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DOE/ER/542-41-169

FRCR #490

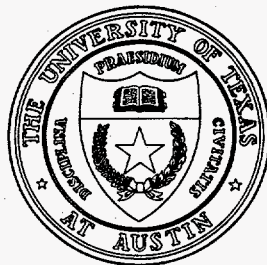
## Compact Tokamak Reactors Part 2 (numerical results)

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October 21, 1996

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## Abstract

We describe a numerical optimization scheme for fusion reactors. The particular application described is to find the smallest copper coil 'spherical' tokamak, although the numerical scheme is sufficiently general to allow many other problems to be solved. The solution to the steady state energy balance is found by first selecting the fixed variables. The range of all remaining variables is then selected, except for the temperature. Within the specified ranges, the temperature which satisfies the power balance is then found. Tests are applied to determine that remaining constraints are satisfied, and the acceptable results then stored. Results are presented for a range of auxiliary current drive efficiencies and different scaling relationships; for the range of variables chosen the machine encompassing volume increases or remains approximately unchanged as the aspect ratio is reduced.

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# 1 Introduction

A previous paper [1] presented an analytic model describing the minimum tokamak reactor size as a function of aspect ratio  $A$ . In particular the dependence of major radius  $R$  and machine encompassing volume  $V_m$  with  $A$  was discussed. In those analytic studies only two auxiliary current drive efficiencies ( $\eta_{CD} = 0$  and  $\infty$ ) and one scaling law were considered. In addition, the power to the grid  $P_{gr}$  remained unspecified (but  $\geq 0$ ). The analysis attempts to find plasma densities and temperatures, and corresponding reactor parameters which are consistent with the steady-state power balance equation and with a number of physical constraints. The power balance equation relates the power lost as described by the ratio of total contained energy to the energy containment time computed from an assumed scaling law to the power input due to fusion power, current drive, and auxiliary heating. While the equation is conceptually simple, it is nonlinear and contains over two dozen parameters which must simultaneously satisfy a number of additional and often nonlinear constraints. In the analytic model, a single scaling law and certain limiting assumptions were made to obtain an analytically tractable equation. Here we use a directly numerical search procedure which allows more general scaling laws and constraints to be considered. Figure 1 illustrates the power flows considered; detailed definitions are found in [1], and at the end of this document.

The mathematical problem is to solve a nonlinear equation, power balance, which contains approximately 24 variables subject to roughly seven nonlinear constraints to minimize either the major radius or volume. The exact number of variables and constraints depends on the particular problem and scaling law considered. The analytic technique in the previous paper fixed some of the parameters and looked at limiting cases in order to obtain an analytically tractable equation. Here we give up the closed analytic result in favor of examining the problem and constraints directly. In both the analytic and numerical treatment some parameters are held fixed for this study. The resulting problem requires a search in an eight or larger dimensional parameter space. We examine this space directly using a relatively coarse mesh rather than following a gradient using a differential approach. This coarse mesh search can be considered as an initial procedure to find good starting conditions for a minimization method; however, the coarse mesh search was adequate for the problem at hand. To date the only plasma losses considered

are those described by the scaling relationships considered (see Appendix 1). Appendix 2 lists the definitions and relations, with Appendix 3 defining the symbols used. Constants are found in Appendix 4.

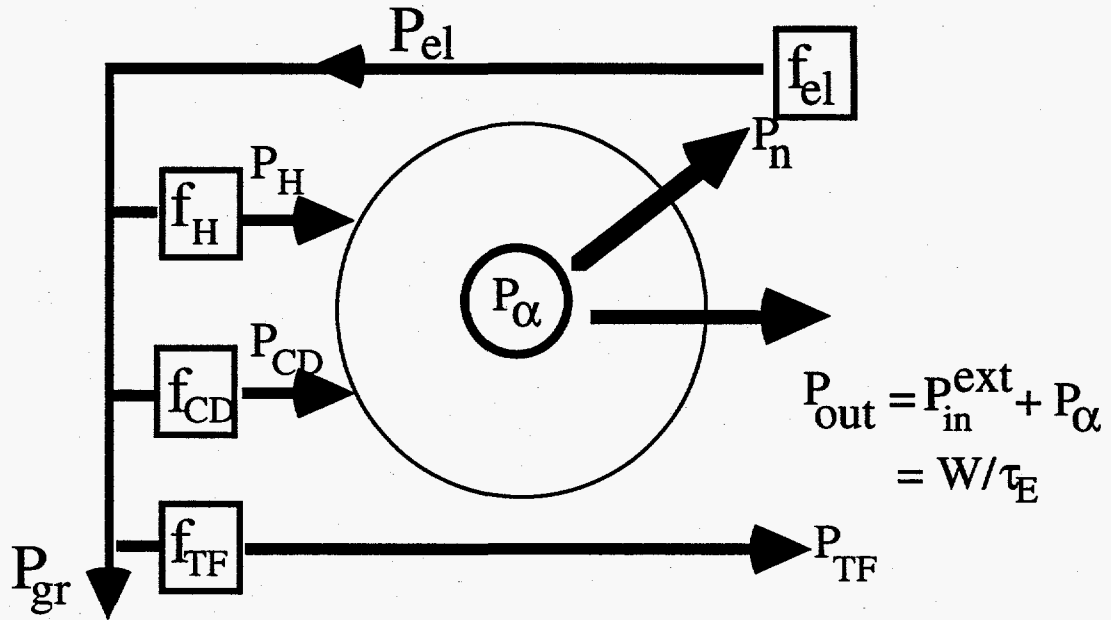


Figure 1. Power flows considered.

