

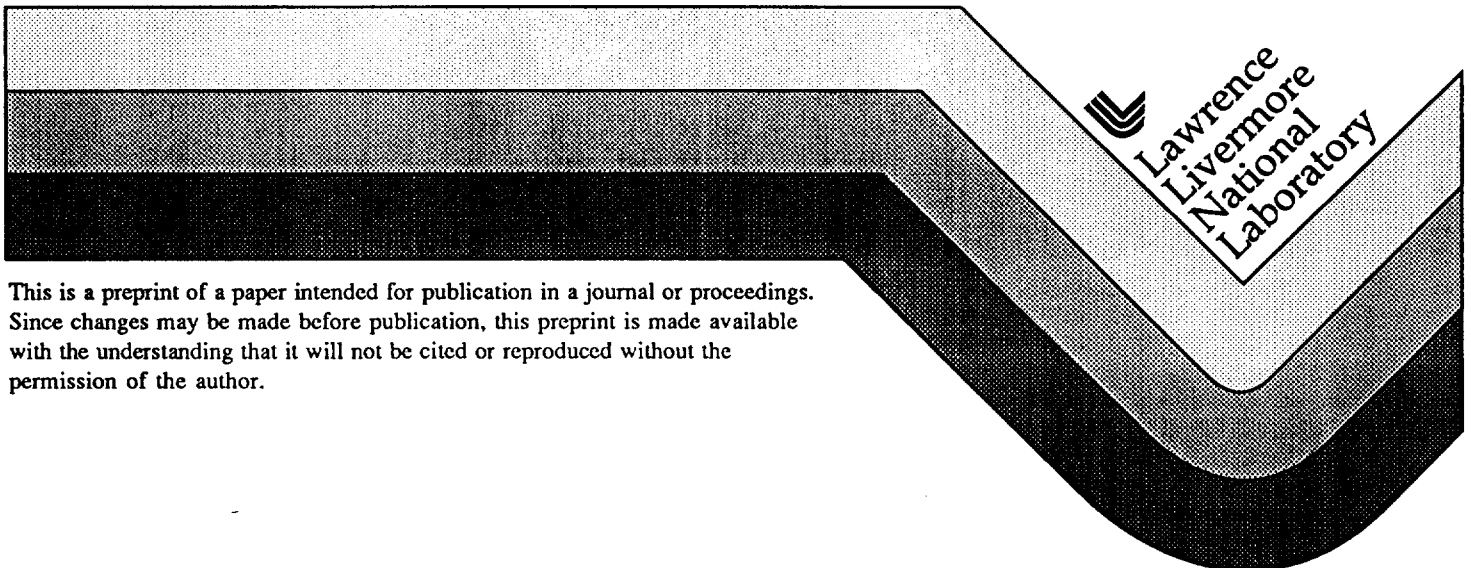
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# Displacements and Rotations of a Body Moving About an Arbitrary Axis in a Global Reference Frame

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## DISPLACEMENTS AND ROTATIONS OF A BODY MOVING ABOUT AN ARBITRARY AXIS IN A GLOBAL REFERENCE FRAME

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### INTRODUCTION

Measurement of human joint motion frequently involves the use of markers to describe joint motion in a global reference frame. Results may be quite arbitrary if the reference frame is not properly chosen with respect to the joint's rotational axis(es). In nature, joint axes can exist at any orientation and location relative to an arbitrarily chosen global reference frame. An arbitrary axis is any axis that is not coincident with a reference coordinate. We calculate the errors that result when joint motion occurs about an arbitrary axis in a global reference frame.

### REVIEW AND THEORY

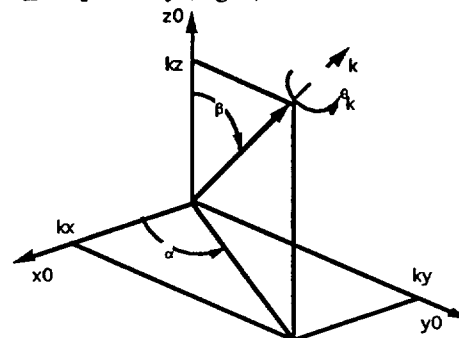
Kinematic and anatomic analyses of a number of human joints have suggested that they move about fixed revolute that are not parallel to the anatomic reference planes (Hollister, et al. 1991, Hollister, et al. 1993, Inman, 1976, London, 1981). Analysis of kinematic data has traditionally been done using joint coordinate systems aligned with the anatomic reference planes (Grood, et al., 1983, Kinzell, et al., 1983, Lundborg, 1988) These analyses use Euler rotations about the coordinate reference frame with or without calculated displacements. This method of describing motion is subject to error (de Lange, et al., 1990) and intra-observer variability. Furthermore, these methods for describing motion do not relate directly to the kinematic mechanism of the joint itself.

### PROCEDURES AND RESULTS

Rotations of a body moving about an arbitrary axis in a reference frame are determined by the axis'  $\alpha$  and  $\beta$  angles of offset from the reference frame and the  $\theta_k$  angle of rotation about the arbitrary axis,  $k$  (Fig. 1).

Displacements of a body moving about an arbitrary axis in a reference frame are determined by the axis'  $\alpha$  and  $\beta$  angles, the  $\theta_k$  angle of rotation,  $r$ , the distance of the body from the axis of rotation, and  $d$ , the distance from the axis to the reference frame (Fig. 2).

