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GLOBAL OPTIMIZATION FOR MULTISENSOR FUSION IN SEISMIC IMAGING*

Jacob Barhen, Vladimir Protopopescu, David Reister

Center for Engineering Systems Advanced Research
Oak Ridge National Laboratory
P.O. Box 2008
Building 6010, MS-6355
Oak Ridge, TN 37831-6355

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ABSTRACT

The accurate imaging of subsurface structures requires the fusion of data collected from large arrays of seismic sensors. The fusion process is formulated as an optimization problem and yields an extremely complex "energy surface". Due to the very large number of local minima to be explored and escaped from, the seismic imaging problem has typically been tackled with stochastic optimization methods based on Monte Carlo techniques. Unfortunately, these algorithms are very cumbersome and computationally intensive. Here, we present TRUST - a novel deterministic algorithm for global optimization that we apply to seismic imaging. Our excellent results demonstrate that TRUST may provide the necessary breakthrough to address major scientific and technological challenges in fields as diverse as seismic modeling, process optimization, and protein engineering.

INTRODUCTION

In many geophysical tasks, seismic energy is detected by receivers which are regularly spaced along a grid that covers the explored domain. A source is positioned at some grid node to produce a shot. Time series data is collected from the detectors for each shot; then the source is moved to another grid node for the next shot. A major degradation of seismic signals usually arises from near-surface geologic irregularities [1, 2]. These include uneven soil densities, topography, and significant lateral variations in the velocity of seismic waves. The most important consequence of such irregularities is a *distorted image of the subsurface structure*, due to misalignment of signals caused by unpredictable delays in recorded travel times of seismic waves in a vertical neighborhood of every source and receiver. To improve the quality of the seismic analysis, timing adjustments (called "statics corrections") must be performed. One typically distinguishes between "*field statics*", which correspond to corrections that can be derived directly from topographic and well measurements, and "*residual statics*", which incorporate adjustments that must be inferred statistically from the seismic data. The common occurrence of severe residual statics (where the dominant period of the recorded data is significantly exceeded), and the significant noise contamination render the automatic identification of large static shifts extraordinarily difficult. Thus, *multisensor fusion* must be invoked [3]. This problem has generally been formulated in terms of global optimization and, to date, Monte-Carlo techniques (e.g., simulated annealing, genetic algorithms) have provided the primary tools for seeking a potential solution.

The objective function associated with the task of fusing data from an array of seismic sensors depends on a very large number of parameters. Finding the extrema and, in particular, the absolute extremum of such a function turns out to be painstaking difficult. The primary difficulty stems from the fact that the global extremum, say minimum, of a real function is - despite its name - a local property. In other words, significant alteration of the location and magnitude of the global minimum can be carried out without affecting at all the locations and magnitudes of the other minima. Short of exhaustive search, it would then appear extraordinarily unlikely to design unfallible methods to locate the absolute minimum for an arbitrary function. In recent years there has been a remarkable surge of interest in global optimization [5 - 8]. Although significant progress has been achieved in breaking new theoretical ground [9 - 19], the need for efficient and reliable global optimization methods remains as urgent as ever. In particular, a major need exists for a breakthrough paradigm which would enable the accurate and efficient solution of *large-scale* problems. In response to that need, we have been focusing, at ORNL's Center for Engineering Systems Advanced Research (CESAR), on two innovative concepts, namely subenergy tunneling and non-Lipschitzian terminal repellers, to ensure escape from local minima in a fast, reliable, and computationally efficient manner. The generally applicable methodology is embodied in the TRUST algorithm [4], which is deterministic, scalable, and easy to implement. Benchmark results show that TRUST is considerably faster and more accurate than previously reported global optimization techniques. Hence, TRUST may provide the enabling means for addressing major scientific and technological challenges in fields as diverse as seismic modeling, process optimization, and protein engineering.

