

DOE/ER/25145-4<sup>2</sup>

HIGH-RESOLUTION NUMERICAL METHODS FOR COMPRESSIBLE  
MULTI-PHASE FLOW IN HIERARCHICAL POROUS MEDIA

Final Report

November 1992 - August 1996

John A. Trangenstein

Department of Mathematics

Duke University

Box 90320

Durham, NC 27708-0320

RECEIVED  
FEB 20 1997  
OSTI

November 25, 1996

PREPARED FOR THE DEPARTMENT OF ENERGY  
UNDER GRANT DE-FG05-92ER25145

**DISCLAIMER**

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

**MASTER**

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

**DISCLAIMER**

**Portions of this document may be illegible  
in electronic image products. Images are  
produced from the best available original  
document.**

## Technical Information

This research program was conducted by several people:

**John Trangenstein** was the principal investigator, and collaborates on all of the projects.

**Sameer Khan** was a post-doctoral fellow in Mathematics at Duke, and is now a full-time employee at Exxon Production Research Company in Houston. He was supported jointly by this grant and its companion DOE grant.

**Richard Hornung** was a graduate student in Mathematics at Duke, and is presently an NSF Industrial Mathematics Post-Doctoral Fellow on a project jointly sponsored by Duke University and Mobil Exploration and Producing Technology Company in Dallas.

## Background

The following issues were addressed in this research:

**Computational efficiency:** Field-scale simulation of enhanced oil recovery, whether for energy production or aquifer remediation, is typically highly under-resolved. This is because rock transport properties vary on many scales, and because current numerical methods have low resolution.

**Effective media properties:** Since porous media are formed through complex geologic processes, they involve significant uncertainty and scale-dependence. Given this uncertainty, knowledge of ensemble averages of flow in porous media can be preferable to knowledge of flow in specific realizations of the reservoir. However, current models of effective properties do not represent the observed behavior very well. Relative permeability models present a good example of this problem. In practice, these models seldom provide realistic representations of hysteresis, interfacial tension effects or three-phase flow; there are no models that represent well all three effects simultaneously.

**Wave propagation:** It is common in the petroleum industry to assume that the models have the same well-posedness properties as the physical system. An example of this fallacy is given by the three-phase relative permeability models; they were widely assumed by the petroleum community to produce hyperbolic systems for the Buckley-Leverett equations, but later the mathematics community proved that these models inherently produce local elliptic regions. Since numerical methods must use the models for computations, oscillations could erroneously be attributed to numerical error rather than modeling difficulties. When wave propagation is well-understood by analysis, it can provide useful information about the expected behavior of the flows,

and can assist in the development of high-resolution numerical methods.

The surfactant model used in this project was developed over the past decade by Gary Pope and his students at the University of Texas Department of Petroleum Engineering. It models the micellar-polymer flooding process, which typically involves at least five stages. Since salinity can significantly lower the effectiveness of surfactants, if necessary the reservoir is first flushed with a fluid, such as water, to lower the salinity. In the second stage, the injection of surfactant lowers the interfacial tension between oil and water, leading to a nearly complete sweep of oil from the pore spaces. This in turn leads to the development of an oil bank ahead of the surfactant. Often alcohol is injected with the surfactant to adjust the phase behavior. In order to drive the slug across the reservoir, in the third stage a fluid with viscosity comparable to that of the hydrocarbon is injected. Typically, this fluid is a mixture of water and polymer. This serves to prevent the fluids in the remaining stages from developing viscous instabilities that channel through the slug and destroy the efficiency of the recovery process. In the fourth stage, the polymer in the mobility buffer is gradually reduced. The engineering principle here is based on numerical simulation with low-order methods, which showed the possibility of recovery failure due to viscous instabilities that develop at the trailing edge of narrowly-tapered mobility buffers and punch through the slug. There is evidence from higher-resolution simulations that these tapers do not need to be as large as first thought. Finally an inexpensive injection fluid, such as water is injected for as long as necessary. This process is being used for enhanced oil recovery by Statoil, and has been examined by a number of the domestic oil companies in order to continue production from the mature domestic reservoirs. This process is also under study by Linda Abriola, of the Civil Engineering Department at the University of Michigan, for the removal of hydrocarbon contaminants from aquifers. Drs. Abriola and Pope are currently funded jointly by the EPA to study the potential success of this process. Currently, a pilot project employing surfactants to remove dense chlorinated hydrocarbons from an aquifer is underway at Hill Air Force Base, under the advise of Gary Pope.

The surfactant model has led to the development of the UTCHEM simulator, which is distributed to essentially all of the domestic oil companies, as well as a number of foreign companies, through the Center for Petroleum and Geosystems Engineering (CPGE) at UT. This simulator is based on a fixed mesh, finite difference scheme using higher-order differencing in at most one or two variables (the relative permeabilities) along coordinate directions. The scheme is first-order in multiple dimensions, and whenever there are waves in addition to the Buckley-Leverett waves. It also requires timesteps that are significantly smaller than the Courant-Friedrich-Levy condition for numerical stability. This simulator is being modified for parallelization under a DOE HPCC grant jointly awarded to Gary Pope, and Mary Wheeler at the Department of Mathematical Sciences at Rice University.

We chose to study this model because it is one of the few enhanced oil recovery techniques that are being considered for use in aquifer remediation. (Another is steam flooding.) Thus we have the potential to influence two DOE missions at the same time, namely environment cleanup and energy production.

