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Title: FEEDBACK LINEARIZATION APPLICATION FOR LLRF CONTROL SYSTEM

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Feedback linearization Application for LLRF Control System

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Abstract-The Low Energy Demonstration Accelerator(LEDA) being constructed at Los Alamos National Laboratory will serve as the prototype for the low energy section of Acceleration Production of Tritium(APT) accelerator. This paper addresses the problem of the LLRF control system for LEDA. We propose a control law which is based on exact feedback linearization coupled with gain scheduling which reduces the effect of the deterministic klystron cathode voltage ripple that is due to harmonics of the high voltage power supply and achieves tracking of desired set points. Also, we propose an estimator of the ripple and its time derivative and the estimates based feedback linearization controller.

I. Introduction

The low energy demonstration accelerator(LEDA) for the Production of Tritium(APT) is being built at Los Alamos National Laboratory. The primary function of the low level RF(LLRF) control system of LEDA is to control RF fields in the accelerating cavity and maintain field stability within $\pm 1\%$ peak to peak amplitude error and $1°$ peak to peak phase error[8].

This paper addresses the problem of the LLRF control system used for LEDA. We propose a control law which is based on exact feedback linearization[3] coupled with gain scheduling[10]. The purpose of exact feedback linearization coupled with gain scheduling is to reduce the effect of the deterministic cathode ripple that is due to harmonics of high voltage power supply[12] and is to achieve tracking of desired set points. Low frequency ripple does not deteriorate the current LLRF control system based on PID control method. As frequency of ripple increases, the effect of the ripple on the performance increases too. Simulation shows that 0.3% high voltage power supply ripple yields $1.05°$ phase error at $72kHz$[5] and 1.0% high voltage power supply ripple yields $3.6°$ phase error at $120kHz$[12]. In order to suppress the high frequency ripple, the proposed controller makes use of not only the ripple but also the time derivative of the ripple. The usage of time derivative of the ripple improve the controller performance[10]. First, we assume that the deterministic cathode ripple is measurable and derive the controller. Second, we propose the ripple estimator which estimates the ripple signal itself and the time derivative of the ripple as well and derive the controller coupled with the ripple estimator. As is well known, in order to design the exact feedback linearization controller, the given system to be controlled must be well defined. Previous works[6],[12] modeled the klystron and RF cavity used for LEDA. Our current work is based on the klystron model and RF cavity model set up in Matlab/Simulink environment.

II. Klystron Model

The klystron is the most commonly used linear accelerator RF power source. The klystron used in LEDA has two inputs, LLRF_I and LLRF_Q and two output HPRF_I and HPRF_Q. As intermediate outputs, klystron has the normalized amplitude y_1^k and the normalized phase y_2^k. Let u_1=LLRF_I and let u_2=LLRF_Q. The klystron in LEDA is modeled as

$$\dot{z}_1 = -a_1 z_1 + a_1 cos(z_2) u_1 + a_1 sin(z_2) u_2 \quad (1)$$

$$\dot{z}_2 = -a_1 \frac{sin(z_2)}{z_1} u_1 + a_1 \frac{cos(z_2)}{z_1} u_2 \quad (2)$$

$$y_1^k = \sum_{i=1}^{N} c_i e^{-f_i w(t) z_1} \quad (3)$$

$$y_2^k = \sum_{i=1}^{N} d_i e^{-f_i w(t) z_1} + z_2 + 3 \cdot \frac{\pi}{180} \cdot R(t) \quad (4)$$

$$HPRF\_I = 10\sqrt{KP_m} \cdot y_1^k \cdot cos(y_2^k) \quad (5)$$

$$HPRF\_Q = 10\sqrt{KP_m} \cdot y_1^k \cdot sin(y_2^k). \quad (6)$$

where

$$w(t) = \frac{K_g}{10\sqrt{KP_m}}(0.01R(t)+1)^{1.25} \quad (7)$$

and f_i, $i = 1, 2, \cdots, N$ and parameters c_i, $i = 1, 2, \cdots, N$, d_i, $i = 1, 2, \cdots, N$ are are given in Table 1.

f_1	0.50	c_1	0.0568043e+006	d_1	-0.1412074e+005
f_2	0.75	c_2	-0.3926436e+006	d_2	0.8308426e+005
f_3	1.00	c_3	1.1280559e+006	d_3	-2.0177823e+005
f_4	1.25	c_4	-1.7241855e+006	d_4	2.5844141e+005
f_5	1.50	c_5	1.4787824e+006	d_5	-1.8368060e+005
f_6	1.75	c_6	-0.6748367e+006	d_6	0.6845313e+005
f_7	2.00	c_7	0.1280230e+006	d_7	-0.1039925e+005

Table 1. Klystron Parameters

The details of the klystron model is given in [6] of this proceeding.