The classical theory of optimization started to develop almost concomitantly with classical mechanics by trying to find extremal values (minima or maxima) of certain functions that bear special physical meaning and practical significance. For instance, Newton studied projectile trajectories and obtained their maximum range by taking into account the friction with the atmosphere. He was also interested in minimizing resistance by modifying the shape of an object propelled through water. The Bernoulli brothers, who were active in Switzerland between 1670 and 1720, discovered that the shortest time of descent between two points under gravity is achieved not on the straight line joining the two points, but on a convex curve, called brachistocrone. Another famous optimization problem is to find the greatest area enclosed between a straight line and an arbitrary curve of fixed length joining two points on the line. By Virgil's account (Aeneid, Book I, line 367), Queen Dido solved the problem by determining the shape of the curve and the position of the points, thereby founding Carthage.

The completion of the main body of classical physics around the turn of the century came with the realization that many natural processes take place according to extremal principles such as: (i) the principle of stationary (minimum) action in mechanics and electrodynamics; (ii) the principle of minimal potential energy in stable mechanical equilibrium states; (iii) the principle of maximal entropy in isolated thermodynamic systems at equilibrium; and (iv) the principle of motion along geodesics (Fermat's principle in geometrical optics and Einstein's principle in relativity theory). Thenceforth extremal principles and, more generally, optimization problems have been perceived as a systematic and elegant framework for addressing and solving more complex problems with applications to economy, investment policies, and social or political negotiations. In these domains, optimization is, in turn, used to determine "the best" model for a complex situation , to make "the best" choice within a given model, and to solve the associated, purely technical, sub-problems that occur in the mathematical analysis and implementation of the model. In this context, optimality is, almost always, to be obtained under certain constraints and/or at the expense of a certain price.

The generic *global optimization* problem can be stated as follows. The overall performance of a system is described by a multivariate function, called the objective function. Optimality of the system is reached when the objective function attains its global extremum, which can be a maximum or a minimum, depending on the problem under consideration. From an algorithmic perspective, however, there is essentially no difference between the two.

THE TRUST ALGORITHM

We now define the global optimization problem considered in more rigorous terms. Let $f(\mathbf{x}) : \mathcal{D} \rightarrow \mathcal{R}$ be a function with a finite number of discontinuities, and $\mathbf{x}$ be an n-dimensional state vector. At any discontinuity point, $\mathbf{x}^{\delta}$, the function $f(\cdot)$ is required to satisfy the inequality $\lim_{\mathbf{x}\rightarrow\mathbf{x}^{\delta}} \inf f(\mathbf{x}) \geq f(\mathbf{x}^{\delta})$ (lower semicontinuity condition). Hereafter, $f(\mathbf{x})$ will be referred to as the objective function, and the set $\mathcal{D}$ as the set of feasible solutions (or the solution space). The goal is to find location of the global minimum, i.e. the value $\mathbf{x}^{gm}$ of the state variables which minimizes $f(\mathbf{x})$,

$$f(\mathbf{x}^{gm}) = \min\{f(\mathbf{x}) \mid \mathbf{x} \in \mathcal{D}\} \ . \tag{1}$$

Without loss of generality, we shall take $\mathcal{D}$ as the hyperparallelepiped

$$\mathcal{D} = \{x_i \mid \beta_i^- \leq x_i \leq \beta_i^+ \ ; \ \ i = 1, 2, \ldots, n\} \ . \tag{2}$$

where β_i^- and β_i^+ denote, respectively, the lower and upper bound of the i-th state variable.

We define the *subenergy tunneling* transformation of the function $f(\mathbf{x})$ by the following nonlinear monotonic mapping:

$$E_{sub}(\mathbf{x}, \mathbf{x}^*) = \log(1/[1 + \exp(-\hat{f}(\mathbf{x}) - a)]) \ . \tag{3}$$

In Eq. (3), $\hat{f}(\mathbf{x}) = f(\mathbf{x}) - f(\mathbf{x}^*)$, a is a constant that affects the asymptotic behavior, but not the monotonicity, of the transformation, and $\mathbf{x}^*$ is a fixed value of $\mathbf{x}$, whose selection will be discussed in the sequel. Whenever f is differentiable, the derivative of $E_{sub}(\mathbf{x}, \mathbf{x}^*)$ with respect to $\mathbf{x}$ is given by

$$\partial E_{sub}(\mathbf{x}, \mathbf{x}^*)/\partial \mathbf{x} = (\partial f(\mathbf{x})/\partial \mathbf{x})(1/[1 + \exp(\hat{f}(\mathbf{x}) + a)]) \ , \tag{4}$$

which yields

$$\partial E_{sub}(\mathbf{x}, \mathbf{x}^*)/\partial \mathbf{x} = 0 \quad \Leftrightarrow \quad \partial f(\mathbf{x})/\partial \mathbf{x} = 0 \ . \tag{5}$$

