Presented at: Micro Bunches Workshop
Upton, NY
September 28-30, 1995

CONF-9509227--17

BNL- 63852

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December 1996

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Work performed under the auspices of the U.S. Department of Energy,
under contract DE-AC02-76CH00016

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Abstract. The effect of the synchrotron radiation free space wakefield on the equilibrium bunch length in an electron storage ring is explored. The equilibrium bunch length, which is obtained numerically, is shown to increase for $\alpha < 0$ and decrease for $\alpha > 0$.

INTRODUCTION

An electron circulating in a storage ring emits synchrotron radiation in the bending magnets. The incoherent emission which is proportional to the number of electrons in a bunch is simply a resistive loss (U_0) that is compensated for by the RF system. The coherent emission of synchrotron radiation can distort the longitudinal potential well and thereby change the electron bunch profile. We begin with a discussion of the synchrotron radiation wakefield[1,2] and then discuss its effect on the bunch length in electron storage rings. This work is an extension of some early work that appeared in the literature in the late 1960s.[3,4]

THE SYNCHROTRON RADIATION WAKEFIELD

For an electron moving on a circular orbit the tangential electric field on the circular orbit can be written in CGS units as the sum of a Coulomb term and a term due to synchrotron radiation, $\tilde{E}_\phi$:[1,2]

$$E_\phi = \frac{e}{4\rho^2\gamma^2}\frac{\cos\xi}{\sin^2\xi} + \tilde{E}_\phi . \tag{1}$$

Work performed under U.S. Dept. of Energy contract no. DE-AC02-CH00012

Here, ρ is the bending radius, γ is the electron energy measured in units of its rest mass and $2\xi = s/\rho$ is the angle between the electron and the observation point. Directly in front of the electron $\xi = 0$ and directly behind $\xi = \pi$. Introducing the scaled angle, $\mu \equiv 3\gamma^3\xi$, the radiation term is given by

$$\tilde{E}_\phi(\mu) = -\frac{2U_0}{2\pi\rho e}\cdot\frac{d\Psi(\mu)}{d\mu} = -\frac{4e\gamma^4}{3\rho^2}\begin{cases} 0, & \mu < 0 \\ \frac{1}{2}, & \mu = 0 \\ \frac{d}{d\mu}\left[\frac{9}{4}\cdot\frac{\cosh\left[\frac{5}{3}\sinh^{-1}\mu\right] - \cosh\left[\sinh^{-1}\mu\right]}{\sinh\left[2\sinh^{-1}\mu\right]}\right], & \mu > 0 \end{cases} \quad (2)$$

where $U_0 = 4\pi e^2\gamma^4/3e\rho$ is the energy loss per turn for a single electron. Since $\Psi(\mu)$ vanishes at $\mu = 0$ and $\mu = \infty$, the function $d\Psi(\mu)/d\mu$ has the additional property, $\int_0^\infty d\Psi(\mu) = 0$. For $\mu >> 1$, $\tilde{E}_\phi$ is given in CGS units by[1,2]

$$\tilde{E}_\phi(s) \approx -\frac{2e}{3^{1/3}\rho^{2/3}}\frac{d}{ds}s^{-1/3}. \quad (3)$$

Note that to convert Equation (3) to MKS one must multiply the right hand side by $Z_0c/4\pi$, where c = speed of light and Z_0 = 377 ohms.

In contrast to the usual notion of a wakefield the synchrotron radiation wake is in front of the exciting charge and not behind! In what follows we shall ignore the Coulomb term and focus our attention on the effects due to the synchrotron radiation term alone.

The discussion above was for the wakefield of a single electron. This wake can be used as the Green's function to obtain the induced voltage per turn due to a bunch of electrons as follows:

$$\tilde{V}(s) = \frac{2\pi\rho}{e}\int_{-\infty}^{s}\tilde{E}_\phi(s-s')\cdot I(s')ds'. \quad (4)$$

For a Gaussian electron beam the induced voltage due to synchrotron radiation is[3,5]

$$\tilde{V}(s) = \frac{2\sqrt{2\pi}Ne\rho^{1/3}}{\left(3\sigma_0^4\right)^{1/3}}F(x), \quad (5)$$

where $x \equiv s/\sigma_0$ and

$$F(x) \equiv \int_0^{\infty} \frac{(x-x')}{x'^{1/3}} \exp\left[-\frac{(x-x')^2}{2}\right] dx' . \quad (6)$$

A plot of F(x) is given in Figure 1. Note that particles at the back of the bunch lose energy and particles at the front of the bunch gain energy. It is also possible to write F(x) in terms of a parabolic cylinder function, $D_{1/3}(x)$,

$$F(x) = -\Gamma\left(\tfrac{2}{3}\right) \exp\left[-\frac{x^2}{4}\right] D_{1/3}(-x) . \quad (7)$$