III. The RF Cavity

Figure 1 shows the RF cavity model.

RF cavity has four inputs, HPRF_I, HPRF_Q, BEAM_I, and BEAM_Q, two outputs, CAV_FLD_I and CAV_FLD_Q. Let u_1^c=HPRF_I, u_2^c=HPRF_Q, u_3^c=BEAM_I, u_4^c=BEAM_Q and let y_1^c=CAV_FLD_I, y_2^c=CAV_FLD_Q. Then, the RF cavity can be expressed in the state space form.

$$\dot{x} = Ax + Bu^c \quad (8)$$

$$y^c = Cx. \quad (9)$$

System matrices A, B, C of RF cavity are given in [5],[11].

Also, FLD_I and FLD_Q of the cavity Field Sample System are given by

$$FLD\_I = FA \cdot cos(GD) \cdot y_1^c - FA \cdot sin(GD) \cdot y_2^c \quad (10)$$

$$FLD\_Q = FA \cdot sin(GD) \cdot y_1^c + FA \cdot cos(GD) \cdot y_2^c \quad (11)$$

and FLD_AMP and FLD_PHS of the cavity Field Sample System are given by

$$FLD\_AMP = \sqrt{FLD\_I^2 + FLD\_Q^2}$$

$$FLD\_PHS = tan^{-1}\left(\frac{FLD\_Q}{FLD\_I}\right)$$

where

$$FA = 0.00037809$$
$$GD = \frac{\pi}{180}\cdot(-0.039455).$$

The RF cavity as given in (8), (9) is Hurwitz stable and is inverse stable as well.

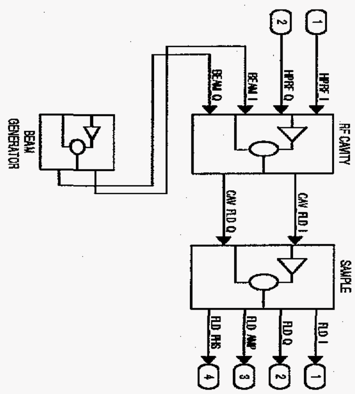

Figure 1. A RF cavity Model and a cavity Field Sample System

IV. The Feedback Linearization Controller

Consider the klystron equation in z-coordinate given in previous section.

Define a coordinate transformation given by

$$\overline{z}_1 = (0.01R(t)+1)^{1.25}z_1 \tag{12}$$
$$\overline{z}_2 = z_2 + 3\cdot\frac{\pi}{180}\cdot R(t). \tag{13}$$

In $\overline{z}$-coordinate, the klystron is expressed by

$$\dot{\overline{z}}_1 = -\overline{a}_1\overline{z}_1 + b_{z11}(\overline{z},R(t),\dot{R}(t))u_1 + b_{z12}(\overline{z},R(t),\dot{R}(t))u_2 + E_{z1}(R(t),\dot{R}(t)) \tag{14}$$
$$\dot{\overline{z}}_2 = b_{z21}(\overline{z},R(t),\dot{R}(t))u_1 + b_{z22}(\overline{z},R(t),\dot{R}(t))u_2 + E_{z2}(R(t),\dot{R}(t)) \tag{15}$$
$$y_1^k = \sum_{i=1}^{N} c_i e^{-f_i M\overline{z}_1} \tag{16}$$
$$y_2^k = \sum_{i=1}^{N} d_i e^{-f_i M\overline{z}_1} + \overline{z}_2. \tag{17}$$