It is clear that $E_{sub}(\mathbf{x}, \mathbf{x}^*)$ has the same discontinuity and critical points as $f(\mathbf{x})$, and the same relative ordering of the local and global minima. In other words, $E_{sub}(\mathbf{x}, \mathbf{x}^*)$ is a transformation of $f(\mathbf{x})$ which preserves all properties relevant for optimization. In addition, this transformation is designed to ensure that: (i) $E_{sub}(\mathbf{x}, \mathbf{x}^*)$ quickly approaches zero for large positive $\hat{f}(\mathbf{x})$; and (ii) $E_{sub}(\mathbf{x}, \mathbf{x}^*)$ rapidly tends toward $\hat{f}(\mathbf{x})$, whenever $\hat{f}(\mathbf{x}) \ll 0$.

An equilibrium point $\mathbf{x}_{eq}$ of the dynamical system $\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x})$ is termed an attractor (repeller) if no (at least one) eigenvalue of the $n \times n$ matrix $\mathcal{M}$, $\mathcal{M} = \partial \mathbf{g}(\mathbf{x}_{eq})/\partial \mathbf{x}$ has a positive real part. Typically, a certain amount of regularity (Lipschitz condition) is required to guarantee the existence of a unique solution for each initial condition $\mathbf{x}(0)$ and, in those cases, the system's relaxation time to an attractor, or escape time from a repeller, is theoretically infinite. If the regularity condition at equilibrium points is violated, singular solutions are induced, such that each solution approaches a *terminal attractor* or escapes from a *terminal repeller* in finite time. The above concepts are at the foundation of our Terminal Repeller Unconstrained Subenegy Tunneling (TRUST) global optimization algorithm.

Let $f(\mathbf{x})$ be a function one wishes to globally minimize over $\mathcal{D}$. We define the TRUST *virtual objective function*

$$\begin{aligned} E(\mathbf{x}, \mathbf{x}^*) &= \log(1/[1 + \exp(-\hat{f}(\mathbf{x}) - a)]) - (3/4)\rho(\mathbf{x} - \mathbf{x}^*)^{4/3}\theta(\hat{f}(\mathbf{x})) \\ &= E_{sub}(\mathbf{x}, \mathbf{x}^*) + E_{rep}(\mathbf{x}, \mathbf{x}^*). \end{aligned} \tag{6}$$

In the above expression $\theta(\cdot)$ denotes the Heaviside function, that is equal to one for positive values of the argument and zero otherwise. The first term on the right-hand side of Eq. (6) corresponds to the subenergy tunneling function; the second term is referred to as the repeller energy term. The parameter $\rho > 0$ quantifies the strength of the repeller. Application of gradient descent to $E(\mathbf{x}, \mathbf{x}^*)$ results in the dynamical system$(i = 1, ..n)$

$$\dot{x}_i = -(\partial f(\mathbf{x})/\partial x_i)(1/[1 + \exp(\hat{f}(\mathbf{x}) + a)]) + \rho(x_i - x_i^*)^{1/3}\theta(\hat{f}(\mathbf{x})) \ . \tag{7}$$

Figure 1 illustrates the main characteristics of TRUST for a one-dimensional problem objective function $E(x, x^*)$. A schematical representation of a sufficiently smooth $f(x)$ is shown, which has three local minima, one of which is the global minimum. We assume that the solution flows in the positive direction (i.e., away from the left boundary), and that the local minimum at $x = {}^{\mu}x$ is encountered by a local minimization method, gradient descent for example. The task under consideration is to escape this local minimum, in order to reach the valley of another minimum with a lower value. We set $x^* = {}^{\mu}x$; then the objective function in Eq. (6) performs the following transformation (see Figure 1):

- the offset function $\hat{f}(x) = f(x) - f(x^*)$ creates the curve parallel to $f(x)$, such that the local minimum at $x = x^*$ intersects with the x-axis tangentially;
- the term $E_{sub}(x, x^*)$ forms the portion of the thick line denoted by **II** (i.e., the lower valley) as a result of the properties of the subenergy transformation;
- the repeller energy term $E_{rep}(x, x^*)$ essentially constitutes the portion of the thick line denoted by **I**;
- finally, as the complete thick line (i.e., **I** and **II**) shows, the virtual objective function $E(x, x^*)$, which is a superposition of these two terms, creates a discontinuous but well-defined function with a *global maximum* located at the previously specified local minimum ${}^{\mu}x$.

