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Fireball Yield from Fractional Intensity Diameters

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FIREBALL YIELD FROM FRACTIONAL INTENSITY DIAMETERS

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Object: It is desired to develop an empirical formula of the type

$$Y = KD^n$$

where Y is the yield in kilotons and D is the "effective diameter" in feet corrected for temperature and pressure variations, if necessary.

Procedure: The values of D were obtained from pinhole photographs of the events made with a 48 inch long pinhole camera having a forty mil pinhole in front (therefore an f/1200 camera). It was necessary to use an N.D. 3.0 filter (attenuation 1000) to diminish the image brightness.[1]

From the diameter of the photographic image, d, the focal distance f = 48 inches, and the slant range R, the fireball diameter was obtained by the simple relation $D = dR/f$, where d and f are in consistent units. In order to improve the results obtained by D. R. Born and E. C. Woodward[2] the effective diameters were taken as the distances between points whose intensities were a certain fixed fraction of the time integrated fireball intensity (5 and 20%), rather than the diameters measured from the photographic image by eye.

Except where the fireball was obscured or blurred in one direction, four equally spaced vertical, horizontal, and diagonal densitometer traces were taken of the photographic image.

A step tablet had not been put on the photographs, nor had the development time been recorded because the photographs were originally intended only for alignment purposes. Therefore, in order to determine the development time the gross fog of the pinhole photograph was measured

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1 D. R. Born and E. C. Woodward - "Instant Fireball Yield" - COPBA-62-7, Sept. 14, 1962.

2 Ibid

and compared with the fog on similar films developed for a known time. Film density was then converted into light intensity values by interpolating between the points of the calibration curves of the standard films.

Diameters corresponding to 1/20 of the maximum light intensity were measured and recorded. However, it was found that the photograph of one event (Alma) was not sufficiently exposed for the 1/20 intensity to be distinguishable from the background, so that no diameter could be recorded for it.

Using the values of <u>d</u> obtained by the method just described, the method of least squares was applied to the equation $Y = KD^n$ in the logarithmic form

$$\text{Log } Y = \text{Log } K + n \log D$$

to determine the parameters K and n. The same procedure was carried out again, with the exception that D was replaced by Do, where $Do = (\frac{P}{Po})^{1/3}D$, the pressure corrected diameter. In another calculation the exponent of the pressure correction was allowed to be an unknown, that is, the method of least squares was applied to the equation:

$$\text{Log } Y = \text{Log } K + n\,(\text{Log } D + q \text{ Log } P/Po)$$

In a fourth calculation an arbitrary temperature correction was also introduced so that the equation to which least squares was applied is

$$\text{Log } Y = \text{Log } K + n \text{ Log } D + r \log \frac{P}{Po} + s \text{ Log } \frac{T}{To}$$

After these calculations were carried out the data was slightly improved by re-measuring fog densities, rescanning the photograph of one event (Truckee) which appeared somewhat suspicious because of the considerable variation in maximum net density (2.06 to 2.31), having more test films made in order to avoid extrapolation of the H-D curve by more than one order of magnitude, and making several other minor improvements. From this data the first calculation, in which we set $Y = KD^n$, was again carried out as well as the similar calculation where D was replaced by Do.

The first calculation was also carried out for the 1/5 intensity diameters. Diameters associated with intensities considerably smaller than 1/20 of the maximum time integrated intensity could not be obtained, because for several of the events, Bighorn, Sunset and Yeso, any lower intensity would not produce an image that was distinguishable from the background.

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Results: Empirical Equations Used

1. Log Y = Log K + n Log D $D = dR/f$
2. Log Y = Log K + n Log Do $D_o = (P/Po)^{1/3} D$
3. Log Y = Log K + n (Log D + q Log (P/Po))
4. Log Y = Log K + n Log D + r Log (P/Po) + s Log (T/To)

I. 1/20 Intensity Diameters

Equation Used	Av.% Dev. between Y calc.& Y egg	Log K	K	n	q	r	s
1	6.1*	-6.52151	3.010×10^{-7}	2.63447	......	....	....
2	7.5	-6.46222	3.450×10^{-7}	2.65055	......	....	....
3	6.4*	-6.51821	3.033×10^{-7}	2.63661	.03136	....	....
4	5.0	-6.44456	3.593×10^{-7}	2.64316	......	6.26496	26.57143

II. 1/5 Intensity Diameters

1	12.7	-6.19223	6.423×10^{-7}	2.59080	......	....	....

