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Title: PHASE SYNCHRONIZATION OF MULTIPLE KLYSTRONS IN RF SYSTEMS

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Remark: The second sections of the paper entitled "Adaptive feedforward ..." , the paper entitled "Phase synchronization ...", and the paper entitled "Feedback linearization ..." are describing the klystron model and are almost same. Also, the third sections of the paper entitled "Adaptive feedforward ..." and the paper entitled "Feedback linearization ..." are describing the RF cavity model and almost same. The rest sections of each paper describe the different control techniques and they are derived from the klystron model and the RF cavity model described in the second section and the third section. When at least two papers are accepted for full papers, the second section and the third section will be modified.

Phase Synchronization of Multiple Klystrons in RF System

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Abstract-The Low Energy Demonstration Accelerator(LEDA) being constructed at Los Alamos National Laboratory will serve as the prototype for the low energy section of the Acceleration Production of Tritium(APT) accelerator. The first LEDA RF system includes three, 1.2 MW, 350 MHz, continuous wave, klystrons driving a radio frequency quadrapole(RFQ). A phase control loop is necessary for each individual klystron in order to guarantee the phase matching of these klystrons. To meet this objective, we propose adaptive PI controllers which are based on simple adaptive control. These controllers guarantee not only phase matching but also amplitude matching.

1 Introduction

The Low Energy Demonstration Accelerator(LEDA) being constructed at Los Alamos National Laboratory will serve as the prototype for the low energy section of Acceleration Production of Tritium(APT) accelerator. The APT accelerator requires over 244 RF klystrons each with a continuous wave output power of 1 MW. The reliability and availability of these RF systems is a critical to the successful operation of APT plant and prototypes of these systems are being developed and demonstrated on LEDA. The first LEDA RF system includes three, 1.2 MW, 350 MHz, continuous wave, klystrons driving a radio frequency quadrapole(RFQ)[11],[12]. The LEDA 350 MHz klystrons are designed to dissipate the full beam power(1.85MW) in the klystron collector in steady state.

In this paper, we address the problem of the control of a multi-klystron RF system. We present the control of amplitudes and phases of multiple klystrons. Previous investigation[16] shows that closed loop amplitude control around each individual klystron is unnecessary and the gain mismatches in klystrons do not affect to the field amplitude control significantly. However, the gain mismatches have influence on the field phase control. A suggestion to counteract this problem is to synchronize multiple klystrons' phases. This paper proposes a method to deal with the synchronization of the phases of multiple klystrons. In order to achieve the objective, we make use of the simple adaptive control(SAC) concept[8].

SAC was first proposed in the work of Sobel, Kaufman, and Mabius[17] for linear plants. This approach uses a control structure which is a linear combination of feedforward of the model states, and inputs and feedback of the error between plant and model outputs. Asymptotic stability is ensured provided that the plant is almost strictly positive real(ASPR). That is, for a plant represented by the triple (A, B, C) which is 1)minimum phase, 2)relative degree one, and 3)positive high frequency gain, there exists a feedback gain K_e such that the closed loop system $(A - BK_eC, B, C)$ is strictly positive real(SPR). The ease of its implementation, non-necessity of identification, and its robustness properties make SAC attractive to the practitioner. SAC was extended to nonlinear plants[2], nonlinear servomechanisms with time varying uncertainties[4], non ASPR plants[3],[6],[7]. Its relation to high gain adaptive control([14] and reference therein) was described in the works of Bar-Kana[1],[2] where high gain adaptive control was described as a special class of SAC.

A klystron is modeled as a two input two output(TITO) nonlinear system where the state equation is linear but the output equation is nonlinear with respect to phase. In order to overcome the nonlinearity of phase in output, a coordinate transformation of the klystron model is addressed. In the new coordinate, the state equation is nonlinear with respect to state and linear with respect to input, and the output equation is linear with respect to phase. Based on the model in the transformed coordinate, we design an adaptive PI controller by applying SAC. In order to set up SAC for multiple klystrons, one of klystrons is taken as the model klystron and the rest of them are taken as plant klystrons. The selected model klystron's input is taken as model input, its output as model output, and its state as model state. For each plant klystron, SAC is designed so that plant klystron's output follows the output of the model klystrons.

2 The Klystron Model

We consider a klystron model.

Figure 1 shows a klystron model in Matlab/Simulink. It has 1)**FILTER AND AMPLIFIER**, 2)**AMPLITUDE DEPENDENCE** and **PHASE DEPENDENCE** of ripple, 3) lookup tables, **AMPLITUDE SATURATION** and **PHASE SATURATION**. Also, it has two inputs, LLRF_I and LLRF_Q and two outputs HPRF_I and HPRF_Q.

Let u_1=LLRF_I and let u_2=LLRF_Q. Let x_1 and x_2 be outputs of the system called **FILTER AND AMPLIFIER** whose transfer function are given by

$$\frac{X_1(s)}{U_1(s)} = \frac{1}{3.54e^{-7}s + 1} \tag{1}$$

$$\frac{X_2(s)}{U_2(s)} = \frac{1}{3.54e^{-7}s + 1}. \tag{2}$$

In state space, transfer functions (1) and (2) are represented as

$$\dot{x}_1 = -a_1 x_1 + a_1 u_1 \tag{3}$$

$$\dot{x}_2 = -a_1 x_2 + a_1 u_2 \tag{4}$$

where $a_1 = \frac{1e+007}{3.54}$.

A klystron is affected by a high voltage power supply ripple. Influence of the ripple on a klystron is represented by **AMPLITUDE DEPENDENCE** and **PHASE DEPENDENCE**. **AMPLITUDE DEPENDENCE** has the form of

$$(c_{rppl}R(t) + 1)^{d_{rppl}}$$

and **PHASE DEPENDENCE** has the form of

$$\phi_{rppl}\frac{\pi}{180}R(t)$$

where c_{rppl}, d_{rppl}, and ϕ_{rppl} are characteristic parameters of a high voltage power supply and a klystron. In this paper, $c_{rppl} = 0.01$, $d_{rppl} = 1.25$, and $\phi_{rppl} = 3.00$.

A klystron has two look-up tables, called **AMPLITUDE SATURATION** and **PHASE SATURATION**. The input of the two look-up tables is given by

$$A = \frac{K_g}{10\sqrt{KP_m}}(0.01R(t) + 1)^{1.25} \cdot \sqrt{x_1^2 + x_2^2} \tag{5}$$

where $R(t)$ is the ripple, K_g is the klystron gain, and KP_m is the maximum klystron power. $R(t)$, K_g, and KP_m are specified for a given klystron. For given A, the output of the look-up table **AMPLITUDE SATURATION** can be represented by