where
$\overline{a}_1 = a_1 - 0.0125(0.01R(t)+1)^{-1}\dot{R}(t)$
$b_{z11}(\overline{z},R(t),\dot{R}(t)) = a_1(0.01R(t)+1)^{1.25}cos(\overline{z}_2 - 3\cdot\frac{\pi}{180}\cdot R(t))$
$b_{z12}(\overline{z},R(t),\dot{R}(t)) = a_1(0.01R(t)+1)^{1.25}sin(\overline{z}_2 - 3\cdot\frac{\pi}{180}\cdot R(t))$
$b_{z21}(\overline{z},R(t),\dot{R}(t)) = -a_1\frac{sin(\overline{z}_2-3\cdot\frac{\pi}{180}\cdot R(t))}{\overline{z}_1}$
$b_{z22}(\overline{z},R(t),\dot{R}(t)) = a_1\frac{cos(\overline{z}_2-3\cdot\frac{\pi}{180}\cdot R(t))}{\overline{z}_1}$
$E_{z1}(R(t),\dot{R}(t)) = 0$
$E_{z2}(R(t),\dot{R}(t)) = 3\cdot\frac{\pi}{180}\dot{R}(t)$
$M = \frac{K_g}{10\sqrt{KP_m}}$.

Let $\overline{z} = [\ \overline{z}_1 \quad \overline{z}_2\ ]^T$ and let $u_z = [\ u_1 \quad u_2\ ]^T$.
Define

$$\overline{A}_z(R(t),\dot{R}(t)) = \begin{bmatrix} -\overline{a}_1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$E_z(R(t),\dot{R}(t)) = \begin{bmatrix} 0 \\ 3\cdot\frac{\pi}{180}\dot{R}(t) \end{bmatrix}, \tag{18}$$

$$\overline{B}_z(\overline{z},R(t),\dot{R}(t)) = \begin{bmatrix} b_{z11}(\overline{z},R(t),\dot{R}(t)) & b_{z12}(\overline{z},R(t),\dot{R}(t)) \\ b_{z21}(\overline{z},R(t),\dot{R}(t)) & b_{z22}(\overline{z},R(t),\dot{R}(t)) \end{bmatrix}.$$

Then, (14) and (15) are represented by

$$\dot{\overline{z}} = \overline{A}_z(R(t),\dot{R}(t))\overline{z} + \overline{B}_z(\overline{z},R(t),\dot{R}(t))u_z + E_z(R(t),\dot{R}(t)). \tag{19}$$

Note that $\overline{B}_z(\overline{z},R(t),\dot{R}(t))$ is invertible for any nonzero $\overline{z}_1$. In $\overline{z}$-coordinate, state equations are dependent upon the ripple $R(t)$ but the output equations are independent upon the ripple $R(t)$. Assume that $\overline{z}_1 \neq 0$. Consider a controller

$$u_z = \overline{B}_z^{-1}(\overline{z},R(t),\dot{R}(t))(\overline{u} - E_z) \tag{20}$$
$$\overline{u} = K\overline{z} + R_z \tag{21}$$
$$= \begin{bmatrix} k_1 + 0.0125(0.01R(t)+1)^{-1}\dot{R}(t) & 0 \\ 0 & k_2 \end{bmatrix}\overline{z} + \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

where R_z is the control input that drives the steady state value $\overline{z}_s$ of $\overline{z}$ to the desired value of $\overline{z}$[2].
Then, (19) is reduced to

$$\dot{\overline{z}} = \overline{A}_{zc}\overline{z} + I_{2\times2}R_z \tag{22}$$

where

$$\overline{A}_{zc} = \begin{bmatrix} -a_1 + k_1 & 0 \\ 0 & k_2 \end{bmatrix}. \tag{23}$$

$\overline{A}_{zc}$ is a constant matrix and is independent of $R(t)$ and $\dot{R}(t)$. By proper choice of k_1 and k_2, we can locate the eigenvalues of the matrix $\overline{A}_{zc}$ that what we want.

R_z is the solution of the following equation[2].

$$0 = \overline{A}_{zc}\overline{z}_s + I_{2\times2}R_z. \tag{24}$$

The solution of (24) can be explained by the steady state value of the transfer function from R_z to $\overline{z}$.

$$T_{\overline{z}R_z}(s) = (sI - \overline{A}_{zc})^{-1}I_{2\times2}.$$

For constant R_z, steady state value $\overline{z}_s$ of $\overline{z}$ is given by the equation

$$\overline{z}_s = (0I - \overline{A}_{zc})^{-1}I_{2\times2}R_z$$

Equivalently,

$$0 = \overline{A}_{zc}\overline{z}_s + I_{2\times2}R_z.$$

The solution R_z of (24) is represented by the steady state value $\overline{z}_s$ of $\overline{z}$ and it is obtained as follow.
First, we consider the equations for FLD_I and FLD_Q as given in (10) and (11).