III. 1/20 Intensity Diameters - Rechecked and Improved Data

1	5.9	-6.21723	6.064×10^{-7}	2.54819	......	....	....
2	6.6	-6.16936	6.7771×10^{-7}	2.54665	......	....	....

Therefore: Equation selected (III-1)

$$Y = 6.064 \times 10^{-7} D^{2.54819}$$

or

$$\text{Log } Y = -6.21723 + 2.54819 \text{ Log } D$$

*Note: Of course the sum of the squares of the difference in the logarithims is less for I-3 than I-1, .00886 as compared with .00893, as required by the method of least squares.

DETAILS OF CALCULATION FOR THE EQUATION SELECTED

(Eq. III-1)

$$Y = 6.064 \times 10^{-7} D^{2.54819}$$

or $$\text{Log } Y = -6.21723 + 2.54819 \text{ Log } D$$

Intensity Data

Event	Max.NT.[1] Dns.	Max. GR[1] Dns.	Log I Max.in m.c.s.	Log $\frac{1}{20}$ I in m.c.s.	Corros Gross Dns.	Corros Net Dns.	Net D/D max. Used to obtain Diameters
Truckee	2.3542	2.5835	+.77	-.53	1.85	1.62	.69
Harlem	2.5479	2.8535	.14	-1.16	1.70	1.39	.55
Riconada	2.3054	2.5497	.32	-.98	1.65	1.41	.61
Bluestone	2.3509	2.6066	1.63	.33	2.04	1.18	.76
Yeso	1.6040	1.9607	-1.05	-2.35	.41	.05	.03
Alma	.2024	.7181	-1.97	-3.27	....	<0	<0
Sunset	1.9134	2.2294	-.73	-2.03	.58	.26	.14
Otowi	2.7619	2.9560	1.76	.46	2.36	2.17	.79
Bighorn	1.5012	1.7067	-.85	-2.15	.41	.20	.13
Dulce	2.3534	2.5317	1.13	-.17	1.89	1.71	.73

1 Net density is density above background. Gross density is the total film density.

- 1/20 Intensity - Measured Photographic Diameters (in m.m.)

Event	1	2	3	4	Average	Corresponding Fireball Diameter (feet)
Truckee	48.5	49.5	50.5	50.4	50.3	2,232
Harlem	60.8	60.3	60.3	....	60.4	4,449
Riconada	50.8	50.6	51.4	....	50.9	3,761
Bluestone	(50.7)[2]	53.1	(51.0)[2]	(50.8)[2]	53.1	4,258
Yeso	74.5	74.5	74.5	....	74.5	6,515
Sunset	57.8	57.4	....	....	57.6	4,198
Otowi	33.5	33.7	33.8	32.9	33.5	1,534
Bighorn	71.4[3]	71.6[3]	....	....	71.5[3]	9,099
Dulce	29.8	30.1	29.9	(29.8)[4]	30.0	1,331

( )^2 - Image smeared out due to the motion of the camera - neglect these values.

3 - One side of the density curve had to be extrapolated on the basis of assumed slope symmetry of both sides of the density curve.

( )^4 - Interpolated value - cross-hairs in way - neglect in average.