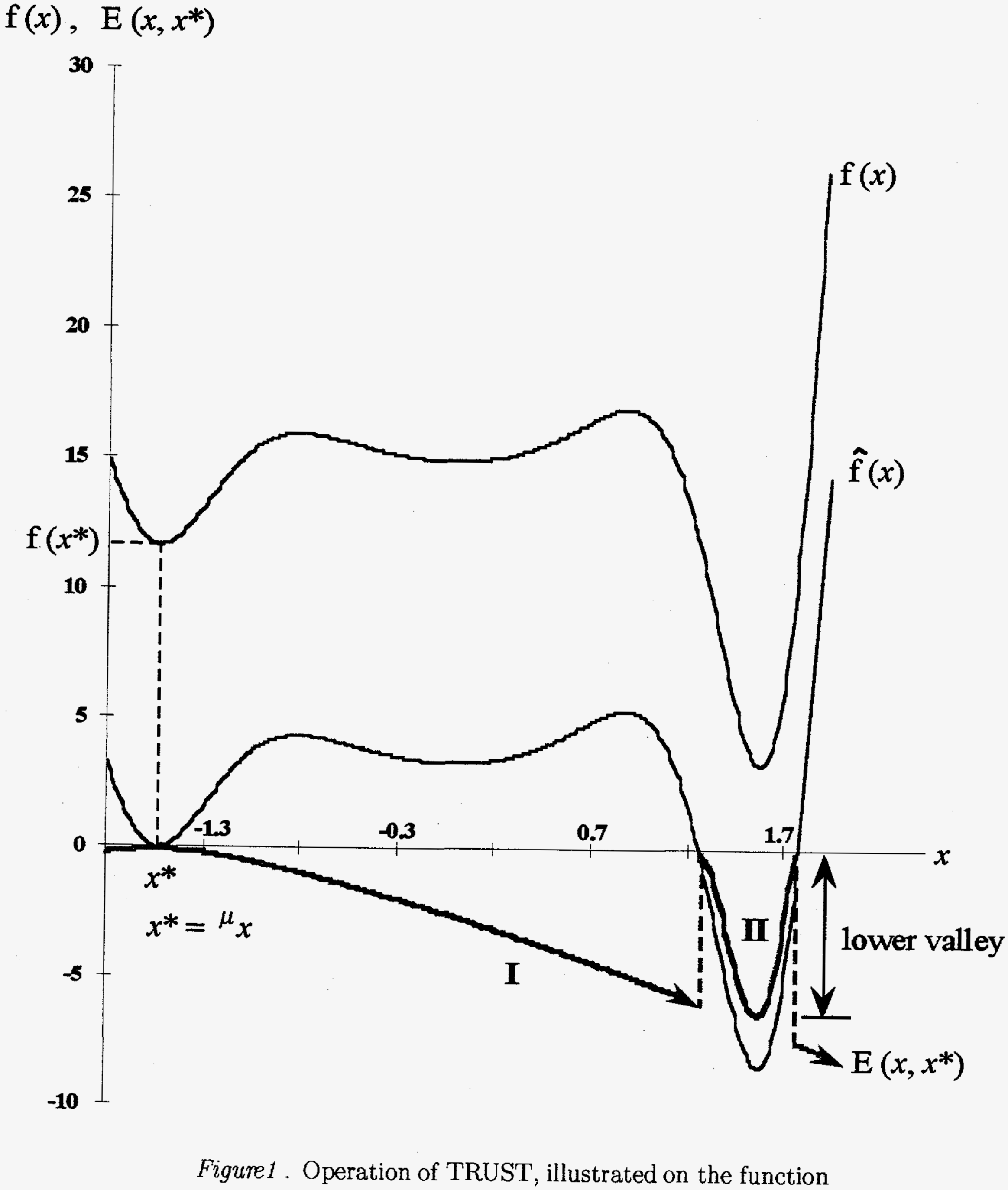

Figure1 . Operation of TRUST, illustrated on the function
$f(x) = 4x^2 e^{2\alpha(x-1)} \sin[\frac{\pi}{8}(4x^2+3)]$, with $\alpha \simeq 1.22$.