$$A_N = I_1(A) \tag{6}$$

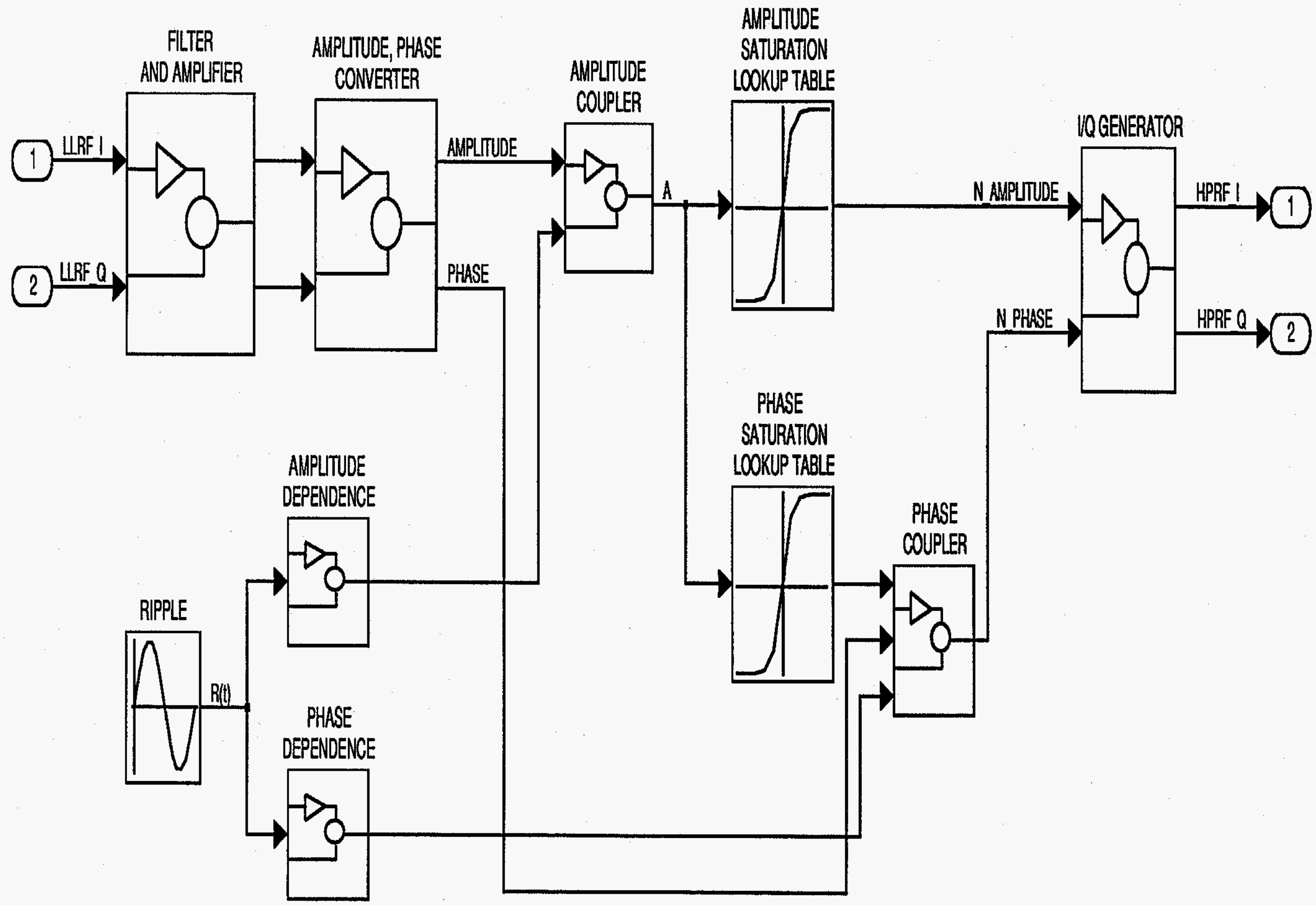

Figure 1: A Klystron Model

and the output of the look-up table **PHASE SATURATION** can be represented by

$$\theta_N = I_2(A) \tag{7}$$

Table 1 and table 2 show data of look-up table **AMPLITUDE SATURATION** and data of look-up table **PHASE SATURATION**, respectively. These data are measured data.

A	A_N	A	A_N	A	A_N	A	A_N
-0.1000	0.0000	0.0700	0.0000	0.1400	0.1900	0.5700	0.7500
0.7100	0.8700	0.8600	0.9800	0.9000	1.0000	0.9100	1.0000

Table 1. AMPLITUDE SATURATION Measured Data

A	θ_N	A	θ_N	A	θ_N	A	θ_N
-0.1000	0.0000	0.0700	0.0000	0.6400	-0.0150	0.7100	-0.0350
0.6400	-0.0150	0.7100	-0.0350	1.0000	-0.4770		

Table 2. PHASE SATURATION Measured Data

In figure 1, the normalized amplitude **N_AMPLITUDE**, defined by y_1, and the normalized phase **N_PHASE**, defined by y_2 are expressed as

$$\begin{aligned} y_1 &= A_N = I_1(A) && (8) \\ y_2 &= \theta_N + tan^{-1}(\frac{x_2}{x_1}) + 3 \cdot \frac{\pi}{180} \cdot R(t) \\ &= I_2(A) + tan^{-1}(\frac{x_2}{x_1}) + 3 \cdot \frac{\pi}{180} \cdot R(t). && (9) \end{aligned}$$

In addition, for given y_1 and y_2, HPRF_I and HPRF_Q are given by

$$HPRF\_I = 10\sqrt{KP_m} \cdot y_1 \cdot cos(y_2) \tag{10}$$

$$HPRF\_Q = 10\sqrt{KP_m} \cdot y_1 \cdot sin(y_2). \tag{11}$$

Since the look-up tables have the limited number of data, we need to approximate the look-up tables by linear or nonlinear curve fitting equations. Considering the characteristic curve of a klystron, we choose nonlinear equations. We choose curve fitting equations of **AMPLITUDE SATURATION** and **PHASE SATURATION** having the forms

$$A_N = \sum_{i=1}^{N} c_i e^{-f_i A} \tag{12}$$

$$\theta_N = \sum_{i=1}^{N} d_i e^{-f_i A} \tag{13}$$

where f_i, $i = 1, 2, \cdots, N$ and parameters c_i, $i = 1, 2, \cdots, N$, d_i, $i = 1, 2, \cdots, N$ are to be determined.

A higher order curve fitting equation may yield more accuracy. For simplicity, we choose $N = 7$. Also, in order to reduce the number of coefficients to be determined, f_i, $i = 1, 2, \cdots, N$ are given in Table 3.

f_1	f_2	f_3	f_4	f_5	f_6	f_7
0.50	0.75	1.00	1.25	1.50	1.75	2.00

Table 3. Exponents of curve fitting equations

In order to increase the accuracy of curve fitting, we need more data. These data are generated by applying a different set of input pairs (LLRF_I, LLRF_Q). Look-up tables generate output data by linear interpolation. Data as given in Table 4 and Table 5 are used for fitting.

By using data given in Table 4 and Table 5, we obtain coefficients c_i, $i = 1, 2, \cdots, N$ and d_i, $i = 1, 2, \cdots, N$, of the curve fitting equations (12) and (13). Coefficients c_i and d_i obtained are given in Table 6. Figure 2 shows plots of data points as given in Table 4, Table 5 and plots of curve fitting equations (12) and (13) whose coefficients, f_i, $i = 1, 2, \cdots, N$, c_i, $i = 1, 2, \cdots, N$, d_i, $i = 1, 2, \cdots, N$, are given in Table 3 and Table 6 with appropriate domain of A.