$$\begin{bmatrix} FLD\_I \\ FLD\_Q \end{bmatrix} = \begin{bmatrix} FA\cdot cos(GD) & -FA\cdot sin(GD) \\ FA\cdot sin(GD) & FA\cdot cos(GD) \end{bmatrix}\begin{bmatrix} y_1^c \\ y_2^c \end{bmatrix}. \tag{25}$$

Let FLD_I$_d$ and FLD_Q$_d$ be the desired values of FLD_I and FLD_Q. Then, the desired values y_{1d}^c, y_{2d}^c of y_1^c, y_2^c are given by the algebraic equation

$$\begin{bmatrix} y_{1d}^c \\ y_{2d}^c \end{bmatrix} = \begin{bmatrix} FA\cdot cos(GD) & -FA\cdot sin(GD) \\ FA\cdot sin(GD) & FA\cdot cos(GD) \end{bmatrix}^{-1} \cdot \begin{bmatrix} FLD\_I_d \\ FLD\_Q_d \end{bmatrix}. \tag{26}$$

Second, we consider the cavity equation.

$$\dot{x} = Ax + Bu^c \tag{27}$$
$$y^c = Cx \tag{28}$$

Let $H_{CAV}(s)$ be the transfer function from u_1^c = HPRF_I and u_2^c = HPRF_Q to y_1^c = CAV_FLD_I and y_2^c = CAV_FLD_Q, assuming that u_3^c = BEAM_I and u_4^c = BEAM_Q are given and constants. Then, we obtain the relation represented by the transfer function $H_{CAV}(s)$

$$\begin{bmatrix} y_1^c(s) \\ y_2^c(s) \end{bmatrix} = H_{CAV}(s) \begin{bmatrix} u_1^c(s) \\ u_2^c(s) \end{bmatrix}. \quad (29)$$

Note that $H_{CAV}(s)$ has no zeros at the origin in the complex plane. Let $\overline{y}_1^c$, $\overline{y}_2^c$ be the steady state value of y_1^c, y_2^c, respectively and let $\overline{u}_1^c$, $\overline{u}_2^c$ be the steady state value of u_1^c, u_2^c, respectively. Then,

$$\begin{bmatrix} \overline{y}_1^c \\ \overline{y}_2^c \end{bmatrix} = H_{CAV}(0) \begin{bmatrix} \overline{u}_1^c \\ \overline{u}_2^c \end{bmatrix}. \quad (30)$$

$H_{CAV}(0)$ can be obtained by applying any steady state value test[1],[7]. One method is step input test[7]. Select u_1^{c1} as nonzero constant, u_2^{c1} as zero, and obtain y_1^{c1} and y_2^{c1}. Next, select u_1^{c2} as zero, u_2^{c2} as nonzero constant and obtain y_1^{c2} and y_2^{c2}. Then, $H_{CAV}(0)$ satisfies

$$\begin{bmatrix} y_1^{c1} & y_1^{c2} \\ y_2^{c1} & y_2^{c2} \end{bmatrix} = H_{CAV}(0) \begin{bmatrix} u_1^{c1} & u_1^{c2} \\ u_2^{c1} & u_2^{c2} \end{bmatrix}. \quad (31)$$

Since cavity has no zeros at the origin in the complex plane,

$$\begin{bmatrix} \overline{u}_1^c \\ \overline{u}_2^c \end{bmatrix} = H_{CAV}^{-1}(0) \begin{bmatrix} \overline{y}_1^c \\ \overline{y}_2^c \end{bmatrix}. \quad (32)$$

Since the input u_1^c is the klystron output HPRF_I and the input u_2^c is the klystron output HPRF_Q and since cavity has no zeros at the origin in the complex plane, plugging (5) and (6) to (29), we obtain

$$\begin{bmatrix} y_1^k \cdot cos(y_2^k) \\ y_1^k \cdot sin(y_2^k) \end{bmatrix} = \frac{1}{10\sqrt{KPm}} H_{CAV}^{-1}(s) \begin{bmatrix} y_1^c \\ y_2^c \end{bmatrix}. \quad (33)$$

Let $\overline{y}_1^k$, $\overline{y}_2^k$ be the steady state values of y_1^k, y_2^k, respectively and let $\overline{y}_1^c$, $\overline{y}_2^c$ be the steady state values of y_1^c, y_1^c, respectively. Then, the steady state relation is given by

$$\begin{bmatrix} \overline{y}_1^k \cdot cos(\overline{y}_2^k) \\ \overline{y}_1^k \cdot sin(\overline{y}_2^k) \end{bmatrix} = \frac{1}{10\sqrt{KPm}} H_{CAV}^{-1}(0) \begin{bmatrix} \overline{y}_1^c \\ \overline{y}_2^c \end{bmatrix}. \quad (34)$$

