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An Integrated Approach for Multi-Level Sample Size Determination*

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Abstract:

This paper gives an integrated approach to sampling problems involving a series of increasingly sensitive measurements on correspondingly smaller sample sizes which proceeds logically from level to level with increasing accuracy. The methods and results are consistent with those presently used by the International Atomic Energy Agency (IAEA), serve to elucidate the underlying concepts, and often simplify the calculations.

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Summary:

This paper presents a unified approach to sampling problems, particularly related to nuclear safeguards questions, in which the various levels of sampling, involving increasingly sensitive measurements and correspondingly smaller samples, are treated in a logically connected and perspicuous manner.

The first step in the method described here is (the usual one of) attributes sampling, i.e., taking a sample of size n from a population of size N, determined from the formula

$$\frac{n}{N} = 1 - \beta_G^{\frac{1}{m}}$$

where β_G is the probability of not including any defect in the sample, G being the detection goal, in kg of special nuclear material, and m is the number of defected items when the total defect is G.

The next step is to note that when a fraction γ of the material has been diverted, the probability that the item is classified as a defect when it is measured with an instrument with standard deviation σ is

$$\Phi\left(\frac{\gamma - r\sigma}{(1-\gamma)\sigma}\right)$$

$\Phi(z)$ being the cumulative normal distribution. The overall non-detection probability β for a defect of m items then becomes

$$\beta_s + (1-\beta_s)F(1)$$

where β_s is the sample non-detection probability, and F(1)is an upper bound of the probability that none of the defects in the sample is detected by the instrument. β thus satisfies the relation

$$1-\beta \geq (1-\beta_s)\Phi(\frac{\gamma - r\sigma}{(1-\gamma)\sigma}).$$

If

$$\Phi\left(\frac{\gamma - r\sigma}{(1-\gamma)\sigma}\right)$$

is very close to 1, the above expression with $\beta_s = \beta_G$ can attain the desired detection goal $1-\beta_G$. As long as

$$\Phi\left(\frac{\gamma - r\sigma}{(1-\gamma)\sigma}\right)$$

is greater than the detection goal, the sample size can be increased to attain the required detection goal using the above equation.

If more accurate measurement methods are available, one can proceed further, considering β as a function of γ, so that

$$\beta(1,\gamma) \leq 1-(1-\beta_1^{\frac{1}{\gamma}})\Phi(\frac{\gamma - r\sigma_1}{(1-\gamma)\sigma_1})$$

where β_1 is the first level non-detection probability described above.
The remainder of this paper deals with the extension of this procedure to further levels of more accurate instrumentation and to discussion of the algorithms associated with the use of such more accurate instrumental methods.

For given $\sigma_1 > \sigma_2$ one finds that the local maximum of Q (given below) as a function of γ

$$Q=\left\{1-\left(\left(\frac{\beta_G}{1-(1-\beta_G^{\frac{1}{\gamma}})}-(1-\Phi_2)\right)/\Phi_2\right)^{\frac{\gamma x}{G}}\right\}\left(1-\frac{G}{2xN\gamma}+\frac{1}{2N}\right)$$

yields the level 2 sample size n_2 via the formula $n_2 = NQ$, while the level 1 sample size becomes $n-n_2$.

Small defects, with a fractional size (approximately) less than the standard deviation, σ_1, will not be detected. In this case, a level 3 sample size can be determined (involving a more sensitive instrument), using an analogous formalism with one more step of calculation.

The method described implicitly assumes that the errors are systematic. The less conservative case, when errors are random, is also treated. Numerical examples are given to show the effects as the various parameters are changed.