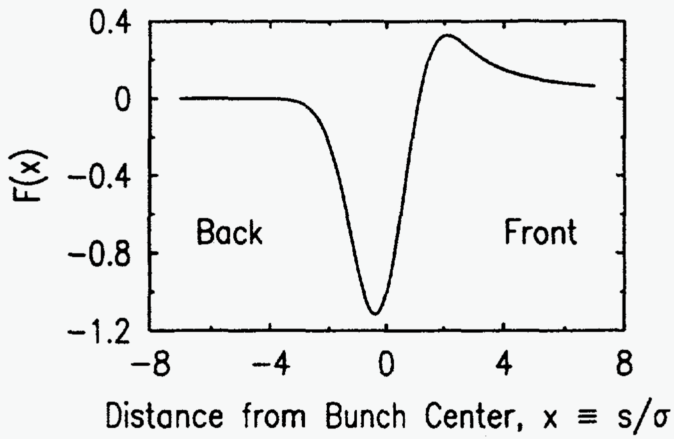

Figure 1: Synchrotron Radiation Wake Potential for a Gaussian Bunch

LONGITUDINAL MOTION IN A STORAGE RING

The longitudinal equations of motion for an electron in a storage ring are[6]

$$\frac{ds}{dt} = -\alpha\varepsilon c , \quad (8a)$$

$$\frac{d\varepsilon}{dt} = \frac{eV_{RF}(s) - U(\varepsilon)}{E_0 T_0} , \quad (8b)$$

where α is the momentum compaction, $V_{RF}(s)$ is the RF voltage, E_0 is the energy of the synchronous particle, T_0 is the revolution time of the synchronous particle, $\varepsilon \equiv \Delta p/p$ and s are the momentum deviation and distance of the electron from the synchronous particle respectively. Note that s is positive when an electron arrives at each azimuth ahead of the synchronous particle. $U(\varepsilon)$ is the energy radiated by the particle; if the particle loses energy $U > 0$. If $V_{RF}(s)$ and $U(\varepsilon)$ are linearized about $s = 0$ and $\varepsilon = 0$ respectively, and $eV_{RF}(0) = U(0)$ for the synchronous particle, Equation (8b) can be written as

$$\frac{d\varepsilon}{dt} = \frac{1}{E_0 T_0}\left[e\frac{dV_{RF}}{ds}s - \frac{dU}{d\varepsilon}\varepsilon\right]. \tag{9}$$

Ignoring the second term in Equation (7) which is responsible for damping, the linearized equations of motion are derivable from a Hamiltonian,

$$H(s,\varepsilon) = -\alpha c\frac{\varepsilon^2}{2} - \frac{\Omega_s^2}{\alpha c}\frac{s^2}{2}, \tag{10}$$

where the synchrotron oscillation frequency is given by

$$\Omega_s \equiv \left(\frac{e\alpha c}{E_0 T_0}\frac{dV_{RF}}{ds}\right)^{1/2}. \tag{11}$$

To have stable linear oscillations requires that

$$\alpha V'_{RF} > 0, \tag{12}$$

where $V'_{RF} \equiv dV_{RF}/ds$. For the typical electron ring, where $\alpha > 0$, if a particle loses more energy it moves toward a larger value of s to ride higher on the RF waveform to compensate for the additional energy loss.

It can be shown that the equilibrium electron distribution, which is the stationary solution of the Fokker-Planck equation, is given by[7,8]

$$\Pi(s,\varepsilon) \propto \exp\left[-\frac{H}{\alpha c\sigma_\varepsilon^2}\right] = \exp\left[-\frac{\varepsilon^2}{2\sigma_\varepsilon^2} - \frac{\Omega_s^2 s^2}{2\alpha^2 c^2 \sigma_\varepsilon^2}\right], \tag{13}$$

where σ_ε is the energy spread of the electron beam. Equation (13) is used to introduce the electron bunch length as follows:

$$\sigma_0 = \frac{\alpha c \sigma_\varepsilon}{\Omega_s}. \tag{14}$$

Integrating over ε, and introducing the proper normalization we can write the equilibrium distribution of the electron beam current as a Gaussian,

$$I(s) = \frac{Nec}{\sqrt{2\pi}\sigma_0}\exp\left[-\frac{s^2}{2\sigma_0^2}\right]. \tag{15}$$