The objective of the calculation is, given the plasma shape (aspect ratio  $A$ , triangularity  $\delta$ , elongation  $\kappa$ ), wall loading limit  $\Gamma_{n0}$ , plasma content,  $Z_{eff}$ ,  $Z_i$ , plasma profiles  $\gamma_n$ ,  $\gamma_T$ , and exponents in confinement time scaling  $\alpha_R$ ,  $\alpha_a$ ,  $c$ ,  $H$ , ..., and certain conversion efficiencies,  $f_{el0}$ ,  $f_{CD}$ , etc., to find the smallest machine, where by smallest we mean either

smallest major radius  $R_0$ ,

smallest machine encompassing volume  $V_m$ ,

with the recirculating power fraction  $\chi \leq \chi_0$  (e.g. 0.6) that satisfies the power balance equation

$$\frac{W}{\tau_E} = P_H + P_{CD} + P_{\alpha}, \quad (1)$$

subject to the restrictions:

Greenwald limit (density)

$$\bar{n}_e < f_n 10^{20} (I_p / 10^6) / (\pi a^2), \quad (2)$$

Beta limit

$$\beta < \frac{\beta_{N_0}}{\sqrt{A}} 10^{-8} \frac{I_p}{a B_{T_0}}, \quad (3)$$

Wall loading limit

$$\Gamma_n < \Gamma_{n0}, \quad (4)$$

Safety factor (stability)

$$q > q_{min}, \quad (5)$$

Power production

$$P_{gr} > P_{gr_{min}}, \quad (6)$$

and other additional constraints such as

maximum allowed field strength on the central column

$$B_{Tleg} < B_{Tleg_{max0}}, \quad (7)$$

and the maximum bootstrap fraction

$$f_{bs} \leq f_{bs_{max}}. \quad (8)$$

We have written a small computer code to perform this task. (The code, written in C++, can be obtained from the authors.) An outline of the computational procedure is as follows (details appear later):

1. Select the set of fixed variables.
2. Select the range of variation for all remaining variables except temperature.
3. Within the given range, solve for the temperature that satisfies the power balance.
4. Test if the remaining constraints are satisfied.
5. Store the acceptable set if  $R_0$  (or  $V_m$ ) less than current  $R_0$  (or  $V_m$ ).

Note that, to speed up the calculations, the energy released by the nuclear reactions is approximated by a simple dependence on temperature, which restricts  $T \leq 25$  keV.

## 2 Computational Procedure

The computational procedure carries out a brute force search. Normally, such a procedure for the large number of variables involved in this study would be computationally prohibitive. By carefully organizing the nested loops and applying the constraints as early as possible, we have been able to examine problems in a reasonable time (a few minutes to a few hours on a desktop personal computer). The problem is coded as a series of nested loops which are outlined as follows:

Specify ranges:  $R_{0min}$ ,  $R_{0max}$ ,  $n_{0min}$ ,  $I_{pmin}$ ,  $\kappa_{min}$ ,  $T_{0min}$ ,  $T_{0max}$ .

Fix  $\alpha_{bs}$ ,  $Z_{eff}$ ,  $Z_i$ ,  $\gamma_n$ ,  $\gamma_T$ ,  $\beta_{N_0}$ ,  $\Gamma_{n_0}$ ,  $\kappa_0$ ,  $\alpha_R$ ,  $\alpha_a$ ,  $H$ ,  $\alpha_P$ ,  $\alpha_{I_p}$ ,  $f_{el}$ ,  $f_H$ ,  $f_{TF}$ ,  $f_{CD}$ ,  $P_{grid_{min}}$ ,  $\delta$ ,  $\alpha_{bs}$ ,  $f_N$ , plus other coefficients needed for scaling law.

Calculate, plasma content factors:

$$g = \frac{(Z_i - Z_{eff})}{(Z_i - 1)}, \quad (9)$$

$$s_p = \frac{g^2 \langle n^2 T^2 \rangle}{\langle nT \rangle^2}. \quad (10)$$

Select an  $A$ , (results plotted as function of  $A$ ).

Begin at  $R_0 = R_{0min}$ , and increment  $R_0$  until solution to energy balance found or  $R_{0max}$  is reached.

Require  $R_0 \in [R_{0min}, R_{0max}]$  so that

$$a = \frac{R_0}{A}. \quad (11)$$

The upper range of  $B_{T_0}$  is limited by  $B_{T_{leg-max}}$  through

$$B_{T_{0max}} = B_{T_{leg-max}} \frac{(A-1)}{A}. \quad (12)$$

Fix  $\kappa_{max} = \kappa_0$  (alternatively one could take, to further advantage low- $A$ ,  $\kappa_{max} = \frac{\kappa_0}{\sqrt{A}}$ ).