A	A_N	A	A_N	A	A_N	A	A_N
-0.1000	0.0000	0.0700	0.0000	0.1400	0.1900	0.5700	0.7500
0.7100	0.8700	0.8600	0.9800	0.9000	1.0000	0.9100	1.0000
0.3122	0.4143	0.3568	0.4724	0.4014	0.5305	0.4461	0.5886
0.4907	0.6467	0.5353	0.7048	0.5799	0.7585	0.5910	0.7680
1.0000	0.9900	0.4461	0.5886	0.6468	0.8158	0.6691	0.8349
0.0446	0.0000	0.0892	0.0521	0.6914	0.8540	0.7360	0.8891
0.1338	0.1732	0.1784	0.2400	0.7806	0.9218	0.8252	0.9545
0.2230	0.2981	0.2676	0.3562	0.8921	0.9961	0.9367	0.9970
0.3122	0.4143	0.3568	0.4724	0.9813	0.9921		

Table 4. AMPLITUDE SATURATION Data

A	θ_N	A	θ_N	A	θ_N	A	θ_N
-0.1000	0.0000	0.0700	0.0000	0.6400	-0.0150	0.7100	-0.0350
0.8600	-0.1370	0.9000	-0.2440	1.0000	-0.4770	0.0446	0.0000
0.4987	-0.0113	0.5445	-0.0125	0.5576	-0.0128	0.6691	-0.0233
0.0892	-5.0552e-4	0.1338	-0.0017	0.5712	-0.0132	0.7140	-0.0377
0.1784	-0.0029	0.2230	-0.0040	0.8921	-0.2229	0.4549	-0.0101
0.2676	-0.0052	0.3122	-0.0064	0.4483	-0.0100	0.4014	-0.0087
0.3568	-0.0075	0.7885	-0.0884	0.9593	-0.3821		

Table 5. PHASE SATURATION Data

c_1	0.05680429876058e+006	d_1	-0.14120739315590e+005
c_2	-0.39264357353961e+006	d_2	0.83084262097993e+005
c_3	1.12805594234952e+006	d_3	-2.01778226478032e+005
c_4	-1.72418545240933e+006	d_4	2.58441412755651e+005
c_5	1.47878241712872e+006	d_5	-1.83680595711727e+005
c_6	-0.67483667002473e+006	d_6	0.68453128529433e+005
c_7	0.12802296547207e+006	d_7	-0.10399245992504e+005

Table 6. Coefficients of curve fitting equations

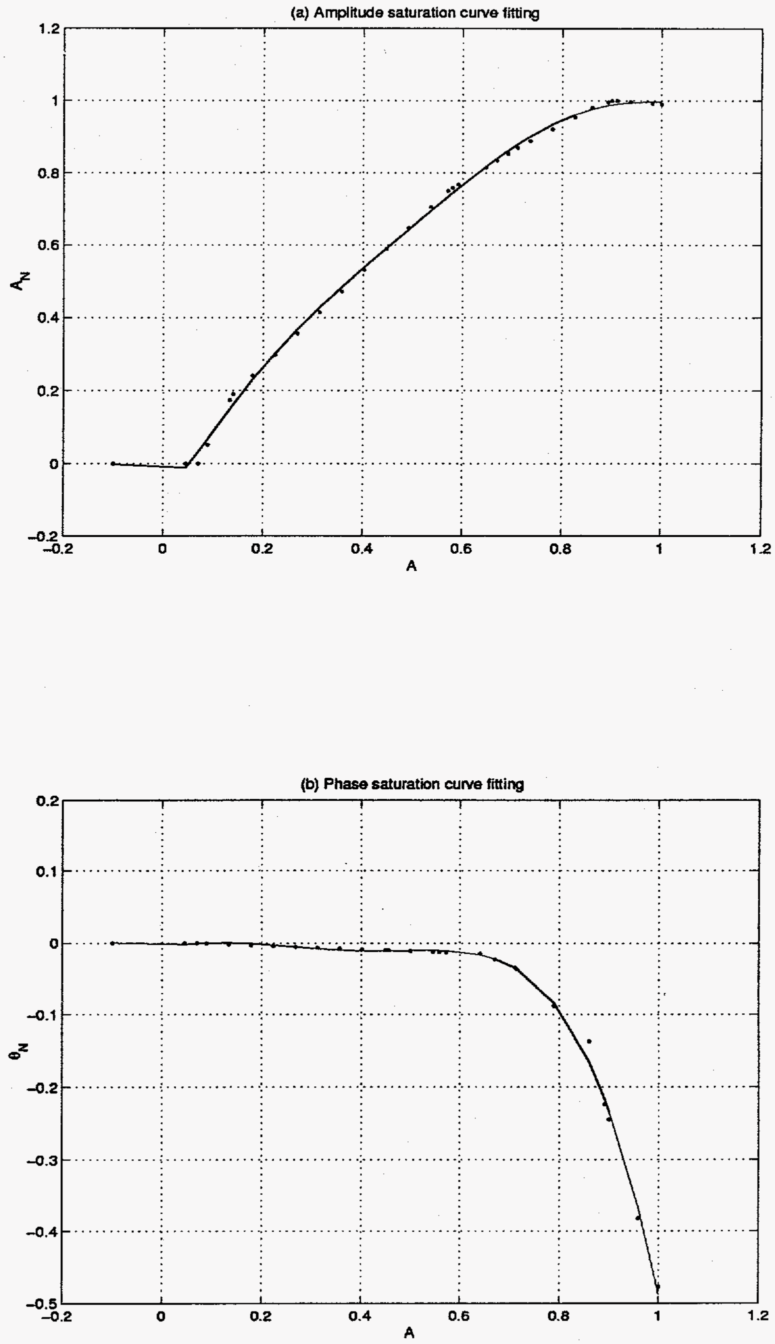

Figure 2: Curve fittings

Plugging (5) into (12) and (13), curve fitting equations (12) and (13) are reduced to

$$A_N = \sum_{i=1}^{N} c_i e^{-f_i w(t)\sqrt{x_1^2+x_2^2}} \tag{14}$$

$$\theta_N = \sum_{i=1}^{N} d_i e^{-f_i w(t)\sqrt{x_1^2+x_2^2}} \tag{15}$$

where

$$w(t) = \frac{K_g}{10\sqrt{KP_m}}(0.01R(t)+1)^{1.25}. \tag{16}$$

The normalized amplitude, **N_AMPLITUDE** y_1 and the normalized phase, **N_PHASE** y_2 of the klystron are now given by

$$y_1 = A_N = \sum_{i=1}^{N} c_i e^{-f_i w(t)\sqrt{x_1^2+x_2^2}} \tag{17}$$

$$\begin{aligned} y_2 &= \theta_N + tan^{-1}(\frac{x_2}{x_1}) + 3 \cdot \frac{\pi}{180} \cdot R(t) \\ &= \sum_{i=1}^{N} d_i e^{-f_i w(t)\sqrt{x_1^2+x_2^2}} + tan^{-1}(\frac{x_2}{x_1}) + 3 \cdot \frac{\pi}{180} \cdot R(t). \end{aligned} \tag{18}$$

The relationship between HPRF_I, HPRF_Q and y_1, y_2 remain the same equations as given in (10), (11).

Now, consider the normalized amplitude y_1 and the normalized phase y_2 as given in (17) and (18).