To summarize, as seen in Figure 1, $E(x, x^*)$ of Eq. (7) transforms the current local minimum of $f(x)$ into a global maximum, but preserves all lower local minima. Thus, when gradient descent is applied to the function $E(x, x^*)$, the new dynamics, initialized at a small perturbation from the local minimum of $f(x)$ (i.e., at $x = x^* + d$, with $x^* = {}^{\mu}x$), will escape this critical point (which is also the global maximum of $E(x, x^*)$) to a lower valley of $f(x)$ with a lower local minimum. It is important to note that the discontinuity of $E(x, x^*)$ does not affect this desired operation, since the flow of the gradient descent dynamics follows the gradient of $E(x, x^*)$, which is well-defined at every point in the region. It is clear that if gradient descent were to be applied to the objective function $f(x)$ under the same conditions, escaping the local minimum at $x = {}^{\mu}x$ would not be accomplished.

Hence, application of gradient descent to the function $E(x, x^*)$ as defined in Eq. (6), as opposed to the original function $f(x)$, results in a system that has a *global descent property*, i.e., the new system escapes the encountered local minimum to another one with a lower functional value. This is the main idea behind constructing the TRUST virtual objective function of Eq. (6). Additional details and formal derivations can be found in [4, 15, 18].

BENCHMARKS AND COMPARISONS TO OTHER METHODS

This section presents results of benchmarks carried out to assess the TRUST algorithm using several standard test functions taken from the literature. A description of each test function is given in Table 1. In Tables 2-3, the performance of TRUST is compared to the best competing global optimization methods, where the term "best" indicates the best widely reported reproducible results the authors could find for the particular test function. The criterion for comparison is the number of function evaluations.

One of the primary limitations of conventional global optimization algorithms is their lack of stopping criteria. This limitation is circumvented in benchmark problems, where the value and coordinates of the global minima are known in advance. The achievement of a desired accurracy (e.g., $\epsilon = 10^{-6}$) is then considered as a suitable termination condition [6]. For consistent comparisons, this condition has also been used in TRUST, rather than its general stopping criterion described earlier. For each function, corners of the domain were taken as initial conditions; each reported result then represents the average number of evaluations required for convergence to the global minimum of the particular function. The TRUST calculations were performed using the value $a = 2$, for which the subenergy tunneling transformation achieves its most desirable asymptotic behavior [15]. The dynamical equations were integrated using an adapitive scheme, that, within the basin of attraction of a local minimum, considers the local minimum as a terminal attractor. Typical base values for the key parameters Δ_t and ρ were 0.05 and 10., respectively.

In Table 2, the benchmark labels, i.e. BR (Branin), CA (Camelback), GP (Goldstein-Price), RA (Rastrigin), SH (Shubert) and H3 (Hartman), refer to the test functions specified in Table 1. The following abbreviations are also used: SDE is the stochastic method of Aluffi-Pentini [9]; EA is the annealing evolution algorithms of Yong, Lishan, and Evans [17] and Schneider [19]; MLSL is the multiple level single linkage method of Kan and Timmer [10]; IA is the interval arithmetic technique of Ratschek and Rokne [19]; TUN is the tunneling method of Levy and Montalvo [11]; and TS refers to the Taboo Search scheme of Cvijovic and Klinowski [16]. The results demonstrate that TRUST is substantially faster than these state-of-the-art methods.

Table 1. Standard Test Functions used for global optimization benchmarks.