Search over range:  $\kappa \in [\kappa_{min}, \kappa_{max}]$ , defining

$$V = 2\pi R_0 \pi a \kappa a \left(1 - \frac{\delta^2}{8} - \frac{\delta}{4A}\right), \quad (13)$$

$$S = 2\pi^2 R_0 a (1 + \kappa) \left(1.0 - 0.13\delta \frac{\kappa^{1/4}}{A}\right), \quad (14)$$

$$f_{el} = f_{elo} \left[ \frac{1}{2} + \frac{\kappa^{0.13}}{\pi A} \right] (1 + 0.34\delta \kappa^{-0.5}). \quad (15)$$

The minimum  $q$  limits the maximum  $I_p$  through

$$I_{pmax} = \frac{g_q R_0 B_{T_0max}}{q_{min}}. \quad (16)$$

Search over range:  $I_p \in [I_{pmin}, I_{pmax}]$ .

The lower range of  $B_{T_0}$  is limited by  $q_{min}$  through

$$B_{T_0min} = \frac{q_{min} I_p}{g_q R_0}. \quad (17)$$

Upper bound on  $n$  is set by the Greenwald limit

$$n_{0max} = \frac{f_n 10^{20} (I_p/10^6)}{(\pi a^2)} \left( \frac{\sqrt{\pi}}{2} \frac{\Gamma(1 + \gamma_n)}{\Gamma(3/2 + \gamma_n)} \right)^{-1}. \quad (18)$$

Search over range:  $n_0 \in [n_{0min}, n_{0max}]$  defining

$$\alpha_0^{CD} = \frac{n_0 R_0}{\eta_{CD} (1 + \gamma_n)} I_p, \quad (19)$$

$$\alpha_1^{CD} = -\alpha_0^{CD} \left( \frac{\alpha_{bs} (2\pi a)^2 2(1 + \kappa^2) k_b}{\sqrt{A} \mu_0 I_p^2} \right) \left( \frac{n_0}{1 + \gamma_n + \gamma_T} g_2 \right), \quad (20)$$

$$\alpha_1^W = 3g_2 \left( \frac{n_0 k_b}{1 + \gamma_n + \gamma_T} \right) V, \quad (21)$$

$$\alpha_2^\alpha = 1.5 \times 10^{-37} \left( \frac{n_0}{1 + \gamma_n + \gamma_T} \right)^2 V s_p. \quad (22)$$

Search over range:  $B_{T_0} \in [B_{T_0min}, B_{T_0max}]$  computing

$$P_{TF} = \frac{8\eta\pi\kappa B_{T_{leg}}^2 R_0}{\mu_0^2 f_{Cu} A} \left( 1 - \exp \left[ -1.3 \left( \frac{3+2\delta}{3-2\delta} \right) \sqrt{A-1} \right] \right). \quad (23)$$

The maximum power available for heating is given by

$$P_{Hmax} = f_H \left( f_{el} 4P_\alpha - \frac{P_{CD}}{f_{CD}} - \frac{P_{TF}}{f_{TF}} - P_{gridmin} \right). \quad (24)$$

Some fraction of this power is supplied as heating. Search over range:  $\alpha_H \in [0, 1]$  calculating

$$\alpha_2^H = \alpha_H f_H f_{el} 4\alpha_2^\alpha, \quad (25)$$

$$\alpha_1^H = -\frac{\alpha_H f_H \alpha_1^{CD}}{f_{CD}}, \quad (26)$$

$$\alpha_0^H = -\alpha_H f_H \left( \frac{\alpha_0^{CD}}{f_{CD}} + \frac{P_{TF}}{f_{TF}} + P_{gridmin} \right). \quad (27)$$

The upper  $T_0$  is given either by the  $\beta$  limit (note the explicit  $A$  dependence has been included) through

$$T_{0max}^\beta = \frac{\beta N_0}{\sqrt{A}} 10^{-8} \frac{I_p B_{T_0} (1 + \gamma_n + \gamma_T)}{a 4\mu_0 k_b n_0 g_2}, \quad (28)$$

or by the wall loading limit

$$T_{0max}^{wall} = \sqrt{\frac{\Gamma_{n_0} S}{4\alpha_2^\alpha}}, \quad (29)$$

so that

$$T_{0max} = \min(T_{0max}^\beta, T_{0max}^{wall}, T_{0max}^{specified}). \quad (30)$$

The inner loop is a search procedure to find a peak temperature that will provide power balance. Solve  $W(T_0) = \tau_E(T_0) P_\Sigma(T_0)$  for  $T_0$  where  $T_0 \in [T_{0min}, T_{0max}]$  and where

$$P_{CD} = \alpha_0^{CD} + \alpha_1^{CD} T_0, \quad (31)$$

$$P_\alpha = \alpha_2^\alpha T_0^2, \quad (32)$$

$$P_H = \alpha_2^H T_0^2 + \alpha_1^H T_0 + \alpha_0^H, \quad (33)$$

$$W = \alpha_1^W T_0, \quad (34)$$

$$P_\Sigma = P_\alpha + P_H + P_{CD}, \quad (35)$$

$$\tau_E = \tau_E(P_\Sigma, T_0). \quad (36)$$

If a solution for  $T_0$  is found, check to see if physically meaningful, i.e.

$$P_H \geq 0, \quad (37)$$

and

$$f_{bs} \leq f_{bsmax}, \quad (38)$$

and that remaining constraints are satisfied. Namely,

$$P_{grid} \geq P_{gridmin}, \quad (39)$$

and

$$\chi < \chi_0, \quad (40)$$

where

$$\chi = \frac{P_{TF}/f_{TF} + P_{CD}/f_{CD} + P_H/f_H}{P_{el}}. \quad (41)$$

Store the result.