Let

$$z_1 = \sqrt{x_1^2 + x_2^2} \tag{19}$$

$$z_2 = tan^{-1}(\frac{x_2}{x_1}) \tag{20}$$

$$z_1 > 0, \quad -\frac{\pi}{2} < z_2 < \frac{\pi}{2}.$$

We consider a transformation from x-coordinate to z-coordinate. In z-coordinate, the state equations (3) and (4) are reduced to

$$\dot{z}_1 = -a_1 z_1 + a_1 cos(z_2) u_1 + a_1 sin(z_2) u_2 \tag{21}$$

$$\dot{z}_2 = -a_1 \frac{sin(z_2)}{z_1} u_1 + a_1 \frac{cos(z_2)}{z_1} u_2. \tag{22}$$

Also, the curve fitting equations (12) and (13) are reduced to

$$A_N = \sum_{i=1}^{N} c_i e^{-f_i w(t) z_1} \tag{23}$$

$$\theta_N = \sum_{i=1}^{N} d_i e^{-f_i w(t) z_1}. \tag{24}$$

The normalized amplitude y_1 and the normalized phase y_2 are represented by

$$y_1 = \sum_{i=1}^{N} c_i e^{-f_i w(t) z_1} \tag{25}$$

$$y_2 = \sum_{i=1}^{N} d_i e^{-f_i w(t) z_1} + z_2 + 3 \cdot \frac{\pi}{180} \cdot R(t). \tag{26}$$

Note that the exponents of the first term of (25) are the same as the exponents of the first term of (26). Also, note that the phase y_2 is linear with respect to z_2.

In the next section, we address the control problem of the normalized amplitudes $y_1's$ and the normalized phases $y_2's$ of multiple(three) klystrons driving RFQ of LEDA.

3 Phase and Amplitude Control of Multiple Klystrons

In this section, we consider the control problem of phases and amplitudes of multiple klystrons which drive the RFQ. As studied in the work of Ziomek [16], phase matching of klystrons is very important. In order to achieve the matching purpose, we develop an adaptive PI controller.

Let K_g^m and KP_m^m be the klystron gain and the klystron maximum power of a model klystron and let $(z^m, y^m, HPRF\_I^m, HPRF\_Q^m)$ be the representation of the model klystron.

$$\dot{z}_1^m = -a_1 z_1^m + a_1 cos(z_2^m) u_1^m + a_1 sin(z_2^m) u_2^m \tag{27}$$

$$\dot{z}_2^m = -a_1 \frac{sin(z_2^m)}{z_1^m} u_1^m + a_1 \frac{cos(z_2^m)}{z_1^m} u_2^m, \tag{28}$$

$$y_1^m = \sum_{i=1}^{N} c_i e^{-f_i w^m(t) z_1^m} \tag{29}$$

$$y_2^m = \sum_{i=1}^{N} d_i e^{-f_i w^m(t) z_1^m} + z_2^m + 3 \cdot \frac{\pi}{180} \cdot R(t) \tag{30}$$

$$z_1^m > 0, \qquad -\frac{\pi}{2} < z_2^m < \frac{\pi}{2},$$

and

$$HPRF\_I^m = 10\sqrt{KP_m^m} \cdot y_1^m \cdot cos(y_2^m) \tag{31}$$

$$HPRF\_Q^m = 10\sqrt{KP_m^m} \cdot y_1^m \cdot sin(y_2^m), \tag{32}$$

where

$$w^m(t) = \frac{K_g^m}{10\sqrt{KP_m^m}} (0.01R(t) + 1)^{1.25}. \tag{33}$$

Similarly, let K_g^l and KP_m^l be the klystron gain and the klystron maximum power of the l^{th} klystron and let $(z^l, y^l, HPRF\_I^l, HPRF\_Q^l)$ be its representation. Hence,

$$\dot{z}_1^l = -a_1 z_1^l + a_1 cos(z_2^l)u_1^l + a_1 sin(z_2^l)u_2^l \tag{34}$$

$$\dot{z}_2^l = -a_1 \frac{sin(z_2^l)}{z_1^l} u_1^l + a_1 \frac{cos(z_2^l)}{z_1^l} u_2^l, \tag{35}$$

$$y_1^l = \sum_{i=1}^{N} c_i e^{-f_i w(t) z_1^l} \tag{36}$$

$$y_2^l = \sum_{i=1}^{N} d_i e^{-f_i w(t) z_1^l} + z_2^l + 3 \cdot \frac{\pi}{180} \cdot R(t) \tag{37}$$

$$z_1^l > 0, \quad -\frac{\pi}{2} < z_2^l < \frac{\pi}{2},$$

and

$$HPRF\_I^l = 10\sqrt{KP_l^m} \cdot y_1^l cos(y_2^l) \tag{38}$$

$$HPRF\_Q^l = 10\sqrt{KP_l^m} \cdot y_1^l sin(y_2^l), \tag{39}$$

where

$$w^l(t) = \frac{K_g^l}{10\sqrt{KP_m^l}} (0.01R(t) + 1)^{1.25}. \tag{40}$$

The purpose of the adaptive PI controller is the perfect follow

$$y_1^l = y_1^m, \tag{41}$$

$$y_2^l = y_2^m, \tag{42}$$

or asymptotic follow

$$y_1^l \to y_1^m, \tag{43}$$

$$y_2^l \to y_2^m. \tag{44}$$

When the perfect follow is achieved,

$$\sum_{i=1}^{N} c_i e^{-f_i w^m(t) z_1^m} = \sum_{i=1}^{N} c_i e^{-f_i w^l(t) z_1^l} \tag{45}$$

and

$$\sum_{i=1}^{N} d_i e^{-f_i w^m(t) z_1^m} + z_2^m + 3 \cdot \frac{\pi}{180} \cdot R(t)$$

$$= \sum_{i=1}^{N} d_i e^{-f_i w^l(t) z_1^l} + z_2^l + 3 \cdot \frac{\pi}{180} \cdot R(t). \tag{46}$$

From (33) and (40), we obtain the relation

$$\begin{aligned} w^m(t)z_1^m &= w^l(t)z_1^l \\ \frac{K_g^m}{\sqrt{KP_m^m}}z_1^m &= \frac{K_g^l}{\sqrt{KP_m^l}}z_1^l. \end{aligned} \tag{47}$$

Hence,

$$z_1^l = \frac{K_g^m}{K_g^l}\sqrt{\frac{KP_m^l}{KP_m^m}}z_1^m. \tag{48}$$

Also, we obtain

$$z_2^l = z_2^m. \tag{49}$$

Similarly, when the asymptotic follow is achieved,

$$z_1^l \to \frac{K_g^m}{K_g^l}\sqrt{\frac{KP_m^l}{KP_m^m}}z_1^m, \tag{50}$$

and

$$z_2^l \to z_2^m. \tag{51}$$

Considering (48), (49), (50), and (51), we construct factitious systems which are parts of adaptive PI controller.

For the model klystron, we construct a factitious model given by

$$\dot{z}_1^m = -a_1 z_1^m + a_1 cos(z_2^m)u_1^m + a_1 sin(z_2^m)u_2^m \tag{52}$$

$$\dot{z}_2^m = -a_1\frac{sin(z_2^m)}{z_1^m}u_1^m + a_1\frac{cos(z_2^m)}{z_1^m}u_2^m, \tag{53}$$

$$v_1^m = z_1^m \tag{54}$$

$$v_2^m = z_2^m \tag{55}$$

where v_1^m and v_2^m are outputs of the factitious model.