Setting

$$\begin{bmatrix} \overline{y}_1^c \\ \overline{y}_2^c \end{bmatrix} = \begin{bmatrix} y_{1d}^c \\ y_{2d}^c \end{bmatrix},$$

and plugging (26) into (34), we obtain

$$\begin{bmatrix} \overline{y}_1^k \cdot cos(\overline{y}_2^k) \\ \overline{y}_1^k \cdot sin(\overline{y}_2^k) \end{bmatrix} = \frac{1}{10\sqrt{KPm}} H_{CAV}^{-1}(0) \cdot \begin{bmatrix} FA \cdot cos(GD) & -FA \cdot sin(GD) \\ FA \cdot sin(GD) & FA \cdot cos(GD) \end{bmatrix}^{-1} \begin{bmatrix} FLD\_I_d \\ FLD\_Q_d \end{bmatrix}. \quad (35)$$

Define the right-hand side of the above equation to be $[\ \psi_1 \quad \psi_2\ ]^T$. Then,

$$\overline{y}_1^k = \sqrt{\psi_1^2 + \psi_2^2} \quad (36)$$

$$\overline{y}_2^k = tan^{-1}(\frac{\psi_2}{\psi_1}) \quad (37)$$

where $\overline{y}_1^k$ and $\overline{y}_2^k$ are the normalized amplitude and the normalized phase of klystron which yield the desired value FLD_I_d of FLD_I and the desired value FLD_Q_d of FLD_Q.

The steady state values $\overline{z}_{1s}$, $\overline{z}_{2s}$, which are the desired values of $\overline{z}_1$, $\overline{z}_2$, respectively are obtained by solving the algebraic equations generated from (16), (17), (36) and (37).

$$\sum_{i=1}^{N} c_i e^{-f_i M \overline{z}_{1s}} = \overline{y}_1^k \quad (38)$$

$$\sum_{i=1}^{N} d_i e^{-f_i M \overline{z}_{1s}} + \overline{z}_{2s} = \overline{y}_2^k. \quad (39)$$

We have to solve $\overline{z}_{1s}$ and $\overline{z}_{2s}$. Instead of obtaining the analytic solution of the system (38) and (39), we obtain the numerical solution by resorting to an optimization method. Our approach is the minimization in the least square sense given as follow:

$$minimize \quad (\sum_{i=1}^{N} c_i e^{-f_i M \overline{z}_{1s}} - \overline{y}_1^k)^2 + (\sum_{i=1}^{N} d_i e^{-f_i M \overline{z}_{1s}} + \overline{z}_{2s} - \overline{y}_2^k)^2. \quad (40)$$

The controller design procedure is as follows:

Controller design procedure

1. Obtain $H_{CAV}(0)$ by applying a step input test.
2. Given desired FLD_I_d and FLD_Q_d, obtain $\overline{y}_1^k$ and $\overline{y}_2^k$.
3. For the solutions $\overline{y}_1^k$ and $\overline{y}_2^k$, solve the optimization problem and obtain $\overline{z}_{1s}$ and $\overline{z}_{2s}$.
4. Obtain k_1 and k_2 so that the matrix $\overline{A}_{zc}$ is stable.
5. Find the solution R_z of $0 = \overline{A}_{zc}\overline{z_s} + R_z$.
6. Obtain the control input u_z as given in (20) and (21).

V Numerical Example

We consider the klystron RF cavity system when there is $20,000Hz$ sinusoidal ripple and $720Hz$, $120Hz$ ripples as well. The maximum power KP_m and the klystron gain K_g are given in Table 2.

K_g	8449.4	KP_m	3.600e+006

Table 2. Klystron gain and klystron maximum power

BEAM_I and BEAM_Q used for the steady state gain simulation is in Figure 2.