### 3 Power balance equation

The procedure requires that the power balance equation,

$$W(T_0) = \tau_E(T_0)P_\Sigma(T_0), \quad (42)$$

which at that point in the calculation is only a function of  $T_0$  (all other parameters fixed) be solved for  $T_0$ . Note that most of the single term scaling laws can be expressed as

$$\tau_E = \tau_{E_0} P_\Sigma^{-(1/2+\gamma)} \quad (43)$$

with

$$P_\Sigma = \alpha_2^\Sigma T_0^2 + \alpha_1^\Sigma T_0 + \alpha_0^\Sigma, \quad (44)$$

where the  $\alpha_i^\Sigma$ 's are obtained from the expressions for  $P_{CD}$ ,  $P_H$ ,  $P_\alpha$ . The  $\gamma$  exponent is either zero or small for most of the scaling laws. The equation for  $T_0$  becomes

$$\left(\frac{\alpha_1^W}{\tau_{E_0}}\right)^2 T_0^2 - (\alpha_2^\Sigma T_0^2 + \alpha_1^\Sigma T_0 + \alpha_0^\Sigma)^{1-2\gamma} = 0, \quad (45)$$

which for  $\gamma = 0$  is a quadratic and can be solved directly. For the case in which  $\gamma \neq 0$ , the quadratic roots are used as initial guesses for a simple iterative Newton's method. The Rebut-Lallia scaling law, which does not fit the pattern, can be expressed in the form

$$\tau_E = (\tau_{E_0} + \tau_{E_1}/P_\Sigma), \quad (46)$$

which also reduces a quadratic equation for  $T_0$

$$\alpha_1^W T_0 = \tau_{E_0}(\alpha_2^\Sigma T_0^2 + \alpha_1^\Sigma T_0 + \alpha_0^\Sigma) + \tau_{E_1}. \quad (47)$$

For the case in which  $\gamma \neq 0$ , Newton's method was applied to the equation

$$f(y) = ay^2 - [by^2 + cy + d]^{1-2\gamma} \quad (48)$$

to obtain a general iterative method for these cases.

## 4 Results

We show here the results of the optimization for tokamak reactors with copper toroidal field coils with no space made available for a nuclear shield or blanket at the inner equator. The objective is to complement the analytic results presented in [1]. The values considered for the optimization are given in Appendix 5 'Nominal values'. The main differences between the numerical results presented here and the analytic results are:

1. Different energy confinement scaling relations are used here.
2. Arbitrary auxiliary current drive efficiencies are used. This means that a driven system is implied.
3. The power to the grid  $P_{gr}$  is specified. In the examples shown below,  $P_{gr} \geq 0.5$  GW was specified.
4. Various parameters are allowed to change, rather than fixed values being chosen.

Figure 2 shows the smallest major radius  $R$  as a function of aspect ratio  $A$  for the four confinement relations used to date (see section 'Scaling Laws'). Figure 3 shows the machine encompassing volume/ $10^3$  for the same data. Results from other relationships can be provided if required. In the legend the first number refers to the scaling law used, with ITER89P = 1, KayeAll = 2, GoldstonL = 3, DIIIJet = 4. For each scaling relationship, results assuming two values of current drive efficiency are shown, namely  $\eta_{CD} = 1. \times 10^{19}$  (pessimistic) and  $10. \times 10^{19}$  (optimistic). In the legend the second number refers to the auxiliary current drive efficiency (1 or 10). For each scaling relationship and current drive efficiency results were obtained with  $\delta = 0.2, 0.3$  and  $0.9$ , but very little difference in the major radius was found. Shown are data obtained with  $\delta = 0.3$ . The results show that there is  $\approx 40$  % reduction in the major radius of the smallest device as  $A$  is reduced from 3 to  $\approx 1$ . However, there is no associated reduction in  $V_m$  even with the most optimistic auxiliary current drive efficiency.

Other general observations include:

1. Optimising on the smallest  $R$  or smallest  $V_m$  gives the same device.
2. There is little dependency of the smallest  $R$  on  $\delta$ .
3. The optimum (i.e. smallest) device generally operates at the maximum allowed elongation and neutron wall loading limit.

The results described above were obtained by first choosing  $A$ , the scaling relationship for energy confinement,  $\eta_{CD}$  and  $\delta$ , and then varying  $R$ ,  $I_p$ ,  $B_T$ ,  $\kappa$ ,  $n$ , etc. Typically for each  $A$ ,  $\eta_{CD}$ , and  $\delta$ , approximately  $10^7$  cases were run, from which those which satisfy the constraints are first chosen, and then within that subset those with the minimum size are recorded.