LONGITUDINAL MOTION WITH A WAKEFIELD

In the presence of an additional wakefield the Hamiltonian must be modified to include the induced voltage of the bunch,[7-9]

$$H(s,\varepsilon) = -\alpha c\frac{\varepsilon^2}{2} - \frac{\Omega_s^2}{\alpha c}\frac{s^2}{2} - \frac{e}{E_0 T_0}\int_0^s \tilde{V}(s')\,ds', \tag{16}$$

where $\tilde{V}(s)$ is given by Equation (4).[3] Writing the Green's function as the derivative of the step response function, S(s),

$$\frac{2\pi\rho}{e} E_{\phi}(s) = \frac{dS(s)}{ds}, \tag{17}$$

the above Hamiltonian can be simplified using Equation (4) to obtain

$$H(s,\varepsilon) = -\alpha c \frac{\varepsilon^2}{2} - \frac{\Omega_s^2}{\alpha c}\frac{s^2}{2} - \frac{e}{E_0 T_0}\int_0^{\infty} ds'\, S(s') I(s-s'). \tag{18}$$

In this case the equilibrium electron beam distribution is given by

$$\Pi(s,\varepsilon) \propto \exp\left[\frac{-H}{\alpha c \sigma_\varepsilon^2}\right] = \exp\left[-\frac{\varepsilon^2}{2\sigma_\varepsilon^2} - \left(\frac{\Omega_s}{\alpha c}\right)^2 \frac{s^2}{2\sigma_\varepsilon^2} - \frac{e}{\alpha c E_0 T_0 \sigma_\varepsilon^2}\int_0^{\infty} ds'\, S(s') I(s-s')\right] \tag{19}$$

Integrating over ε and introducing a normalization constant, K, the equilibrium current distribution is seen to be determined as the solution of the Haïssinski equation,[7]

$$I(s) = K \exp\left[-\frac{s^2}{2\sigma_0^2} - \frac{1}{\sigma_0^2 V'_{RF}}\int_0^{\infty} ds'\, S(s') I(s-s') ds'\right]. \tag{20}$$

For a given wakefield this implicit equation can be solved in some cases analytically but more frequently one must resort to numerical analysis. The location of the bunch centroid and the bunch length can then be computed as moments of the distribution.

EQUILIBRIUM BUNCH DISTRIBUTION IN THE PRESENCE OF THE SYNCHROTRON RADIATION WAKEFIELD

The effect of coherent synchrotron radiation on the equilibrium electron bunch distribution can be investigated by using the Green's function given in Equation (3) to write the step response function in MKS units as

$$S(s) = -Z_0\left(\frac{\rho}{3s}\right)^{1/3}. \tag{21}$$

Substituting this into Equation (20) yields

$$y(x) = K \exp\left[-\frac{x^2}{2} \pm \int_0^{\infty} \frac{y(x-x')}{x'^{1/3}} dx'\right], \tag{22}$$

where $x \equiv \frac{s}{\sigma_0}$, $y(x) \equiv \frac{Z_0}{V'_{RF}\sigma_0}\left(\frac{\rho}{3\sigma_0}\right)^{1/3} I(s)$ and the plus sign(+) is taken for $\alpha > 0$ and the minus sign (-) for $\alpha < 0$.

In Figure 2 we display the equilibrium distribution for $\alpha > 0$ and several values of the normalization constant K. In Table 1 we list the mean, $\langle x \rangle$, and the width of the distribution, $\sqrt{\langle x^2\rangle - \langle x\rangle^2}$, for several values of the normalization constant K.

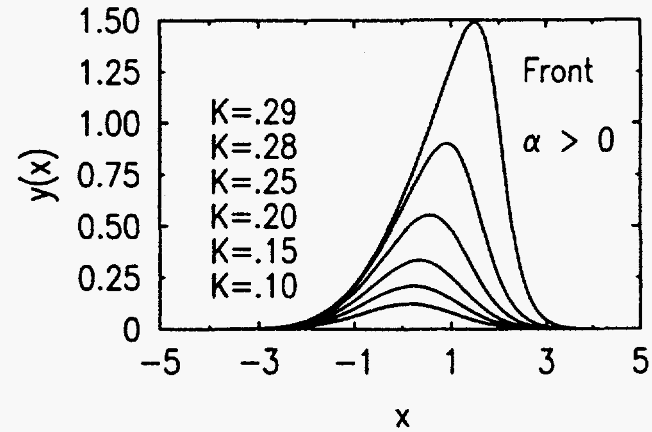

Figure 2: Equilibrium Beam Distribution for Synchrotron Radiation Wake, $\alpha > 0$.

Table 1 Equilibrium beam parameters due to potential well distortion with $\alpha > 0$.