For the l^{th} klystron, we construct a factitious system given by

$$\dot{z}_1^l = -a_1 z_1^l + a_1 cos(z_2^l)u_1^l + a_1 sin(z_2^l)u_2^l \tag{56}$$

$$\dot{z}_2^l = -a_1\frac{sin(z_2^l)}{z_1^l}u_1^l + a_1\frac{cos(z_2^l)}{z_1^l}u_2^l, \tag{57}$$

$$v_1^l = C_v z_1^l \tag{58}$$

$$v_2^l = z_2^l \tag{59}$$

where v_1^l and v_2^l are outputs of the factitious klystron and the coefficient C_v is given by

$$C_v = \frac{K_g^l}{K_g^m}\sqrt{\frac{KP_m^m}{KP_m^l}}. \tag{60}$$

The adaptive PI controller is designed based on two factitious systems in such a way that

$$v_1^l \rightarrow v_1^m \tag{61}$$

$$v_2^l \rightarrow v_2^m \tag{62}$$

If the adaptive PI controller guarantees convergences (61) and (62), by (50), (51), (54), (55), and (58), (59), we obtain the convergences (43) and (44). The convergence (61) is equivalent to

$$w^l(t)z_1^l \rightarrow w^m(t)z_1^m. \tag{63}$$

Hence, we obviously obtain the convergences (43) and (44),

$$y_1^l \rightarrow y_1^m \tag{64}$$

$$y_2^l \rightarrow y_2^m. \tag{65}$$

Now, consider the adaptive PI controller. In order to design the adaptive PI controller, we make use of the simple adaptive controller concept[8]. To develop the adaptive PI controller, we consider the system as given in (56)-(59). The system as given in (56)-(59) is described by

$$\dot{z}^l = A_l z^l + B(z^l)u^l \tag{66}$$

$$v^l = C_l z^l \tag{67}$$

where

$$A_l = \begin{bmatrix} -a_1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$B(z^l) = \begin{bmatrix} a_1 cos(z_2^l) & a_1 sin(z_2^l) \\ -a_1\frac{sin(z_2^l)}{z_1^l} & a_1\frac{cos(z_2^l)}{z_1^l} \end{bmatrix},$$

$$C_l = \begin{bmatrix} C_v & 0 \\ 0 & 0 \end{bmatrix}$$

with appropriate definitions of state z^l and input u^l and output v^l. Similarly, we can describe the model klystron as given in (52)-(55) as

$$\dot{z}^m = A_m z^m + B(z^m)u^m \tag{68}$$

$$v^m = C_m z^m \tag{69}$$

with similar definitions of system matrices $A_m(=A_l)$, $B(z^m)$, C_m and state z^m, input u^m, output v^m.

(66), (67) can be considered as a nonlinearly perturbed linear multivariable system with reference signal v^m generated by the system as given in (68) and (69):

$$\dot{z}^l = A_l z^l + B_l(u + g(z^l, u^l)) \tag{70}$$

$$v^l = C_l z^l \tag{71}$$

$$e_v = v^m - v^l \tag{72}$$

where

$$B_l = \begin{bmatrix} a_1 cos(z_2^m) & a_1 sin(z_2^m) \\ -a_1 C_v \frac{sin(z_2^m)}{z_1^m} & a_1 C_v \frac{cos(z_2^m)}{z_1^m} \end{bmatrix} \tag{73}$$

$$g(z^l, u^l) = \begin{bmatrix} g_{11}(z^l) & g_{12}(z^l) \\ g_{21}(z^l) & g_{22}(z^l) \end{bmatrix} u^l \tag{74}$$

$$g_{11}(z^l) = a_1 cos(z_2^m) cos(z_2^l) + \frac{a_1}{C_v} \frac{z_1^m}{z_1^l} sin(z_2^m) sin(z_2^l) - 1$$

$$g_{12}(z^l) = a_1 cos(z_2^m) sin(z_2^l) - \frac{a_1}{C_v} \frac{z_1^m}{z_1^l} sin(z_2^m) cos(z_2^l)$$

$$g_{21}(z^l) = a_1 sin(z_2^m) cos(z_2^l) - \frac{a_1}{C_v} \frac{z_1^m}{z_1^l} cos(z_2^m) sin(z_2^l)$$

$$g_{22}(z^l) = a_1 sin(z_2^m) sin(z_2^l) + \frac{a_1}{C_v} \frac{z_1^m}{z_1^l} cos(z_2^m) cos(z_2^l) - 1.$$

In (70) and (71),

$$C_l B_l = \begin{bmatrix} a_1 C_v cos(z_2^m) & a_1 C_v sin(z_2^m) \\ -a_1 C_v \frac{sin(z_2^m)}{z_1^m} & a_1 C_v \frac{cos(z_2^m)}{z_1^m} \end{bmatrix}, \tag{75}$$

and $det(C_l B_l) \neq 0$ for any bounded z_1^m. Also,

$$det \begin{bmatrix} sI - A_l & B_l \\ C_l & 0 \end{bmatrix} = \frac{a_1^2 C_v^2}{z_1^l} \neq 0 \tag{76}$$

for any bounded z_1^l. Let $P = P^T$ be a positive definite matrix defined by

$$P = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix}. \tag{77}$$

If we take

$$p_1 = 1 \tag{78}$$
$$p_2 = 0 \tag{79}$$
$$p_3 = z_1^m, \tag{80}$$

then, since $-\pi/2 < z_2^m < \pi/2$,

$$PC_lB_l + (C_lB_l)^T P = \begin{bmatrix} 2a_1C_v cos(z_2^m) & 0 \\ 0 & 2a_1C_v cos(z_2^m) \end{bmatrix} > 0. \tag{81}$$

Hence, there exists a static output feedback that achieves stability and solves servomechanism problem guaranteeing boundedness of e_v[5],[13],[14] and this, in turn, guarantees existence of a simple adaptive control(SAC)[1],[2].

The adaptive PI controller based on SAC is given by the following:

$$r(t) = \begin{bmatrix} v_1^m - v_1^l \\ v_2^m - v_2^l \\ z_1^m \\ z_2^m \\ u_1^m \\ u_2^m \end{bmatrix}, \tag{82}$$

$$K_r(t) = \begin{bmatrix} K_e(t) & K_z(t) & K_u(t) \end{bmatrix}, \tag{83}$$

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} LLRF\_1\_I(t) \\ LLRF\_1\_Q(t) \end{bmatrix} = K_r(t)r(t) \tag{84}$$

$$K_r(t) = Ki_l(t) + Kp_l(t) \tag{85}$$

$$\dot{K}i_l(t) = \begin{bmatrix} v_1^m - v_1^l \\ v_2^m - v_2^l \end{bmatrix} r^T(t)T_i, \qquad T_i = T_i^T > 0 \tag{86}$$

$$Kp_l(t) = \begin{bmatrix} v_1^m - v_1^l \\ v_2^m - v_2^l \end{bmatrix} r^T(t)T_p, \qquad T_p = T_p^T \geq 0. \tag{87}$$

The integral gain $K_e(t)$ is the crucial component of the controller and governs the stability of the controlled system. The integral gain $K_e(t)$ can be viewed as the gain of high gain adaptive control[10],[14],[18]. $K_z(t)$ and $K_u(t)$ help improve the performance of the adaptive model-following system and and even achieve perfect following in ideal environments[3]. The proportional gain $K_{p_l}(t)$ is added for its immediate action and makes the tracking error small quickly.