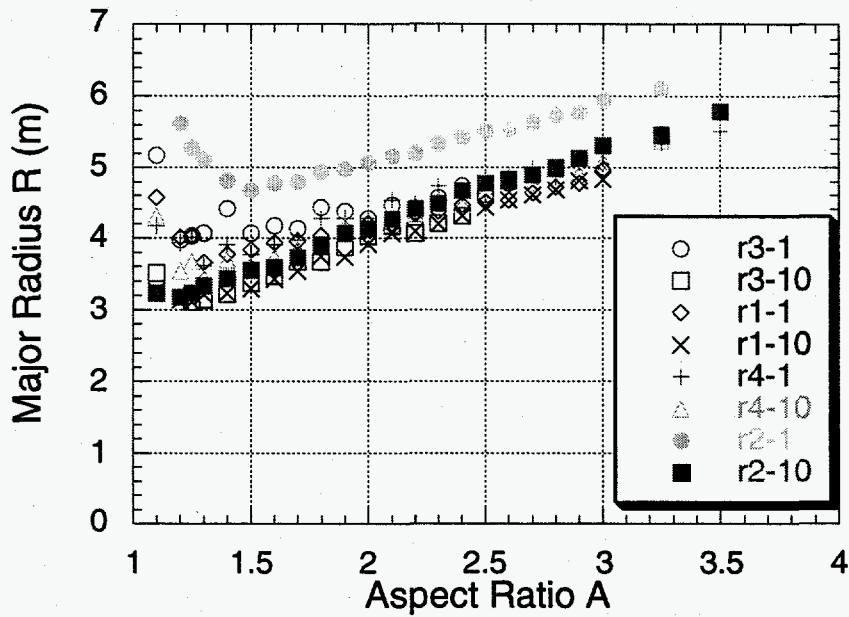


Figure 2. The major radius  $R$  of the smallest copper toroidal field coil tokamak reactor as a function of aspect ratio  $A$ . Results using four different energy confinement scaling relationships are shown. The first digit of the legend refers to the scaling relationship (ITER89P = 1, KayeAll = 2, GoldstonL = 3, DIIIJet = 4) and the second digit refers to the auxiliary current drive efficiency used ( $\eta_{CD} = 1. \times 10^{19}$  and  $10. \times 10^{19}$ ).  $\delta = 0.3$ , with other parameters and variations found in the text (see 'Nominal values').

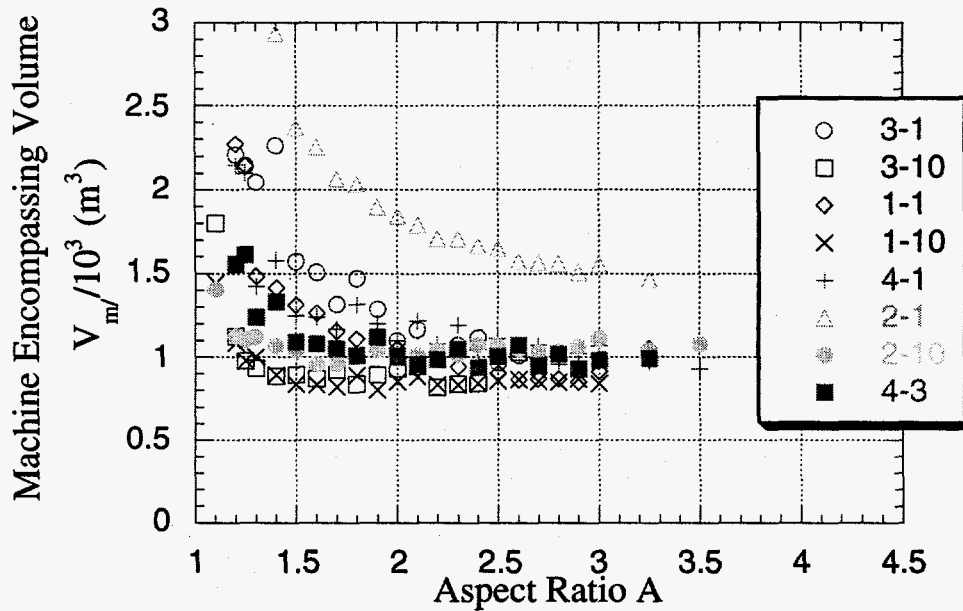


Figure 3. The machine encompassing volume  $V_m/10^3 \text{ m}^3$  of the smallest copper toroidal field coil tokamak reactor as a function of aspect ratio  $A$ . Results using four different energy confinement scaling relationships are shown. The first digit of the legend refers to the scaling relationship (ITER89P = 1, KayeAll = 2, GoldstonL = 3, DIIIJet = 4) and the second digit refers to the auxiliary current drive efficiency used ( $\eta_{CD} = 1. \times 10^{19}$  and  $10. \times 10^{19}$ ).  $\delta = 0.3$ , with other parameters and variations found in the text (see 'Nominal values')



## 5 Summary and Conclusions

We have presented a numerical technique for optimizing a given energy confinement scaling relationship, together with given ranges of variables and fixed constraints, to find the smallest tokamak reactor. Results were presented for a range of auxiliary current drive efficiencies and different scaling relationships; for the range of variables chosen the machine encompassing volume increases or remains approximately unchanged as the aspect ratio is reduced. These results were consistent with the results obtained using the analytic method in [1].

