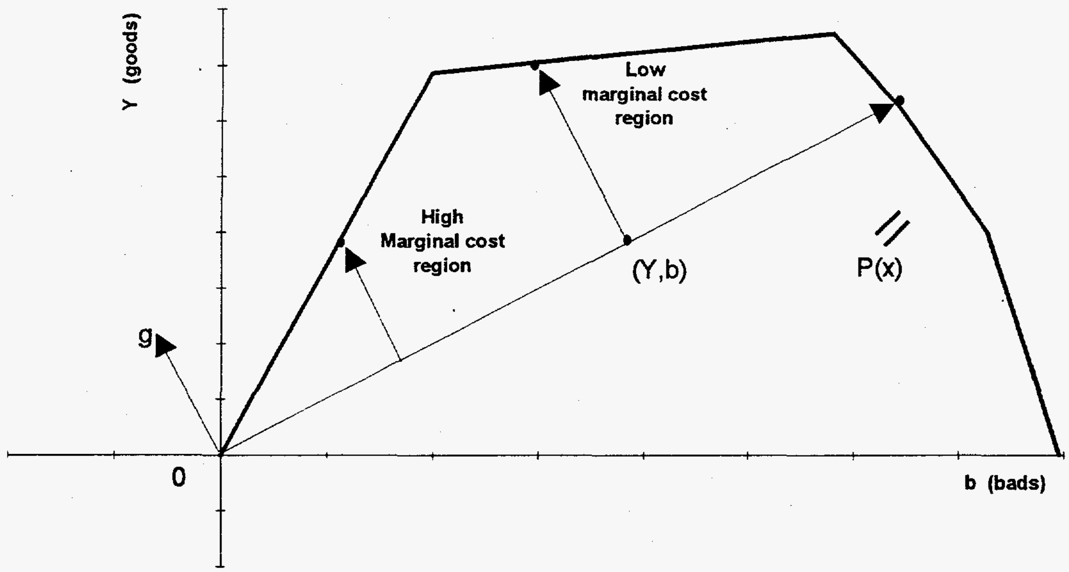


**Figure 5 Comparison of High and Low Marginal Cost Regions of a Piecewise Linear Production Set**

The second reason for positive shadow prices is the potential that the economic regulation of electric generation has created disincentive to optimize fuel choice in the face of changing coal markets. Long term contracts tend to 'lock in' existing coal choice. The presence of fuel adjustment clauses also eliminate any profit incentive to find cheaper coal supply, regardless of environmental concerns. The simple matter is that some plants may actually have opportunities to reduce costs and emissions, if the incentives were in place to do so. (Ellerman 1995) argues that allowance prices have been much lower due to reduction in delivered low-sulfur coal prices, particularly because of falling rail rates.

If these results are not spurious, but reflect underlying opportunities for changing coal choice as an option to reduce costs and emissions, the question arises: How large a coal price differential are implied by these results? To examine this we compare the magnitude of the (total) marginal abatement cost to the total fuel expenditures. The marginal abatement costs estimates for plants with positive shadow prices are multiplied by the total emissions at the plant and divided by fossil fuel expenditures. The average is 36 cents/mbtu and the median is 14 cents/mbtu. In 1989, the average delivered price of coal was 144 cents/mbtu in 1989, so that the median increase is only about 10%.

Implication of Preliminary Analysis and Future Directions

Given the nature of these preliminary results one must answer several questions.

- It is possible to explain the large fraction of positive shadow prices obtained by this methodology and data?
- How do we reconcile the estimates (which are negative) with emission trading that has occurred at $200/ton and lower?

While some modest values of positive shadow prices may be based on actual opportunities to reduce costs and emissions, the primary reason for these positive shadow prices is the lack of uniformity in environmental regulations in 1989 for the plants in the test data set. This is no longer the case under the Title IV of the CAAA. Under more uniform regulatory incentives, we would expect data on recent performance to have fewer positive shadow prices. Nevertheless, non-parametric methods can be sensitive to outliers. A bootstrap procedure might reveal sensitivities in the estimates to particular observations in the data.

The high estimates of shadow prices may be an artifact of the non-parametric methods. If the input data is not sufficiently rich to identify several plants to represent the production set, then the slopes of those segments of the production set are likely to be steep and the resulting shadow prices high, as illustrated in figure 5. A smoothly differentiable, parametric approach might be more desirable. Another simple reason is that the data used by the previous researchers is not detailed enough to construct shadow prices. We consider each of these issues and how to proceed to improve the analysis.

Data quality and completeness is always an issue in empirical analysis. The data used in the test does not include non-labor operating costs. This an important cost of scrubbing which may be underrepresented in the input specification. The other issue is frequently debated economists question on the measurement of capital. The data provided for this test computes capital additions in each year and deflates by the Handy-Whitman index. This procedure to computing capital needs to me examined. Scubber costs can be 10%-15% of the total plant costs, so we do not wish to loose this element of capital in any noise in the data.

Finally, it is important to include all of the elements of pollution in the model specification. $SO_2$ is only one, and a highly variable part, of the plants emission profile. Particulate and Nox control is also important to power plant operations and increasingly the focus on (current and future) emission control regulations.

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