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Introduction

The simplified spherical harmonics (SP_N) method has been used as an approximation to the transport equation in a number of situations.[1,2,3] Recently, the SP_N method has been formulated within the framework of the variational nodal method (VNM). Implementation in the VARIANT[4] code indicated that for many two and three dimensional problems, near P_N accuracy can be obtained at a fraction of the cost.[5] Perturbation methods offer additional computational cost reduction for reactor core calculations and are indispensable for performing a variety of calculations including sensitivity studies and the breakdown by components of reactivity worths. Here, we extend the perturbation method developed for the VNM in the full P_N approximation[6] to treat simplified spherical harmonics. The change in reactivity predicted by both first order and exact perturbation theory using the SP_N approximation is demonstrated for a benchmark problem and compared to diffusion and full P_N estimates.

Theory

The even-parity form of the slab geometry P_N equations may be written in terms of even- and odd-parity vectors of the Legendre flux moments: $\xi_m = \phi_{2m-2}$ and $\chi_m = \phi_{2m-1}$. Since the forward and adjoint equations differ only in the source and by a sign change in the odd-parity term, they may be expressed in the $\pm$ paired form

$$-\frac{d}{dx}\mathbf{E}\frac{1}{\sigma}\mathbf{E}^T\frac{d}{dx}\xi + \sigma\xi = \mathbf{b}(\sigma_s\,\phi_0 + s^{\pm}) \tag{1}$$

and

$$\chi = \mp\frac{1}{\sigma}\mathbf{E}^T\frac{d}{dx}\xi \tag{2}$$

where $\mathbf{E}$ is a two-stripe coefficient matrix, $E_{mm'} = (2m-1)\delta_{mm'}+2m\delta_{m,m'+1}$ and $b_m = \delta_{m1}$.

To obtain the SP_N equations, we simply replace $\frac{d}{dx}$ by $\vec{\nabla}$. The even-parity flux and source become functions in three dimensions: $\xi(x)\rightarrow\xi(\vec{r})$ and $s^{\pm}(x)\rightarrow s^{\pm}(\vec{r})$. The odd-parity flux is a vector in three dimensions: $\chi(x)\rightarrow\vec{\chi}(\vec{r})$.

Expressing the resulting SP_N equation in terms of variational functionals which employ Lagrange multipliers to enforce nodal balance, we obtain

$$F_v[\xi,\chi] = \int_v dV\left[\vec{\nabla}\xi^T\,\mathbf{E}\frac{1}{\sigma}\mathbf{E}^T\,\vec{\nabla}\xi + \xi^T(\sigma-\sigma_s)\,\xi - 2\,\xi^T s^{\pm}\right] \mp 2\int_v d\Gamma\,\xi^T\,E\,\hat{n}\cdot\vec{\chi}\ .$$

Requiring the functional to be stationary with respect to ξ yields the SP_N generalization of Eq. (1) within the node and Eq. (2) on the interfaces. Requiring the functional to be stationary with respect to χ yields the continuity condition on ξ across the interface.

In the VNM, the flux and source moments are approximated by known spatial trial functions with unknown coefficients, and the Ritz procedure is applied. The reduced functionals have the form

$$F_v\,[\zeta,\chi] = \zeta^T\mathbf{A}\,\zeta - 2\,\zeta^T\mathbf{s}^{\pm} \mp 2\,\zeta^T\mathbf{M}\,\chi$$

where the elements of the **A** (symmetric) and **M** matrices contain integrals of the spatial trial functions. The forward and adjoint equations are obtained by requiring stationarity with respect to ζ and χ:

$$\begin{bmatrix} \mathbf{A} & \pm\mathbf{M} \\ \mp\mathbf{M}^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \zeta \\ \chi \end{bmatrix} = \begin{bmatrix} \mathbf{s}^{\pm} \\ \mathbf{0} \end{bmatrix}$$