## Progress Review

- In FY 93 we completely reprogrammed the phase behavior model in UTCHEM. The purpose of the model revision was to remove some obvious discontinuities and to improve the numerical performance. The revised model was made available to Gary Pope, and may be transmitted on to the industrial subscribers of CPGE. We also developed a numerical strategy for dealing with the mass conservation equations, which are subject to the constraint that the sum of the volume-occupying component concentrations is one. This led to the development of a one-dimensional simulator, based on a second-order Godunov method.
- In FY 94 we ran a number of simulations in 1D and 2D for practical surfactant floods. We also completed the model development, by removing a number of discontinuities in the original UTCHEM model, programmed an initial adaptive mesh refinement algorithm and tested it on polymer flooding, and completed our 2D graphics development. Also, in FY94 Richard Hornung completed his Ph.D. thesis at Duke, in which he developed an adaptive multi-level iteration strategy for polymer flooding. The method is based on multi-grid V-cycle iteration between levels, and domain decomposition techniques to handle multiple grid patches within a single level of refinement.
- In FY 95 we extended Hornung's adaptive algorithm to the surfactant model, and improved its computational efficiency. We also ran a number of simulations in 1D and 2D with the second-order Godunov surfactant flooding simulator.

## Progress Summary

- We ran some adaptive simulations of surfactant floods for a problem suggested by Statoil. This problem involved a surfactant injection, followed by the injection of a polymer mobility buffer, followed afterwards by water. Thus there were several injection fronts, and much for adaptive mesh refinement to resolve. The simulations showed that, even for such a complex problem, adaptive mesh refinement has computational complexity that scales linearly with the number of grid cells in the equivalent uniform grid. This is opposed to a uniform grid calculation, in which the computational complexity increases proportional to the number of grid cells times the number of timesteps. Effectively,

the work in 2D adaptive mesh refinement calculations scales as if it were for 1D uniform grid calculations.

- We also found that second-order Godunov methods are not computationally efficient for models as complex as the surfactant model. The Riemann solver is too expensive, even with the use of Engquist-Osher approximation techniques. In fact, it is very expensive to compute all of the characteristic speeds and directions.
- However, it has been worthwhile to perform an analysis of the characteristic structure of the surfactant model. We were able to identify unfortunate aspects of the UTCHEM model that lead to infinite characteristic speeds. This caused previous simulators to select very small timesteps during certain stages of the simulation. By making slight modifications to the phase behavior model, we were able to prevent infinite characteristic speeds and perform far more stable simulations.
- In order to conduct adaptive mesh refinement for flow in porous media, it is necessary to develop various schemes for coarsening and refining variables. The scientific issues in developing these schemes is very much related to ongoing research in "upscaling." As much as possible, we used non-controversial coarsening schemes based on mass and volume conservation (for example, upscale porosity). Other quantities, such as permeability and effective dispersion, are very controversial to upscale. We used iterated homogenization to upscale permeability and scaled dispersion with the refinement ratio. Other quantities have no good theoretical basis for constructing upscaled values (such as relative permeability and capillary pressure); for such variables we used the same functional form on all scales. Clearly, much work remains to be done on this issue.

## Subsequent Research

This grant has led to a subsequent joint project, sponsored by the Army Research Office and the National Science Foundation, to apply adaptive mesh refinement methods and stochastic optimization techniques to surfactant enhanced aquifer remediation. This project will be joint among Duke University, The University of Texas and Texas A & M University.

## Publications

The following papers were directly related to this project and resulted from this research grant:

1. "Adaptive Mesh Refinement and Upscaling for Multicomponent Flow in Porous Media," to be submitted to *Computational Geosciences*. With Richard Hornung and Sameer A. Khan.
2. "High-Resolution Numerical Methods for Micellar-Polymer Flooding and Surfactant Enhanced Aquifer Remediation," to be submitted to *Computational Geosciences*. With Sameer A. Khan.

John A. Trangenstein : DOE DE-FG05-92ER25145

3. "Multiphase Flow and Transport Modeling in Heterogeneous Porous Media: Challenges and Approaches," to appear in *Advances in Water Resources*. With Cass T. Miller, George Christakos, Paul T. Imhoff and John F. McBride.
4. "Adaptive Mesh Refinement and Multilevel Iteration for Flow in Porous Media," to appear in *Journal of Computational Physics*. With Richard D. Hornung.
5. "Micellar/Polymer Physical-Property Models for Contaminant Cleanup Problems and Enhanced Oil Recovery," to appear in *Transport in Porous Media*. With Sameer A. Khan and Gary A. Pope.
6. "Application of Adaptive Mesh-Refinement with a New Higher-Order Method in Simulation of a North Sea Micellar/Polymer Flood," in *Proceedings of the SPE Symposium on Reservoir Simulation*, San Antonio, 1995. SPE 29145.

In addition, the following papers were indirectly related to this research project:

1. "Adaptive Mesh Refinement and Front Tracking for Shear Bands in an Antiplane Shear Model," in preparation. With F. Xabier Garaizar.
2. "Front Tracking for Shear Bands in an Antiplane Shear Model," to appear in *Journal of Computational Physics*. With F. Xabier Garaizar.
3. "Adaptive Mesh Refinement for Wave Propagation in Nonlinear Solids," *SIAM Journal on Scientific and Statistical Computing* **16**(1995): 819-835.
4. "A Second-Order Godunov Algorithm for Two-Dimensional Solid Mechanics," *Computational Mechanics* **13**(1994): 343-359.

Finally, the following Ph.D. theses were completed with indirect assistance from the financial support of this grant:

1. "Simulation of Granular and Fluid Systems Using Combined Continuous and Discrete Methods," Richard Buel Clelland, April 1996.
2. "A Hysteretic Polymer Flooding Model," Khaled Furati, April 1995.
3. "Adaptive Mesh Refinement and Multi-Level Iteration Techniques," Richard D. Hornung, September 1994.