K	$\int_{-\infty}^{\infty} y(x)dx$	$\langle x\rangle$	$\sqrt{\langle x^2\rangle - \langle x\rangle^2}$
.1	.3	.077	.986
.15	.505	.133	.976
.2	.79	.212	.965
.25	1.27	.353	.952
.28	2.0	.573	.944
.29	3.2	.941	.967

From Figure 2 it can be seen that the bunch centroid moves to the right in order to move up the RF waveform to compensate for the energy loss due to synchrotron radiation. The bunch narrows as the current in the bunch increases up until $K \approx 0.28$ after which the bunch begins to widen. For $K > 0.2905$ no equilibrium is found; it is not clear whether our numerical solution is breaking down or if there is a more profound physical effect taking place. More work is needed to explore this regime.

The narrowing of the electron bunch as the current increases is desirable if one is trying to produce coherent synchrotron radiation. The distortion of the bunch from a Gaussian as it develops the sharp leading edge, should produce coherent radiation at higher frequencies than the simply Gaussian bunch.

For computational purposes one can relate the integral of y(x) to the total charge in the bunch, N, as follows:

$$\int_{-\infty}^{\infty} y(x)dx = \frac{Nec}{\sigma_0}\frac{Z_0}{V'_{RF}\sigma_0}\left(\frac{\rho}{3\sigma_0}\right)^{1/3}. \tag{23}$$

As an example we can consider the proposed coherent synchrotron radiation experiment in a small storage ring given in reference 10. In this case, $\rho = 0.6$ meters, $\sigma_0 = 3.3\times10^{-4}$ meters and $V'_{RF} = 9.0\times10^{7}$ volts/meter, hence

$$N = \frac{10^8}{157}\int_{-\infty}^{\infty} y(x)dx. \tag{24}$$

In Figure 3 we display the equilibrium distribution for $\alpha < 0$ and several values of the normalization constant K. In Table 2 we list the mean, $\langle x\rangle$, and the width of the distribution, $\sqrt{\langle x^2\rangle - \langle x\rangle^2}$, for several values of the normalization constant K.

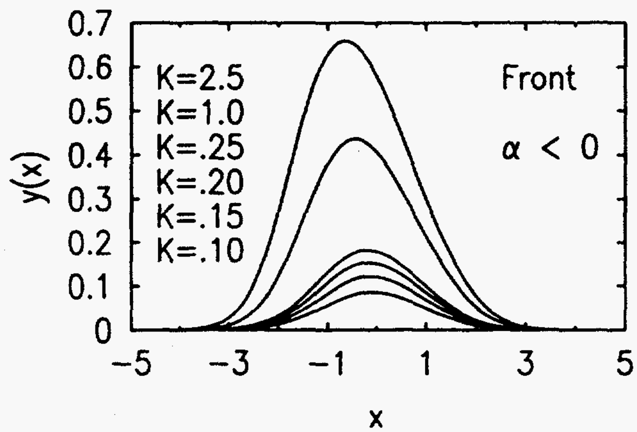

Figure 3: Equilibrium Beam Distribution for Synchrotron Radiation Wake, $\alpha < 0$.

Table 2 Equilibrium beam parameters due to potential well distortion with $\alpha < 0$.

K	$\int_{-\infty}^{\infty} y(x)dx$	$\langle x\rangle$	$\sqrt{\langle x^2\rangle - \langle x\rangle^2}$
.1	.22	-.055	1.01
.15	.31	-.077	1.02
.2	.39	-.096	1.02
.25	.47	-.114	1.024
1.0	1.2	-.273	1.07
2.5	1.89	-.411	1.11

From Figure 3 it can be seen that the bunch centroid moves to the left in order to move up the RF waveform to compensate for the energy loss due to synchrotron radiation. The bunch widens slowly as the current in the bunch increases.

CONCLUSIONS

We have explored the contribution of coherent synchrotron radiation to potential well distortion of the equilibrium electron bunch length. For $\alpha > 0$, the bunch centroid shifts to compensate for the resistive loss and there is primarily bunch shortening. However for large enough currents we are unable to find an equilibrium solution. Whether or not this is simply a numerical problem or indicative of an instability threshold is not known at this time. Additional work is required. The distortion of the bunch from a Gaussian should lead to coherent radiation at higher frequencies than was possible with the simply Gaussian bunch.

For $\alpha < 0$, the bunch centroid shifts toward the rear of the bunch and the bunch lengthens. Even though the overall bunch is widening, the steepening of the bunch should provide higher frequency coherent synchrotron radiation.

The next step in the analysis would be to use the equilibrium distributions given here as the starting point in a stability analysis of the Vlasov or Fokker-Planck equations.

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