We form global multi-group equations by combining the vectors of fluxes and sources from the corresponding nodal vectors for all groups. The matrices **A** and **M** are now block diagonal combinations of the nodal matrices for all nodes and groups. Expanding the source terms into fission and scattering, the unperturbed adjoint equations become

$$\begin{bmatrix} \mathbf{A} & -\mathbf{M} \\ \mathbf{M}^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \zeta^* \\ \chi^* \end{bmatrix} = \frac{1}{k} \begin{bmatrix} \mathbf{F}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \zeta^* \\ \chi^* \end{bmatrix} + \begin{bmatrix} \mathbf{C}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \zeta^* \\ \chi^* \end{bmatrix}$$

while the perturbed forward equations are

$$\begin{bmatrix} \mathbf{A}' & \mathbf{M} \\ -\mathbf{M}^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \zeta' \\ \chi' \end{bmatrix} = \frac{1}{k'} \begin{bmatrix} \mathbf{F}' & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \zeta' \\ \chi' \end{bmatrix} + \begin{bmatrix} \mathbf{C}' & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \zeta' \\ \chi' \end{bmatrix}$$

where the **F** matrix consists of the fission spectrum and cross sections, **C** contains the group-to-group scattering cross sections and $\mathbf{A}' = \mathbf{A} + \delta\mathbf{A}$, etc. Performing the prescribed operations,[6] we obtain the exact change in reactivity:

$$\frac{\delta k}{kk'} = \frac{\frac{1}{k'}\zeta^{*T}\,\delta\mathbf{F}\,\zeta + \zeta^{*T}\,\delta\mathbf{C}\,\zeta - \zeta^{*T}\,\delta\mathbf{A}\zeta}{\zeta^{*T}\,\mathbf{F}\,\zeta}$$

The corresponding first order approximation results from replacing ζ', σ' and k' by ζ, σ and k

$$\frac{\delta k}{k^2} = \frac{\frac{1}{k}\zeta^{*T}\,\delta\mathbf{F}\,\zeta + \zeta^{*T}\,\delta\mathbf{C}\,\zeta - \zeta^{*T}\,\delta\mathbf{A}\zeta}{\zeta^{*T}\,\mathbf{F}\,\zeta}.$$

Since **M** contains no material cross sections, $\delta\mathbf{M} = 0$ vanishes from the perturbation expressions.

Results

Exact and first order SP_N perturbation theory has been implemented as a post processor to VARIANT. Results of perturbations applied to the Takeda Benchmark[7] Models 1 and 4 are given in Figure 1. For the Takeda 1 problem, the standard rods-in problem is used as the base state. The applied perturbations consist of increasing the thermal capture cross section of the control rod material. For the Takeda 4 problem, the base state is the standard rods-out problem. Initially the control rod channel contains sodium. The perturbations consist of sodium voiding of the channel.

In both problems, the SP_3 approximation produces improved eigenvalues compared to diffusion theory. However, the corresponding improvement in the change in reactivity estimated by the SP_3 perturbation theory is highly problem dependent. In the Takeda 1 problem, SP_3 estimates of $|\delta k/kk'|$ are nearly identical to the P_3 predictions. The base *and* perturbed cases can be accurately modeled using the SP_3 approximation. For the Takeda 4 problem, SP_3 theory fails to greatly improve the estimated change in reactivity because full P_N expansions are required to accurately model the flux distribution about the small, nearly-voided control rod channel.