Figure 3 depicts an amplitude and phase controller for a klystron. FACTITIOUS MODEL represents the factitious model given by (52)-(55) for the prototype klystron operated as a model, FACTITIOUS SYSTEM_l is the l^{th} factitious system represented by (56)-(59). LLRF_M_I and LLRF_M_Q are used for inputs of FACTITIOUS MODEL. LLRF_l_I and LLRF_l_Q are generated from the adaptive PI controller and are used for inputs of FACTITIOUS SYSTEM_l. Kpl represents the adaptive proportional gain Kp_l and Kil represents the adaptive integration gain Ki_l. The adaptive controller makes use of both LLRF_M_I and LLRF_M_Q, FACTITIOUS MODEL outputs v_1^m, v_2^m, FACTITIOUS MODEL states z_1^m, z_2^m and FACTITIOUS SYSTEM_l outputs v_1^l, v_2^l.

For a multiple klystron system, we make parallel connections of factitious systems and the corresponding adaptive PI controllers. Of interest is the case when the system is composed of three klystrons. Figure 4 shows the case when the system has three klystrons. One of three klystrons is taken as a model klystron. The closed loop system is composed of a factitious model(FACTITIOUS MODEL) as given in (52)-(55), two adaptive control systems(ADAPTIVE CONTROL SYSTEM 1, ADAPTIVE CONTROL SYSTEM 2), and three klystrons(KLYSTRON_M, KLYSTRON_1, KLYSTRON_2). An adaptive control system is composed of a factitious system and an adaptive PI controller as shown in Figure 3. The first klystron(KLYSTRON_M) is used for model klystron and its klystron gain and klystron maximum power are used for K_g^m and KP_m^m. LLRF_M_I and LLRF_M_Q are inputs to KLYSTRON MODEL. They are inputs of FACTITIOUS MODEL and the adaptive control systems. States and outputs of FACTITIOUS MODEL are feedback to adaptive PI controllers too. Each adaptive control system generates LLRF_I and LLRF_Q which is feedback to the corresponding klystron.

The klystron gains $K_g^l s$ and klystron maximum powers $KP_m^l s$ used in the simulation are given in Table 7[16].

K_g^m	4410	KP_m^m	763980(Watts)
K_g^1	5145	KP_m^1	590640(Watts)
K_g^2	5145	KP_m^2	642000(Watts)

Table 7. Klystron gain and klystron maximum power

The gain matrices T_i and T_p of two adaptive PI controllers are given by

$$T_i = 5000 I_{6\times 6}$$

$$T_p = 500 I_{6\times 6}.$$

Figure 5 shows unit step inputs LLRF_M_I, LLRF_M_Q and inputs $u_1^l s$, LLRF_1_I, LLRF_2_I, and inputs $u_2^l s$, LLRF_1_Q, LLRF_2_Q generated by simple adaptive control systems. Figure

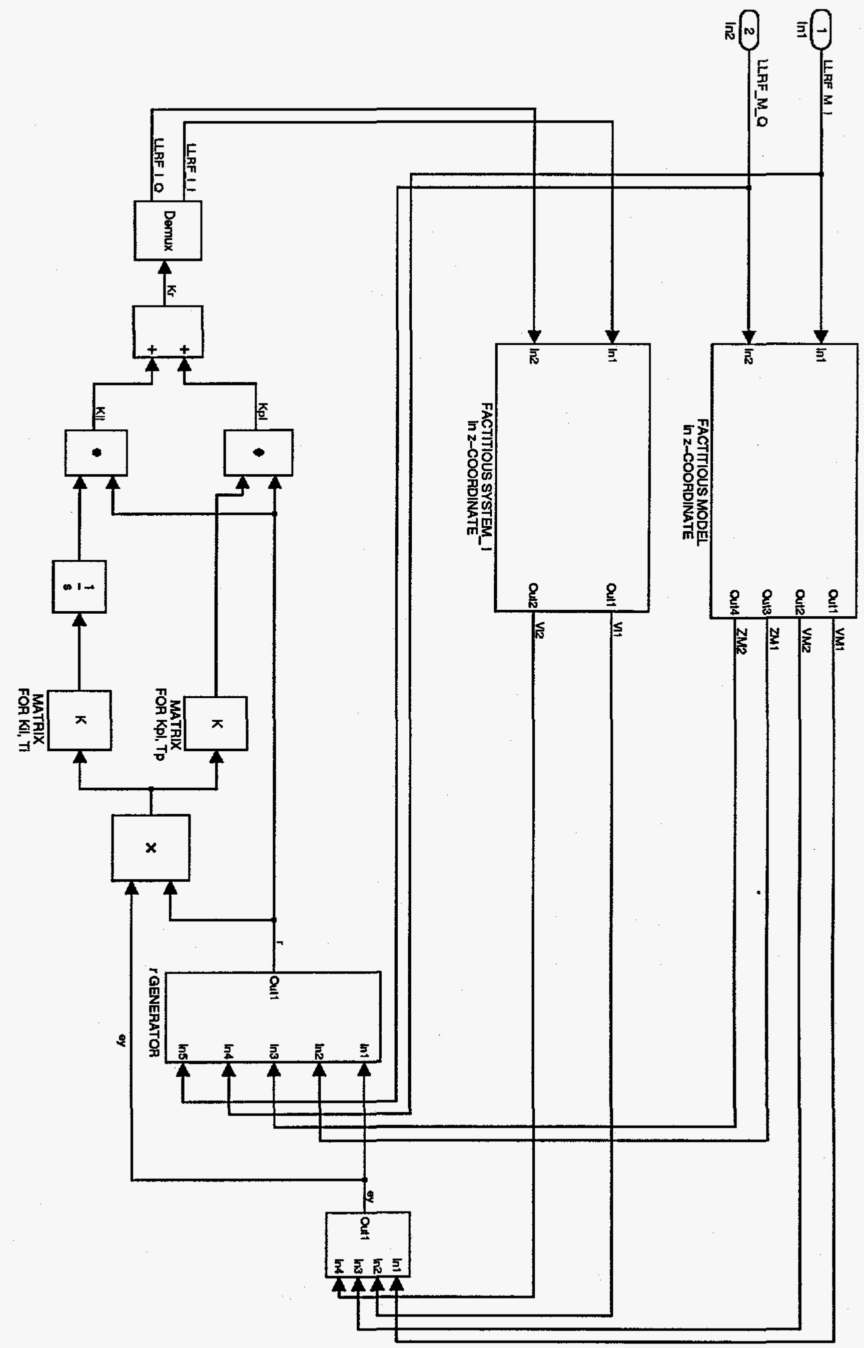

Figure 3: The adaptive PI controller. z_1^m=ZM1, z_2^m=ZM2, v_1^m=VM1, v_2^m=VM2, u_1^m=LLRF_M_I, u_2^m=LLRF_M_Q, u_1^l=LLRF_I_I, u_2^l=LLRF_I_Q.