Name	Definition	Domain
Branin	$f(\mathbf{x}) = [x_2 - (5.1/4\pi^2)x_1^2 + (5/\pi)x_1 - 6]^2 + 10(1 - 1/8\pi)\cos x_1 + 10$	$-5. \leq x_1 \leq +10.$ $0. \leq x_2 \leq +15.$
Camelback	$f(\mathbf{x}) = [4 - 2.1x_1^2 + (x_1^4/3)]\, x_1^2 + x_1x_2 + (-4 + 4x_2^2)x_2^2$	$-3. \leq x_1 \leq +3.$ $-2. \leq x_2 \leq +2.$
Goldstein-Price	$f(\mathbf{x}) = [1 + (x_1 + x_2 + 1)^2\,(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)]$ $\times[30 + (2x_1 - 3x_2)^2\,(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	$-2. \leq x_i \leq +2.$
Rastrigin	$f(\mathbf{x}) = x_1^2 + x_2^2 - \cos(18x_1) - \cos(18x_2)$	$-1. \leq x_i \leq +1.$
Shubert	$f(\mathbf{x}) = \left\{\sum_{i=1}^{5} i\cos[(i+1)x_1 + i]\right\}\left\{\sum_{i=1}^{5} i\cos[(i+1)x_2 + i]\right\}$	$-10. \leq x_i \leq +10.$
Hartman*	$f(\mathbf{x}) = \sum_{i-1}^{i=4} c_i \exp[-\sum_{j=1}^{j=N} a_{ij}(x_j - p_{ij})^2]$	$0. \leq x_i \leq 1.$
Styblinski and Tang	$f(\mathbf{x}) = \frac{1}{2}\sum_{i=1}^{i=2}(x_i^4 - 16x_1^2 + 5x_i) + \sum_{i=3}^{i=5}(x_i - 1)^2$	$-4.6 \leq x_i \leq +4.6$

(*)The values of the parameters are given in ([6], p. 185).

Table 2. Number of function evaluations required by different methods to reach a global minimum of Standard Test Functions.

Method	BR	CA	GP	RA	SH	H3
SDE	2700	10822	5439	–	241215	3416
EA	430	–	460	5917	–	–
MLSL	206	–	148	–	–	197
IA	1354	326	–	–	7424	–
TUN	–	1469	–	–	12160	–
TS	492	–	486	540	727	508
TRUST	55	31	103	59	72	58

Table 3. Number of function evaluations and precision for Styblinski and Tang function. Global minimum FSA and SAS results taken from Ref. [13].

Method	FSA	SAS	TRUST	Exact
Cost	100,000	3,710	89	n/a
x_1	-2.702844	-2.903506	-2.90353	-2.903534
x_2	-3.148829	-2.903527	- 2.90353	-2.903534
x_3	1.099552	1.000241	1.00004	1.
x_4	1.355916	0.999855	0.99997	1.
x_5	1.485936	1.000194	0.99997	1.

In Table 3, FSA is the fast simulated annealing algorithm of Szu [12], and SAS denotes the stochastic approximation paradigm of Styblinski and Tang [13]. As can be observed, TRUST is not only much faster, but produces very consistent and accurate results. Therefore, it seemed the ideal candidate for the solution of the notoriously difficult problem of multisensor fusion for seismic imaging, formulated as residual statics optimiation.

RESIDUAL STATICS CORRECTIONS FOR SEISMIC DATA

Statics optimization is typically done in a surface consistent manner to seismic traces corrected for *normal moveout* [3]; consequently, the correction time shifts depend only on the shot and receiver positions, and not on the ray path from shot to receiver. Shot corrections **S** correspond to wave propagation times from the shot locations to a reference plane, while the receiver corrections **R** are propagation times from the reference plane to receiver locations. From an operational perspective, data D_{ft} are provided by trace ($t = 1, \ldots N_t$), and sorted to midpoint offset coordinates (common midpoint *stacking*). For each trace, the data consist of the complex Fourier components ($f = 1, \ldots N_f$) of the collected time series. Each trace t corresponds to seismic energy travel from a source s_t to a receiver r_t via a midpoint k_t. Assuming the availability of N_k common midpoints, we seek statics corrections **S** and **R** that maximize the total power E in the stacked data:

$$E = \sum_k \sum_f | \sum_t \exp[2\pi i\, f\, (S_{s_t} + R_{r_t})] D_{ft} \delta_{kk_t}|^2 \ . \tag{8}$$

The above expression highlights the multimodal nature of E which, even for relatively low dimensional **S** and **R**, exhibits a very large number of local minima. This is illustrated in Figure 2.