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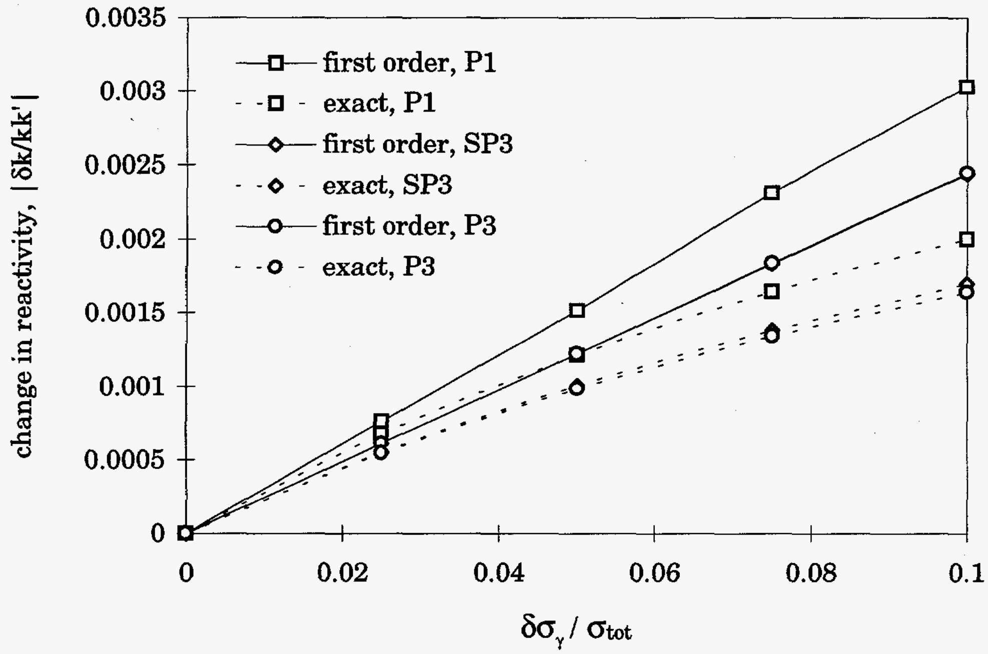

Fig. 1a Takeda Benchmark Model 1 Perturbation

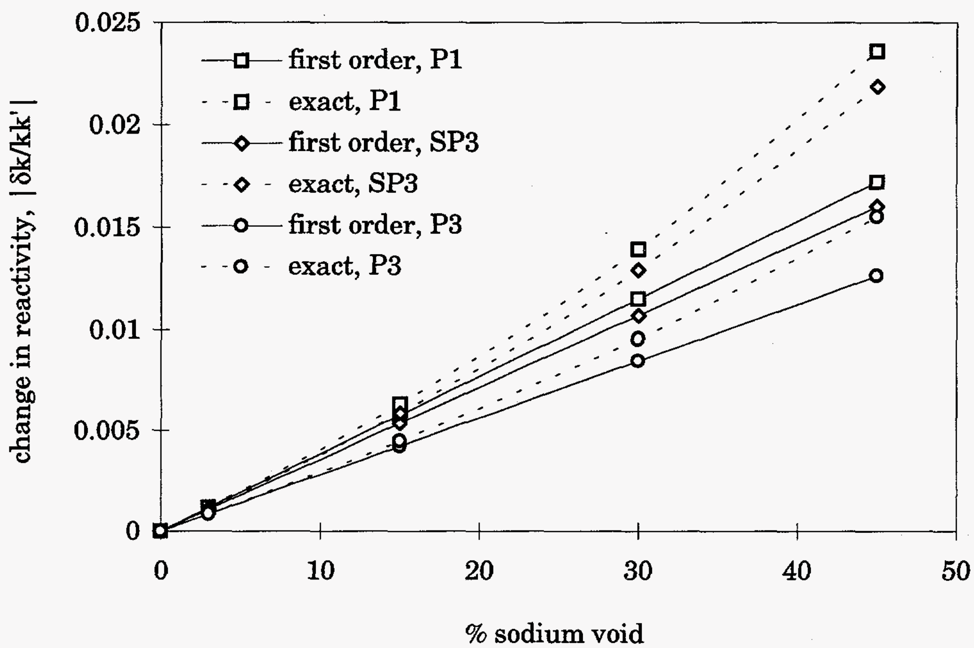

Fig. 1b Takeda Benchmark Model 4 Perturbation