## References

- [1] A. J. Wootton, J. C. Wiley, P. H. Edmonds and D. W. Ross "Compact Tokamak Reactors Part 1 (analytic results)" DOE/ER/542-41-163 and FRCR 484(1996).
- [2] S.C. McCool, A. J. Wootton, R. V. Bravenec, G.G. Castle, P. H. Edmonds, K. W. Gentle, J. W. Jagger, B. Richards, David W. Ross, William L. Rowan, E. R. Solano, P.M. Valanju, R. R. White "Optimization of a Mega-Ampere Spherical Tokamak for Beta-Limit and Confinement Studies", DOE/ER/542-41-145 and FRCR 464(1994).

## Appendix 1. Scaling laws [2]

Data from the first four scaling laws are discussed in this paper, although all those listed (and any others provided) can be used if required.

$$a(m), R(m), B(T), I_p(A), \tau_E(s), P_\Sigma(W), n(m^{-3})$$

ITER89-P (L-mode)

$$\tau_E = \left( 0.048H\sqrt{A_i}a^{0.3}R_0^{1.2} \left( \frac{I_p}{10^6} \right)^{0.85} \kappa^{0.5} B^{0.2} \left( \frac{\bar{n}}{10^{20}} \right)^{0.1} \right) \left( \frac{P_\Sigma}{10^6} \right)^{-0.5} \quad (49)$$

Kaye-all-complex (L-mode)

$$\tau_E = \left( 0.067H\sqrt{A_i}a^{0.3}R_0^{0.85} \left( \frac{I_p}{10^6} \right)^{0.85} \kappa^{0.25} B^{0.3} \left( \frac{\langle n \rangle}{10^{20}} \right)^{0.1} \right) \left( \frac{P_\Sigma}{10^6} \right)^{-0.5} \quad (50)$$

Goldston (L-mode, H/D)

$$\tau_E = \left( 0.037H\sqrt{\frac{A_i}{1.5}}a^{-0.37}R_0^{1.75} \left( \frac{I_p}{10^6} \right) \kappa^{0.5} \right) \left( \frac{P_\Sigma}{10^6} \right)^{-0.5} \quad (51)$$

DIII-Jet (H-mode)

$$\tau_E = \left( 0.053HR_0^{1.48} \left( \frac{I_p}{10^6} \right)^{1.03} \right) \left( \frac{P_\Sigma}{10^6} \right)^{-0.46} \quad (52)$$

Lackner-Gottardi (L-mode)

$$\tau_E = \left( 0.12H\sqrt{\frac{A_i}{2}}a^{0.4}R_0^{1.8} \left( \frac{I_p}{10^6} \right)^{0.8} \frac{\kappa}{(1+\kappa)^{0.8}} q^{0.4} \left( \frac{\bar{n}}{10^{20}} \right)^{0.6} \right) \left( \frac{P_\Sigma}{10^6} \right)^{-0.6}$$

$$q = \frac{2\pi(1+\kappa^2)}{\mu_0} \frac{a^2 B}{2R_0(I_p/10^6)} \quad (53)$$

Kaye-Goldston (L-mode)

$$\tau_E = \left( 0.055H\sqrt{\frac{A_i}{1.5}}a^{-0.49}R_0^{1.65} \left( \frac{I_p}{10^6} \right)^{1.24} \kappa^{0.28} B^{-0.09} \left( \frac{\langle n \rangle}{10^{20}} \right)^{0.26} \right) \left( \frac{P_\Sigma}{10^6} \right)^{-0.58} \quad (54)$$

Rebut-Lallia (L-mode, D)

$$\tau_E = \left( 2H \sqrt{\frac{A_i}{2}} 0.012 \left( \frac{I_p}{10^6} \right) (a^2 R_0 \kappa Z_{eff}^{-1})^{1/2} \right) + \quad (55)$$

$$\left( 2H \sqrt{\frac{A_i}{2}} 0.146 \left( B \left( \frac{I_p}{10^6} \right) \right)^{1/2} (a^2 R_0 \kappa)^{11/12} \left( \frac{\langle n \rangle}{10^{20}} \right)^{3/4} Z_{eff}^{1/4} \right) \left( \frac{P_\Sigma}{10^6} \right)^{-1}$$

Goldston-quadrature (OH, L-mode H/D)

$$\tau_E = \left( \frac{1}{\tau_{E,OH}^2} + \frac{1}{\tau_{E,AUX}^2} \right)^{-1/2} \quad (56)$$

where

$$\tau_{E,OH} = 0.1026 \bar{n}_{20} a^{1.04} R^{2.04} q^{0.50} \quad (57)$$

$$\tau_{E,AUX} = 0.037 H \sqrt{\frac{A_i}{1.5}} a^{-0.37} R_0^{1.75} \kappa^{1/2} \left( \frac{I_p}{10^6} \right) \left( \frac{P_\Sigma}{10^6} \right)^{-0.5} \quad (58)$$