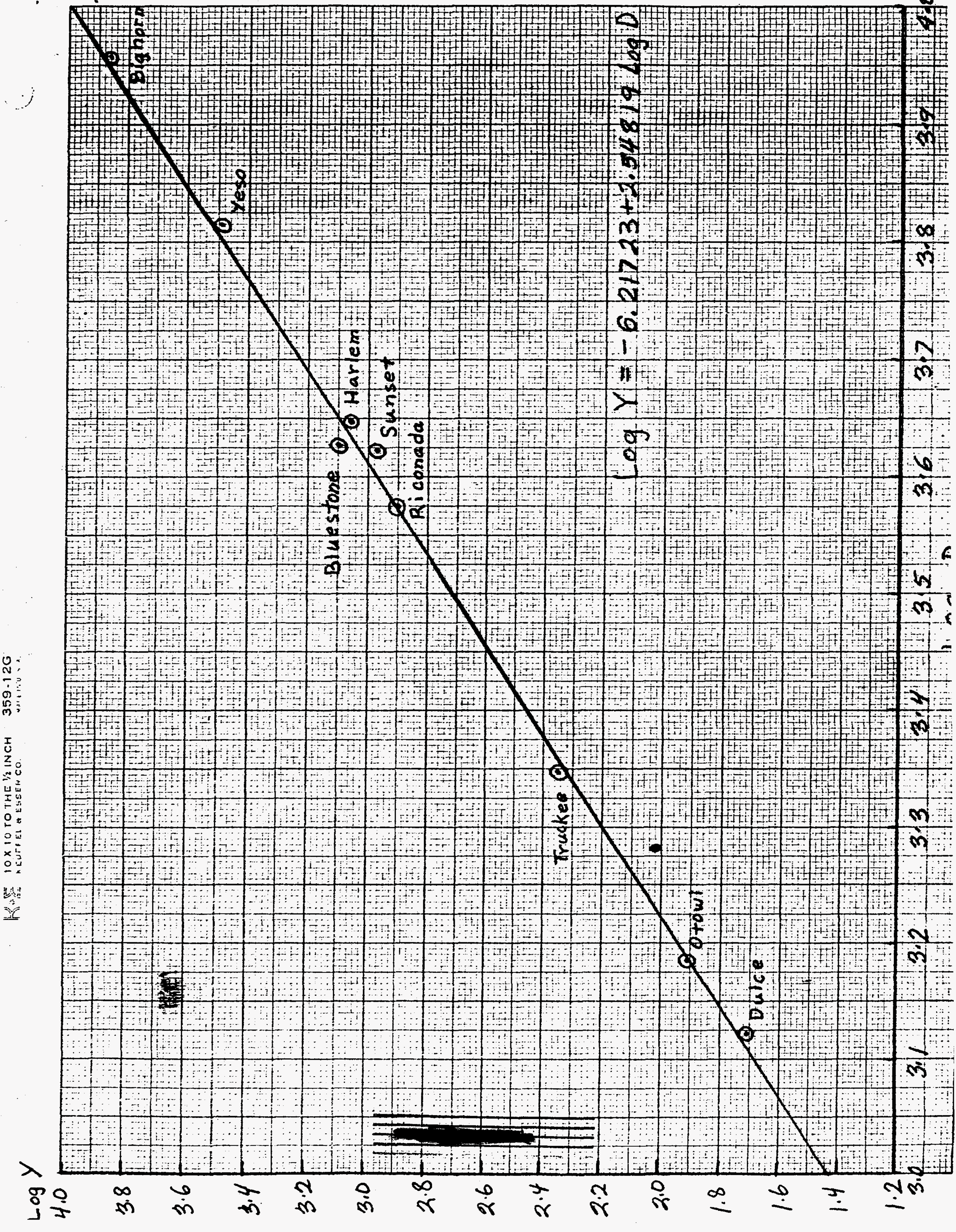
Analysis of the Accuracy of the
Calculated Equation

Event	-6.21723 +2.54819 LogD = Log Y calc.	Y calc.	Y e.g.g.	Difference between Y calc. & Y e.g.g.	Δ as a % of Y e.g.g.
Truckee	2.31572	207	225	-18	-8.0%
Harlem	3.07913	1200	1140	+60	+5.2
Bluestone	3.03066	1070	1260	-190	-15.0
Yeso	3.50139	3170	3100	+10	+2.2
Riconada	2.89344	782	790	-8	-1.0
Sunset	3.01486	1040	930	+110	+11.8
Otowi	1.90107	97.6	80.5	-.9	-1.1
Bighorn	3.87100	7430	7350	+80	+1.0
Dulce	1.74360	55.4	51.5	-3.9	+7.5

Average percentage deviation = 5.9%

Comments:

An examination of the fit of equations 1, 2 and 3 with the observed data clearly shows that the experimental evidence here gives no support to the introduction of a pressure correction factor of the form $(P/Po)^q$. The value of q obtained from equation 3, .03136 is so small that it can be assumed to be due only to statistical fluctuations. Equation 4 seems to indicate that the correction terms P/Po & T/To are quite important, giving them large exponents. However, the exponents are related to each other in such a way that the effects of these two correction terms is practically cancelled. The fact that this gives a somewhat better fit than equations 1 - 3 can be explained purely on the basis of statistical fluctuations since we are now varying four quantities to fit only nine points.