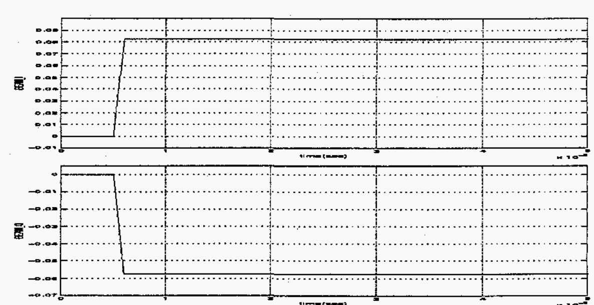

Figure 2. BEAM_I and BEAM_Q

Following the controller design procedure, we obtain the following data.

$$H_{CAV}(0) = \begin{bmatrix} 0.99987 & -0.01452 \\ -0.00013 & 1.31809 \end{bmatrix},$$

$\overline{y}_1^k$	6.97080e-001
$\overline{y}_2^k$	6.22858e-004
$\overline{z}_{1s}$	1.20825
$\overline{z}_{2s}$	0.01088
k_1	-5.64972e+006
k_2	-2.82486e+007

$$R_z = \left[\begin{array}{c} 3.41312e+006 \\ 3.07291e+005 \end{array} \right].$$

Based on numerical values obtained, we implement the controller (20) and (21) to drive the klystron-RF cavity system.

Figure 3 show the simulation results of the klystron-RF cavity system in Matlab/Simulink environment.

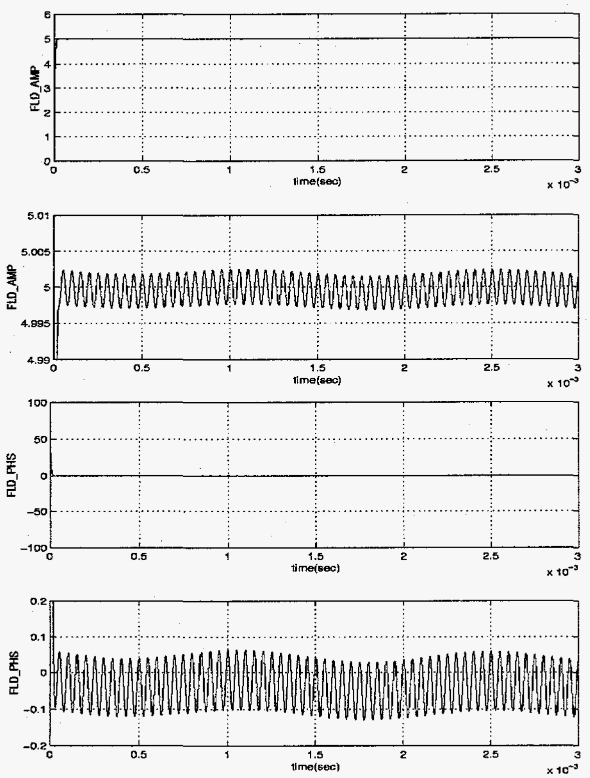

Figure 3. Field Amplitude, FLD_AMP and Field Phase, FLD_PHS

IV. RIPPLE ESTIMATION

The purpose of the low level RF control(LLRF) system is to maintain the field stability within ±1.0% amplitude and 1.0° phase. In the case that there is phase difference, only $\frac{\pi}{8}$ phase difference yields 1.12° field phase error. In the case that there is amplitude difference, 40% gap of amplitude yields 1.156° field phase error.

For the remedy to the poor information on the ripple and its time derivative, we can make use of Lyapunov redesign[4] after we design the exact feedback linearization controller based on the nominal values of the ripple and its time derivative. This additional controller compensates the uncertainties or unmodelled dynamics. Another possible remedy is to design the estimator which yields the estimated ripple and its time derivative and based on the estimated information, we design the controller.

In this section, we address the ripple estimator which estimates the ripple $R(t)$ and its time derivative $\frac{dR(t)}{dt}$, and the feedback linearization controller based on the estimator.

First, we consider equations given by (5) and (6). From (5) and (6), for given HPRF_I and HPRF_Q, we obtain the normalized amplitude y_1^k and the normalized phase y_2^k of the klystron by solving algebraic equations.