## Appendix 2. Definitions and Relations

We assume that the plasma contains one hydrogen-like main ion,  $n_i$  and one impurity,  $n_I$ , with charge  $Z_I$ . Using charge neutrality  $n_i + Z_I n_I = n_e$ , and the definition of  $Z_{eff}$ ,  $Z_{eff} n_e = n_i + n_I Z_I^2$ , we can define two factors  $g = n_i/n_e$  and  $g_2 = (1/2)(1 + n_i/n_e + n_I/n_e)$  in terms of  $Z_I$  and  $Z_{eff}$  as

$$g = \frac{(Z_I - Z_{eff})}{(Z_I - 1)}, \quad (59)$$

$$g_2 = \frac{1 + 2Z_I - Z_{eff}}{2Z_I}, \quad (60)$$

so that the pressure  $p = (n_e + n_i + n_I)kT$ ,

$$P = 2ng_2kT, \quad (61)$$

and energy content  $W = (3/2)(n_e + n_i + n_I)kT$ ,

$$W = 3ng_2kT \quad (62)$$

are expressed only in terms of  $g_2$ . Where we assume  $T_e = T_i = T$  and  $n = n_e$ . The plasma profiles are assumed to have the forms:

$$n, T = n_0, T_0 [1 - (r/a)^2]^{\gamma_n, \gamma_T}, \quad (63)$$

so that

$$\langle n \rangle = \frac{n_0}{1 + \gamma_n}, \quad (64)$$

$$\bar{n} = n_0 \frac{\sqrt{\pi}}{2} \frac{\Gamma(1 + \gamma_n)}{\Gamma(3/2 + \gamma_n)}, \quad (65)$$

$$\langle n^2 T^2 \rangle = \frac{n_0^2 T_0^2}{1 + 2(\gamma_n + \gamma_T)}, \quad (66)$$

$$\langle nT \rangle = \frac{n_0 T_0}{1 + \gamma_n + \gamma_T}. \quad (67)$$

It is convenient to define a plasma profile and impurity content factor as

$$s_p = \frac{g^2 \langle n^2 T^2 \rangle}{\langle nT \rangle^2}. \quad (68)$$

Additional definitions include:

Plasma volume

$$V = 2\pi R_0 \pi a \kappa a \left(1 - \frac{\delta^2}{8} - \frac{\delta}{4A}\right) \quad (69)$$

Machine-encompassing volume

$$V_m = \pi(R_0 + a)^2 2\kappa a \quad (70)$$

First wall surface area

$$S = 2\pi^2 R_0 a (1 + \kappa) \left(1.0 - 0.13\delta \frac{\kappa^{1/4}}{A}\right) \quad (71)$$

Elongation

$$\kappa = \frac{\kappa_0}{\sqrt{A}} \quad (72)$$

Maximum toroidal field on axis

$$B_{T_0} = B_{T_{leg}} \frac{(A-1)}{A} \quad (73)$$

Safety factor(s)

$$q = q_{cyl} \left(1.22 - \frac{0.68}{A}\right) (1 - A^{-2})^{-2} \quad (74)$$

$$q_{cyl} = \frac{2\pi a^2 B_{T_0}}{\mu_0 I_p R_0} \left(\frac{1 + \kappa^2(1 + 2\delta^2 - 1.2\delta^3)}{2}\right) \quad (75)$$

or

$$g_q = \frac{2\pi}{\mu_0} \left(\frac{1 + \kappa^2(1 + 2\delta^2 - 1.2\delta^3)}{2}\right) \frac{(1.22 - \frac{0.68}{A})}{A^2(1 - A^{-2})^2} \quad (76)$$

$$q = g_q B_{T_0} R_0 / I_p \quad (77)$$

Averaged poloidal field

$$B_p = \frac{\mu_0 I_p}{2\pi a} \sqrt{\frac{2}{1 + \kappa^2}} \quad (78)$$

Beta

$$\beta = 4\mu_0 k_b g_2 \langle nT \rangle / B_{T_0}^2 \quad (79)$$

Beta poloidal

$$\beta_p = \beta \left( \frac{B_{T0}}{B_p} \right)^2 \quad (80)$$

Bootstrap fraction

$$f_{bs} = \alpha_{bs} \frac{\beta_p}{\sqrt{A}} \quad (81)$$

Bootstrap current

$$I_{bs} = f_{bs} I_p \quad (82)$$

Total heating power, external plus alpha

$$P_{\Sigma} = P_{\alpha} + P_H + P_{CD} \quad (83)$$

Neutron power

$$P_n = 4P_{\alpha} \quad (84)$$

Alpha power

$$P_{\alpha} = 1.5 \times 10^{-37} \langle nT \rangle^2 V s_p \quad (85)$$

Power consumed by the TF coil

$$P_{TF} = \frac{8\eta\pi\kappa B_{T_{leg}}^2 R_0}{\mu_0^2 f_{Cu} A} \left( 1 - \exp(-1.3 \left( \frac{1+2/3\delta}{1-2/3\delta} \right) \sqrt{A-1}) \right) \quad (86)$$

Power consumed by current drive

$$P_{CD} = \frac{\langle n \rangle R_0}{\eta_{CD}} I_p (1 - f_{bs}) \quad (87)$$

Electric power generated

$$P_{el} = P_n f_{el} \quad (88)$$

Total stored energy

$$W = 3g_2 \langle nT \rangle V \quad (89)$$

Recirculating power fraction

$$\chi = \frac{P_{TF}/f_{TF} + P_{CD}/f_{CD} + P_H/f_H}{P_{el}} \quad (90)$$

Wall loading

$$\Gamma_n = \frac{P_n}{S} \quad (91)$$

Conversion efficiency

$$f_{el} = f_{el0} \left[ \frac{1}{2} + \frac{\kappa^{0.13}}{\pi A} \right] (1 + 0.34\delta\kappa^{-0.5}) \quad (92)$$