To assess the performance of TRUST, we considered a problem involving 77 shots and 77 receivers. A dataset consisting of 1462 synthetic seismic traces folded over 133 common midpoint gathers was obtained from CogniSeis Corporation (J. DuBose). It uses 49 Fourier components for data representation. Even though this set is somewhat smaller than typical collections obtained during seismic surveys by the oil industry, it is representative of the extreme complexity underlying residual statics problems. To derive a quantitative estimate of TRUST's impact, let E_k denote the total contribution to the stack power arising from midpoint k, and let B_k refer to the upper bound of E_k in terms of **S** and **R**. Using a polar coordinates representation for the trace data D_{ft}, i.e., writing $D_{ft} = \alpha_{ft} \exp(iw_{ft})$, we can prove that

$$B_k = \sum_f (\sum_t \alpha_{ft}\, \delta_{kk_t})^2 \ . \tag{9}$$

The TRUST results, illustrated in Figure 3, show the dramatic improvement in the coherence factor of each common gather. This factor is defined as the ratio $\kappa_k = E_k / B_k$, and characterizes the overall quality of the seismic image.

CONCLUSIONS

TRUST is a novel methodology for unconstrained global function optimization, that combines the concepts of subenergy tunneling and non-Lipschitzian "terminal repellers." The evolution of a deterministic nonlinear dynamical system incorporating these concepts provides the computational

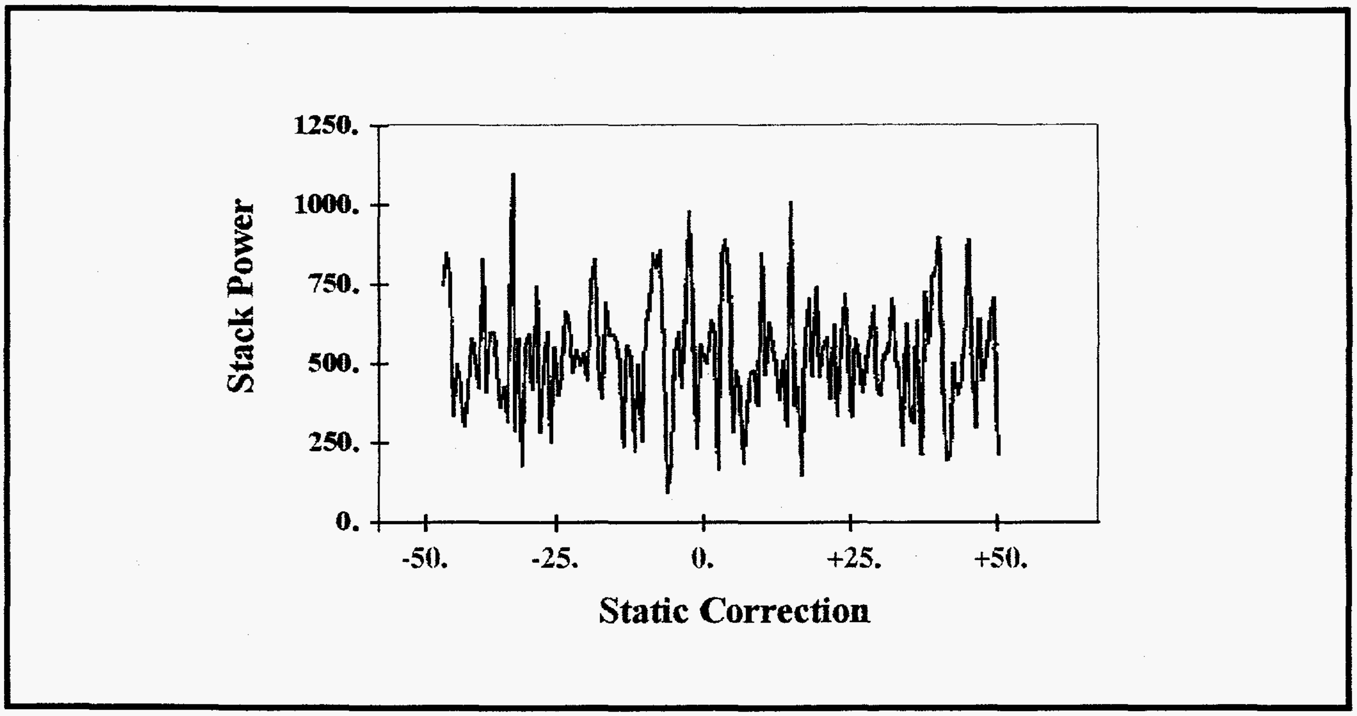

Figure 2. One-dimensional slice through a 154-dimensional objective function associated with a residual statics problem.