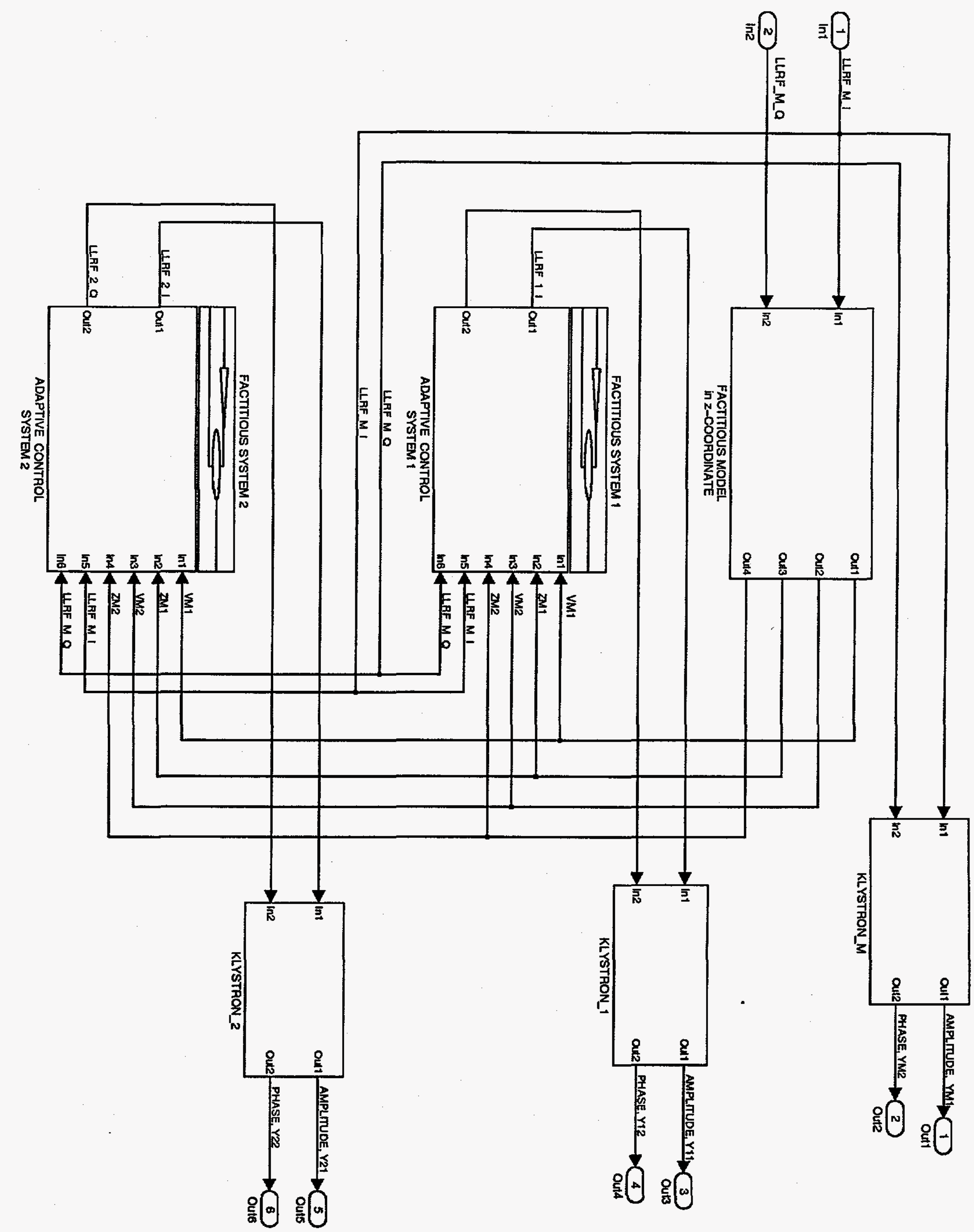

Figure 4: The adaptive PI control closed loop system for three klystrons. Inputs for this closed loop system are LLRF_M_I and LLRF_M_Q. LLRF_1_I, LLRF_1_Q are generated from ADAPTIVE CONTROL SYSTEM 1; and LLRF_2_I, LLRF_2_Q are generated from ADAPTIVE CONTROL SYSTEM 2. These are feedback to KLYSTRON_1 and KLYSTRON_2. z_1^m=ZM1, z_2^m=ZM2, v_1^m=VM1, v_2^m=VM2, u_1^m=LLRF_M_I, u_2^m=LLRF_M_Q, y_1^m=YM1, y_2^m=YM2, y_1^1=Y11, y_2^1=Y12, y_1^2=Y21, y_2^2=Y22.

6 shows klystrons' normalized amplitudes $y_1's$ and normalized phases $y_2's$ when LLRF_M_I and LLRF_M_Q are unit step inputs. Figure 6 shows that amplitudes and phases of KLYSTRON_1 and KLYSTRON_2 converge to amplitude and phase of klystron model, KLYSTRON_M.

Figure 7 shows LLRF_M_I, LLRF_M_Q and inputs $u_1's$, LLRF_1_I, LLRF_2_I, and inputs $u_2's$, LLRF_1_Q, LLRF_2_Q generated by simple adaptive control systems when LLRF_M_I and LLRF_M_Q are

$$LLRF\_M\_I = 1.0e^{-5.0e5t}(sin(2.0e6t) + 1.0)$$
$$LLRF\_M\_Q = 1.0e^{-5.0e5t}sin(1.0e6t).$$

Figure 8 shows klystrons' amplitudes, $y_1's$ and phases, $y_2's$. Figure 8 shows that amplitudes and phases of KLYSTRON_1 and KLYSTRON_2 converge to amplitude and phase of klystron model, KLYSTRON_M.

Figure 9 shows LLRF_M_I and LLRF_M_Q and inputs $u_1's$, LLRF_1_I, LLRF_2_I, and inputs $u_2's$, LLRF_1_Q, LLRF_2_Q generated by simple adaptive control systems when LLRF_M_I and LLRF_M_Q are

$$LLRF\_M\_I = 1.0 + 0.25sin(2.0e6t)$$
$$LLRF\_M\_Q = 0.25sin(1.0e6t).$$

Figure 10 shows klystrons' amplitudes, $y_1's$ and phases, $y_2's$.

4 Conclusion

An adaptive PI controller has been developed for phase synchronization of a multi-klystron driven accelerating cavity. This controller, based on one of the klystrons, modifies the outputs of the others in order to present the *same* drive signals to the cavity.