Power to grid

$$P_{grid} = f_{el} P_n - \frac{P_H}{f_H} - \frac{P_{CD}}{f_{CD}} - \frac{P_{TF}}{f_{TF}} \quad (93)$$

### Appendix 3. Symbols

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$a$	plasma minor radius ( $m$ )
$\kappa$	vertical elongation
$\delta$	triangularity
$R_0$	major radius ( $m$ )
$A$	aspect ratio
$V$	plasma volume ( $m^3$ )
$V_m$	machine encompassing volume( $m^3$ )
$S$	plasma surface area ( $m^2$ )
$B_{T_0}$	vacuum toroidal field at plasma geometric axis ( $T$ )
$B_{T_{leg}}$	magnetic field at inner toroidal-field-coil leg ( $T$ )

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$B_p$	averaged poloidal field at plasma surface ( $T$ )
$I_p$	plasma current ( $A$ )
$I_{bs}$	bootstrap current ( $A$ )
$q$	edge safety factor $q_\psi$
$q_{cyl}$	cylindrical safety factor
$n_0$	central density ( $m^{-3}$ ), $n_e = n_i$
$T_0$	central temperature( $keV$ ), $T_e = T_i$
$\gamma_n$	density profile shaping factor
$\gamma_T$	temperature profile shaping factor
$Z_i$	charge of main impurity ion
$Z_{eff}$	effective charge including impurities
$A_i$	effective mass
$g$	fuel dilution factor
$s_p$	plasma profile and impurity content factor
$W$	total stored energy in plasma ( $J$ )
$\beta$	ratio of average pressure to vacuum toroidal field
$\beta_N$	beta normal
$f_n$	fraction of Greenwald limit
$f_{bs}$	bootstrap fraction
$\alpha_{bs}$	bootstrap fraction coefficient



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$\tau_E$	confinement time (s)
$H$	confinement enhancement factor
$\alpha_R$	confinement-time scaling-law exponent
$\alpha_a$	confinement-time scaling-law exponent

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$P_\Sigma$	total heating power (external plus alpha)(W)
$P_\alpha$	alpha heating power(W)
$P_n$	power in neutrons(W)
$P_{el}$	total electric power generated(W)
$P_{TF}$	power consumed in producing toroidal field (W)
$P_{CD}$	power consumed in generating current drive (W)
$P_H$	power consumed in generating auxiliary heating(W)
$P_{out}$	total heat lost from plasma(W)
$P_B$	power lost to Bremsstrahlung(W)
$P_S$	power lost to cyclotron radiation(W)
$\Gamma_n$	neutron power flux to walls (W/m <sup>2</sup> )
$F$	ratio of alpha power to conduction loss
$Q_n$	ratio of neutron power to auxiliary power
$\chi$	recirculating power fraction
$\eta_{CD}$	conversion efficiency from electric power to plasma current ( $\frac{A}{Wm^{-3}}$ )
$f_{el}$	conversion efficiency from neutrons to electrical power
$f_{TF}$	conversion efficiency from electrical power to toroidal field
$f_{CD}$	conversion efficiency from electrical power to current drive
$f_H$	conversion efficiency from electrical power to auxiliary heating
$f_{Cu}$	fraction of TF area made of conductor

## Appendix 4. constants

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$\mu_0$	permeability of free space $4\pi \times 10^{-7}(Hm^{-1})$
$k_b$	Boltzman constant $1.6022 \times 10^{-16}(J/keV)$
$\eta$	resistivity of copper $2 \times 10^{-8}(\Omega - m)$

## Appendix 5. Nominal values

$\chi_0$	$< 0.62$	(94)
$\Gamma_{n_0}$	$< 5 \times 10^6$	(95)
$H$	$= 2$	(96)
$Z_{eff}$	$= 1.5$	(97)
$Z_i$	$= 6$	(98)
$\gamma_n$	$= 1$	(99)
$\gamma_T$	$= 1$	(100)
$A_i$	$= 1.5$	(101)
$f_{elo}$	$= 0.4$	(102)
$f_{Cu}$	$= 0.7$	(103)
$\delta$	$= 0.2, 0.3, 0.9$	(104)
$\kappa_0$	$< 2$	(105)
$\beta_{N_0}$	$< 9$	(106)
$T_{min}$	$> 5$	(107)
$T_{max}$	$< 25$	(108)
$B_{T_{max_0}}$	$= 13$	(109)
$\alpha_{bs}$	$= 1$	(110)
$f_{bso}$	$< 1.0$	(111)
$\eta_{CD}$	$= 1.0, 3.0, 10.0 \times 10^{19}$	(112)
$q_{min}$	$= 3.2$	(113)
$f_n$	$= 0.9$	(114)
$f_{TF}$	$= 0.8$	(115)
$f_{CD}$	$= 0.8$	(116)
$P_{gr}$	$> 0.5GW$	(117)