Log Y = -6.21723 + 2.54819 Log D
Bighorn
Yeso
Harlem
Sunset
Bluestone
Riconada
Truckee
Otowi
Dulce
Log Y
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
3.0
3.1
3.2
3.3
3.4
3.5
3.6
3.7
3.8
3.9
4.0
10 X 10 TO THE 1/2 INCH
KEUFFEL & ESSER CO.
359-12G
[redacted]

The improved data can be seen to give results in reasonable agreement with the original results. However, since it does incorporate more measurements and less extrapolation, the form of equation one based on this data is selected, giving

$$Y = 6.064 \times 10^{-7} D^{2.54819}$$

The fact that the 1/5 intensity diameters give twice as poor a fit does not indicate that if the 1/5 intensity diameters could be measured as accurately as the 1/20 intensity diameters, we would not get just as good a fit. As one gets closer to the maximum density the diameter varies more steeply with the fractional intensity chosen so that any error in the fractional intensity chosen will be more greatly magnified in this region. Considerable errors in the fractional intensity chosen are quite likely in this case because of the previously mentioned inaccurate way in which one is forced to calibrate this film. As previously mentioned, diameters much lower than 1/20 the maximum intensity would not be chosen because they would be indistinguishable from the fog. Therefore, the only reason for 1/20 intensity diameters were chosen was because of limitations of the photographic processes.

A fit of 6% is fairly good when one considers that the values of the yield calculated by E G and G are only guaranteed to 5%, and that if this method of calculation was as good as E G and G's, and if the errors were independant, than the yield numbers calculated by this method would differ from E G & G's by an average of 7.1%, by the method of combining errors. This does not imply that pressure and temperature effects can be neglected at high altitudes, however, since these events all took place between 5,000 and 15,000 feet.

The empirical equation obtained from this data is in general agreement with the scaling laws relating radius to yield. For example if D is chosen as the diameter at thermal minimum, we obtain [3]

$$\text{Log } Y = -5.638 + 2.5 \text{ Log } D$$

or

$$Y = 2.30 \times 10^{-6} D^{2.5}$$

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3 The effects of Nuclear Weapons. pp-77

Similarily at breakaway

$$\text{Log } Y = -5.856 + 2.5 \text{ Log } D$$

or

$$Y = 1.39 \times 10^{-6} D^{2.5}$$

The value of the exponent for the 1/20 intensity 2.54819, is practically 2.5. If we set it exactly equal to 2.5, then noteing that the average value of D is 3.50138, we would obtain

$$\begin{aligned} \text{Log } Y &\approx -6.21723 + 2.5 \text{ Log } D + K \\ &\quad +0.04819 \text{ Log } 3.50138 \\ &\approx -6.04850 + 2.5 \text{ Log } D. \end{aligned}$$

These scaling laws could only hold if these three diameters were in the same proportions reguardless of the yield of the bomb. By setting the yields of these three sets of equations equal, one finds that the radius at breakaway is 22% greater than the radius at thermal minimum and that the radius at 1/20 intensity is 46 per cent greater than that at thermal minimum and 20 per cent greater than that at breakaway.

Unfortunately, at the time this report was written the author had not yet received the data on breakaway or thermal minimum diameters from E G & G so that the proportionality of the various diameters could not be checked for these events.

The empirical formula obtained from these nine events is another case of the general scaling law of the form

$$Y = CD^{2.5}$$

where D is the diameter, chosen by some consistant method, and C is a constant depending on how D is chosen. The constant C, can be written in the form

$$C = 2.30 \times 10^{-6} \quad \left\{\frac{Dt}{D}\right\}^{2.5}$$

where D_t is the diameter at thermal minima and D is the diameter chosen by some other constant procedure, for example the 1/20 intensity diameter.

Distribution:

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2A D. R. Born
3A J. Carothers
4A A. L. Jorgensen
5A E. C. Woodward
6A L. Wouters
7A J. Howard
8A File
9A File