$$y_1^k = \frac{1}{10\sqrt{KP_m}}\sqrt{HPRF\_I^2 + HPRF\_Q^2} \qquad (41)$$

$$y_2^k = tan^{-1}(\frac{HPRF\_Q}{HPRF\_I}). \qquad (42)$$

Second, we consider the normalized amplitude and the normalized phase as given in (16) and (17) in $\overline{z}$-coordinate. The normalized amplitude of the klystron is the output of the look-up table **AMPLITUDE SATURATION**[6] and the input of the look-up table **AMPLITUDE SATURATION** is given by

$$A = M\overline{z}_1 \qquad (43)$$

in $\overline{z}$-coordinate. Also, there exists a region of (A, y_1^k) pairs where there is an inverse look-up table of **AMPLITUDE SATURATION**. This region can be extracted from data of the look-up table **AMPLITUDE SATURATION**. We can obtain the curve fitting equation for the inverse look-up table of **AMPLITUDE SATURATION**. Based on the data of look-up table **AMPLITUDE SATURATION** where the selected data of y_1^k and $\overline{z}_1$ guarantee invertibility, we obtain the curve fitting equation as follows.

$$\overline{z}_1 = \sum_{i=1}^{N} c_i^z e^{-f_i^z y_1^k} \qquad (44)$$

where $N = 7$, coefficients f_i^z, $i = 1, 2, \cdots, N$ are given and the coefficients c_i^z, $i = 1, 2, \cdots, N$ are obtained by applying the optimization toolbox of Matlab/Simulink. Table 3 gives the data of the coefficients of the curve fitting.

f_1^z	0.50	c_1^z	246379.7012736
f_2^z	0.75	c_2^z	-1633291.8595640
f_3^z	1.00	c_3^z	4505197.5753121
f_4^z	1.25	c_4^z	-6618176.9543979
f_5^z	1.50	c_5^z	5460679.7305035
f_6^z	1.75	c_6^z	-2399431.1109898
f_7^z	2.00	c_7^z	438643.01506646

Table 3. Coefficients of Curve fitting equation for Inverse **AMPLITUDE SATURATION**

The estimate of the ripple $R(t)$ and the estimate of the time derivative $\frac{dR(t)}{dt}$ of the ripple $R(t)$ are obtained by considering the klystron system both in z-coordinate and $\overline{z}$-coordinate. The relation between z-coordinate and $\overline{z}$-coordinate is as given in (12) and (13).

$$\overline{z}_1 = (0.01R(t) + 1)^{1.25} z_1 \qquad (45)$$

$$\overline{z}_2 = z_2 + 3 \cdot \frac{\pi}{180} \cdot R(t). \qquad (46)$$

When z_1 is obtained from the solution (1), (2) and also when y_1^k is obtained from (41), we can obtain $\overline{z}_1$ by using (45). For given z_1 and $\overline{z}_1$, we can obtain the estimate $\hat{R}(t)$ of the ripple $R(t)$ by solving algebraic equation (45).

$$\hat{R}(t) = 100((\frac{\overline{z}_1(t)}{z_1(t)})^{0.8} - 1.0). \qquad (47)$$

Also, the estimate $\dot{\hat{R}}(t)$ of time derivative $\dot{R}(t)$ of the ripple $R(t)$ is obtained by differentiation of $\hat{R}(t)$.

The feedback linearization controller based on the estimate $\hat{R}(t)$ and $\dot{\hat{R}}(t)$ is given by

$$u_z = \hat{\overline{B}}_z^{-1}(\overline{z}, \hat{R}(t), \dot{\hat{R}}(t))(\overline{u} - E_z(\hat{R}(t), \dot{\hat{R}}(t))) \qquad (48)$$

$$\overline{u} = \hat{K}(\hat{R}(t), \dot{\hat{R}}(t))\overline{z} + R_z, \qquad (49)$$

where

$$\hat{\overline{B}}_z(\overline{z},\hat{R}(t),\dot{\hat{R}}(t)) = \begin{bmatrix} b_{z11}(\overline{z},\hat{R}(t),\dot{\hat{R}}(t)) & b_{z12}(\overline{z},\hat{R}(t),\dot{\hat{R}}(t)) \\ b_{z21}(\overline{z},\hat{R}(t),\dot{\hat{R}}(t)) & b_{z22}(\overline{z},\hat{R}(t),\dot{\hat{R}}(t)) \end{bmatrix},$$

$$E_z(\hat{R}(t),\dot{\hat{R}}(t)) = \begin{bmatrix} 0 \\ 3\cdot\frac{\pi}{180}\dot{\hat{R}}(t) \end{bmatrix},$$

$$\hat{K}(\hat{R}(t),\dot{\hat{R}}(t)) = \begin{bmatrix} k_1 + 0.0125(0.01\hat{R}(t)+1)^{-1}\dot{\hat{R}}(t) & 0 \\ 0 & k_2 \end{bmatrix},$$

$$R_z = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}.$$

Figure 4 shows the feedback linearization system in Matlab/Simulink environment. KLYSTRON is the klystron model, RIPPLE is the equivalent system which generates high voltage power supply ripple. RF_CAVITY is the RF cavity with Beam. Inputs of the ripple estimator are HPRF_I, HPRF_Q, LLRF_I and LLRF_Q which are measurable. The ripple estimator estimates both the ripple and its time derivative. The time derivative information is used in the feedback linearization controller and the usage of the time derivative information improves the closed loop performance[10].