When the arbitrary axis,  $k$ , is coincident with the reference  $z$ -axis ( $d, \alpha, \beta = 0$ ) and with  $r = 1$ , the  $x$  and  $y$  positions of the point trace out cosine and sine waves, respectively, and the  $z$  position remains at zero for varying  $\theta_k$ . For perfect alignment, but with  $r \neq 1$ , the amplitude of the cosine and sine curves is scaled accordingly. When the arbitrary axis is parallel to the reference  $z$ -axis, but is translated by non-zero  $x_k, y_k$ , and/or  $z_k$ , the corresponding measured  $x, y$ , and  $z$  trajectories are shifted by  $x_k, y_k$ , and  $z_k$ , respectively (Fig. 3).



$$R_{k\theta} = \begin{bmatrix} k_x^2 v_\theta + c_\theta & k_x k_y v_\theta - k_z s_\theta & k_x k_z v_\theta + k_y s_\theta \\ k_x k_y v_\theta + k_z s_\theta & k_y^2 v_\theta + c_\theta & k_y k_z v_\theta - k_x s_\theta \\ k_x k_z v_\theta - k_y s_\theta & k_y k_z v_\theta + k_x s_\theta & k_z^2 v_\theta + c_\theta \end{bmatrix}$$

where  $v_\theta = 1 - c_\theta$

Figure 1: Rotation about an arbitrary axis.

With the arbitrary axis offset from the reference frame's axes, but still passing through the origin ( $d = 0$ ),

the xyz trajectories vary significantly with the offset angles,  $\alpha$  and  $\beta$  (Fig. 4). The trajectories can be made to vary qualitatively as well as quantitatively depending on the choice of  $\alpha$  and  $\beta$ .

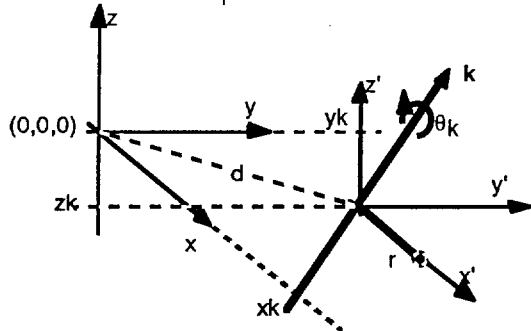


Figure 2: Rotation,  $\theta_k$ , about an arbitrary axis,  $k$ , with translation (by amounts  $x_k, y_k, z_k$ ) from the reference frame,  $x, y, z$ . The distance of the arbitrary axis is given by:  $d = (x_k^2 + y_k^2 + z_k^2)^{1/2}$ . The point moved about the arbitrary frame is a distance,  $r$ , from the arbitrary axis.

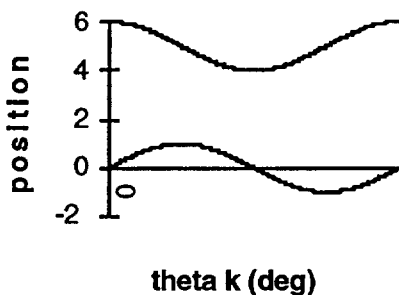


Figure 3: xyz positions with a shift of  $x_k = 5$ , for  $\theta_k$  varying from zero to 90 deg. The z trajectory is zero; the y trajectory traces out the cosine function; the x trajectory traces out an offset sine, centered about  $x_k=5$ .



Figure 4: xyz positions for axes of rotation with varying offset angles,  $\alpha$  and  $\beta$  ( $d=0, r=1$ ): shown are x trajectories (thick lines;  $\alpha, \beta = 0$ , solid;  $\alpha, \beta = 10$  and  $20$  deg, dashed;  $\alpha=80$  deg,  $\beta = 0$ , dashed), y trajectories (thin dashed), and z trajectories (thin, solid).

Euler angle rotations are also affected by the  $\alpha$  and  $\beta$  offsets. With zero offsets, the Euler angles correspond with the  $\theta_k$  rotations (with variation in only one of the Euler angles) With rotation about a single

arbitrary axis with 10 deg offset, this relationship breaks down:  $\alpha$  and  $\beta$  offset angles produce rotations about all three Euler coordinates, and the relationships are nonlinear (Fig. 5). Different results are obtained if the  $\alpha$  and  $\beta$  offsets are  $80^\circ$ , even though the axis lies within  $10^\circ$  of a reference coordinate.

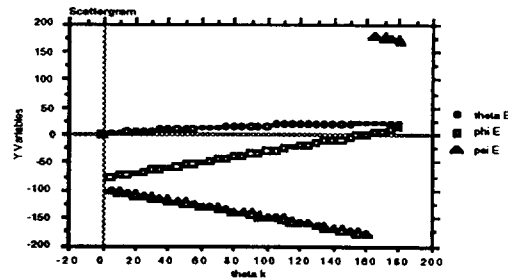


Figure 5: Euler angle rotations calculated from rotations about an arbitrary axis with  $\alpha$  and  $\beta = 10$  deg.

## DISCUSSION

Slight offsets of the global reference frame from the axis of rotation produce significant errors in recording displacements and rotations for motion about the joint axis. We conclude that the relationship of the reference frame to the axis of rotation must be known in order to perform accurate kinematic measurements.

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