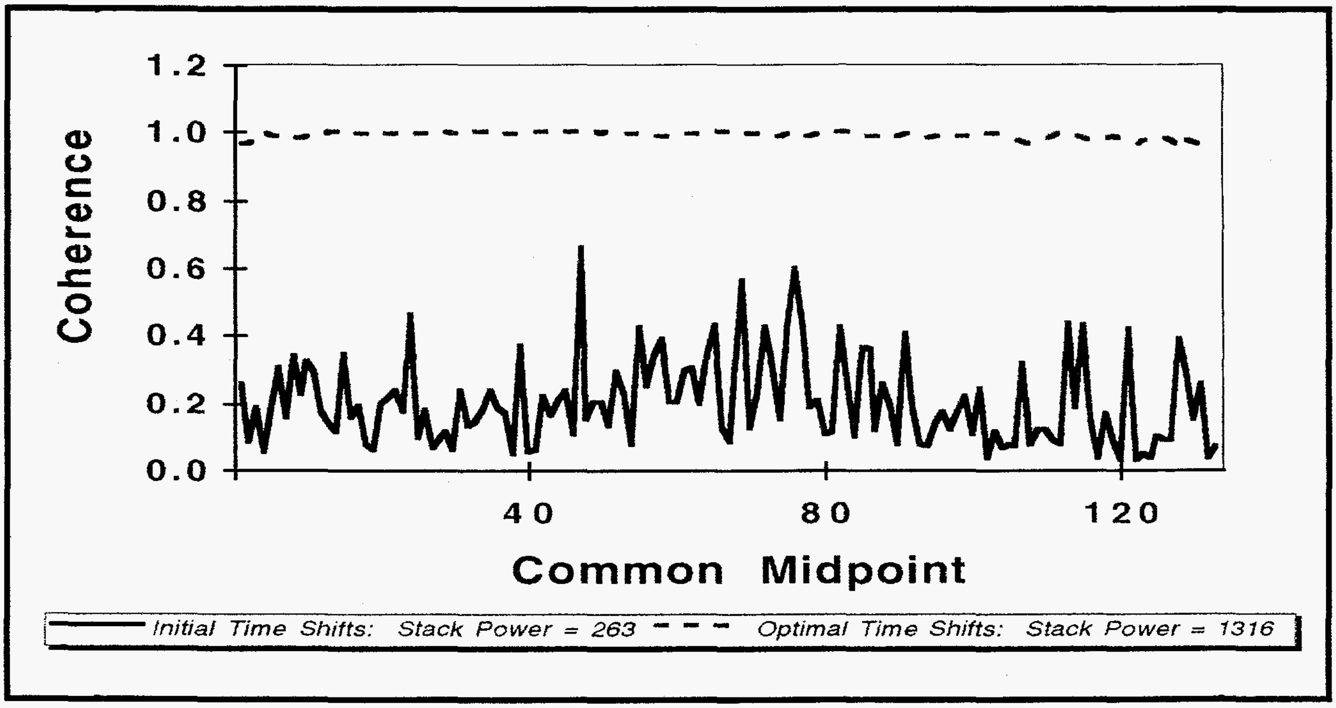

Figure 3. The coherence factors, i.e., the dimensionless ratios E_k/B_k, are plotted for each common gather using the initial and the optimal time shifts ("residual statics"). Ideally, at the global optimum, these ratios should be equal to one.

mechanism for reaching the global minima. The benchmark results demonstrate that TRUST is substantially faster, as measured by the number of function evaluations, than other global optimization techniques for which reproducible results have been published in the open literature. The application of TRUST to the problem of multisensor fusion for accurate seismic imaging (residual statics corrections) proves that the method is not a mere academic exercise for toy problems, but has the robustness and consistency required by large-scale, real-life applications.

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