Figures 5-6 show the simulation when the ripple is

$$R(t) = 1.0sin(2\pi f_1 t) + 1.0sin(2\pi f_2 t + \frac{3\pi}{8})$$

where $f_1 = 120kHz$, $f_2 = 80kHz$.

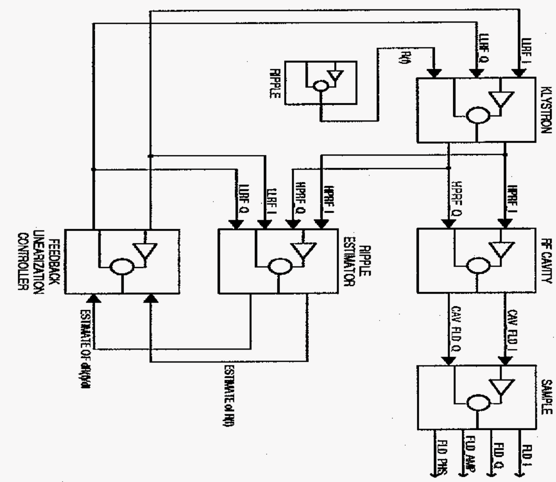

Figure 4. The feedback linearization system with ripple estimator

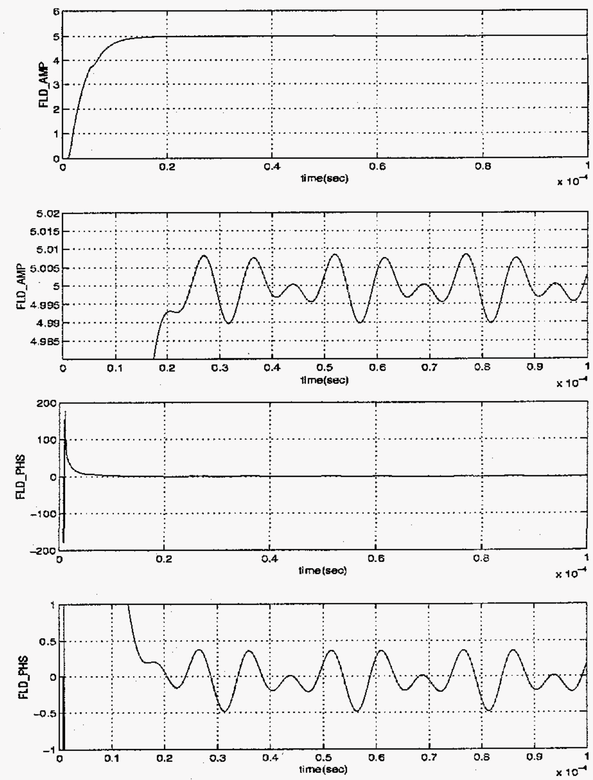

Figure 5. Amplitude of Field, FLD_AMP and Phase of Field, FLD_PHS

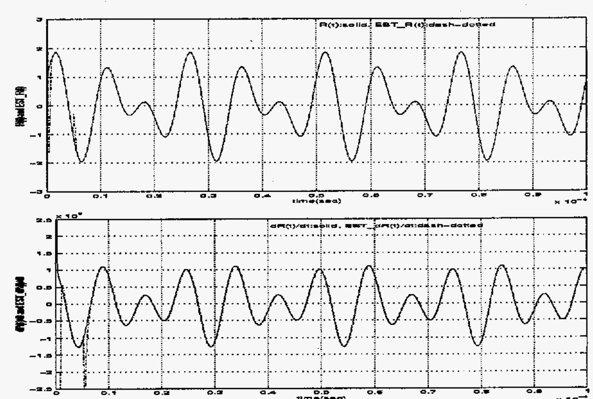

Figure 6. Ripple $R(t)$ and its time derivative $dR(t)/dt$ and their estimates $EST\_R(t)$ and $EST\_dR(t)/dt$

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