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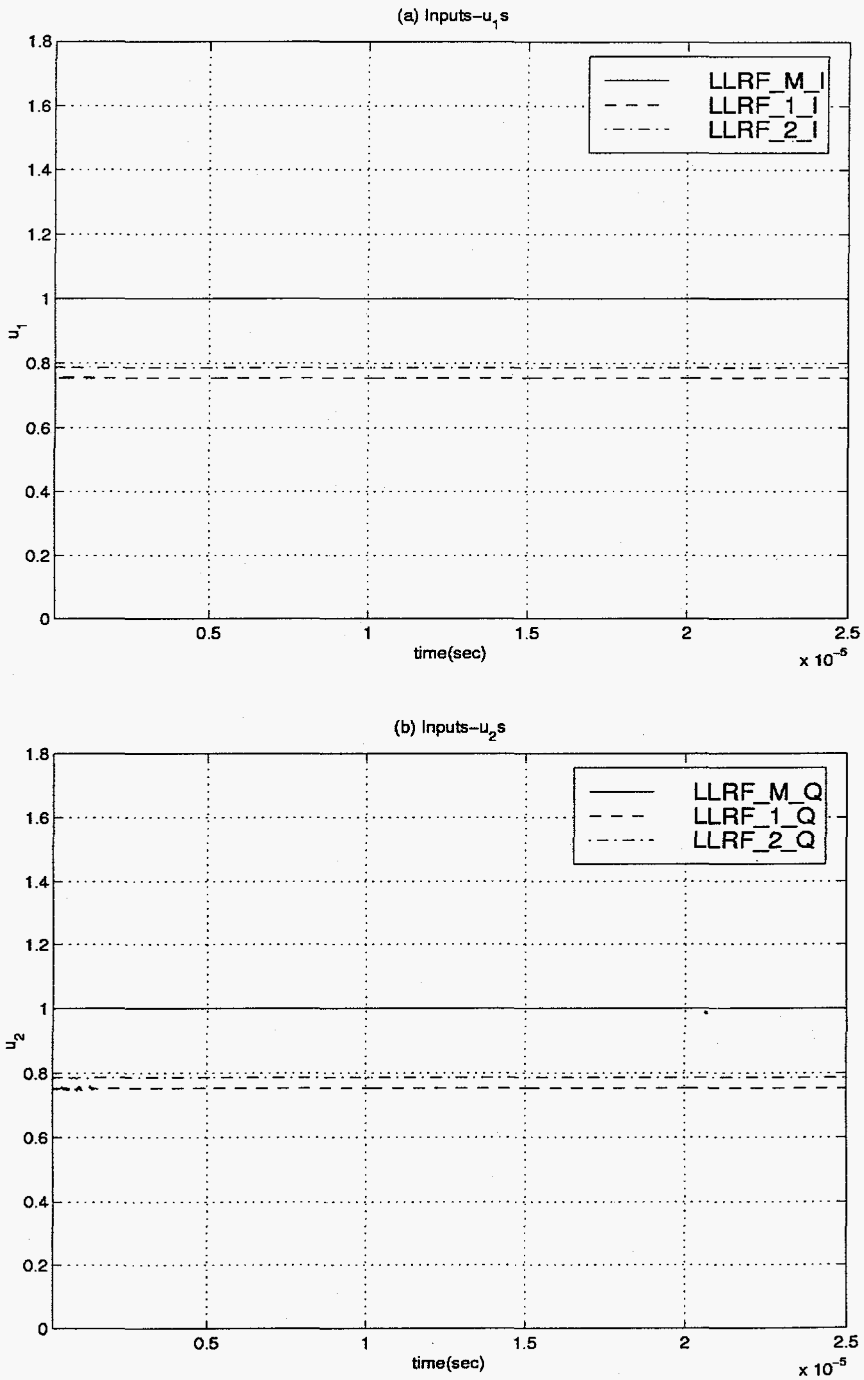

Figure 5: Inputs for three klystrons.

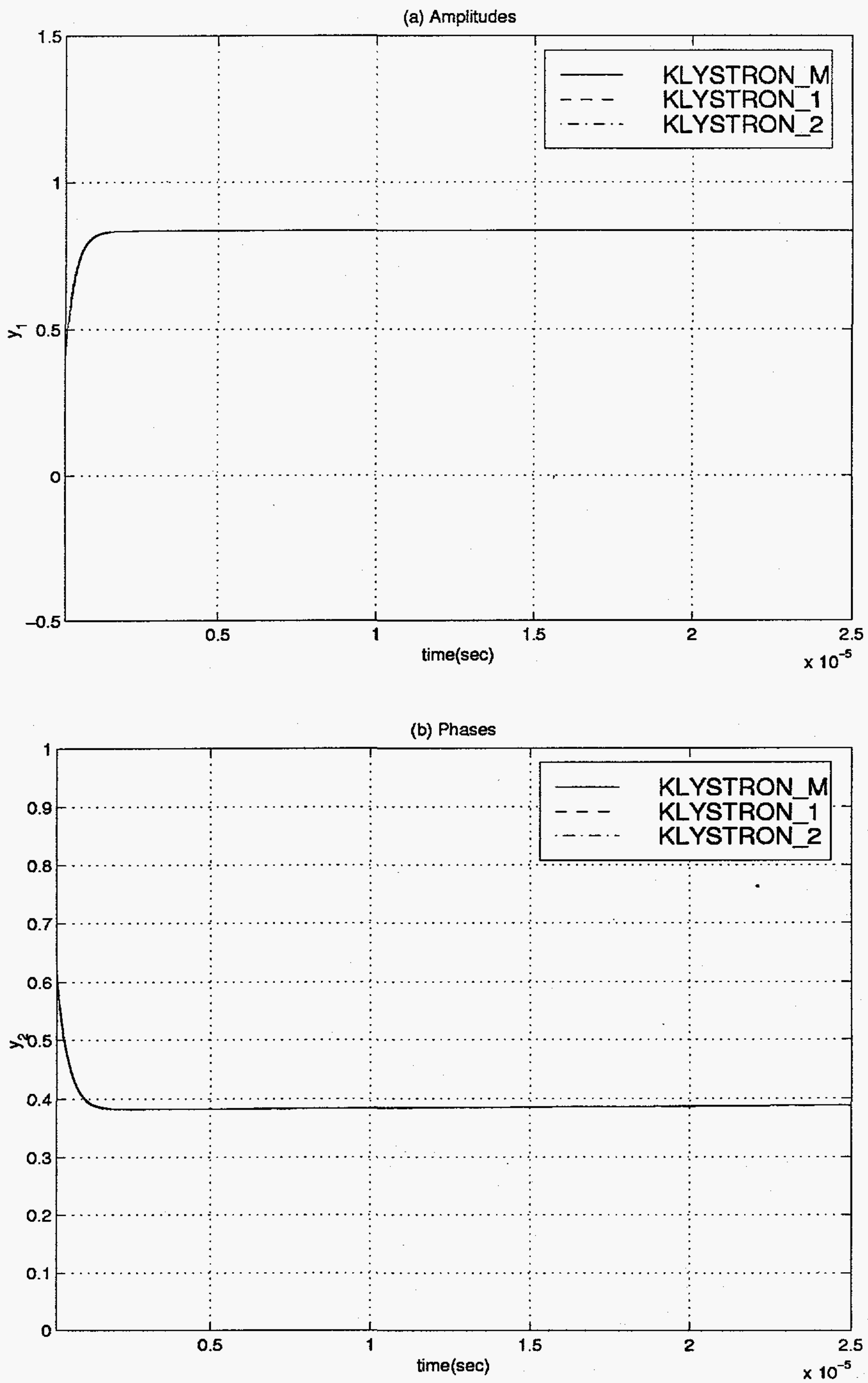

Figure 6: Amplitudes y_1 and Phases y_2 for three klystrons.

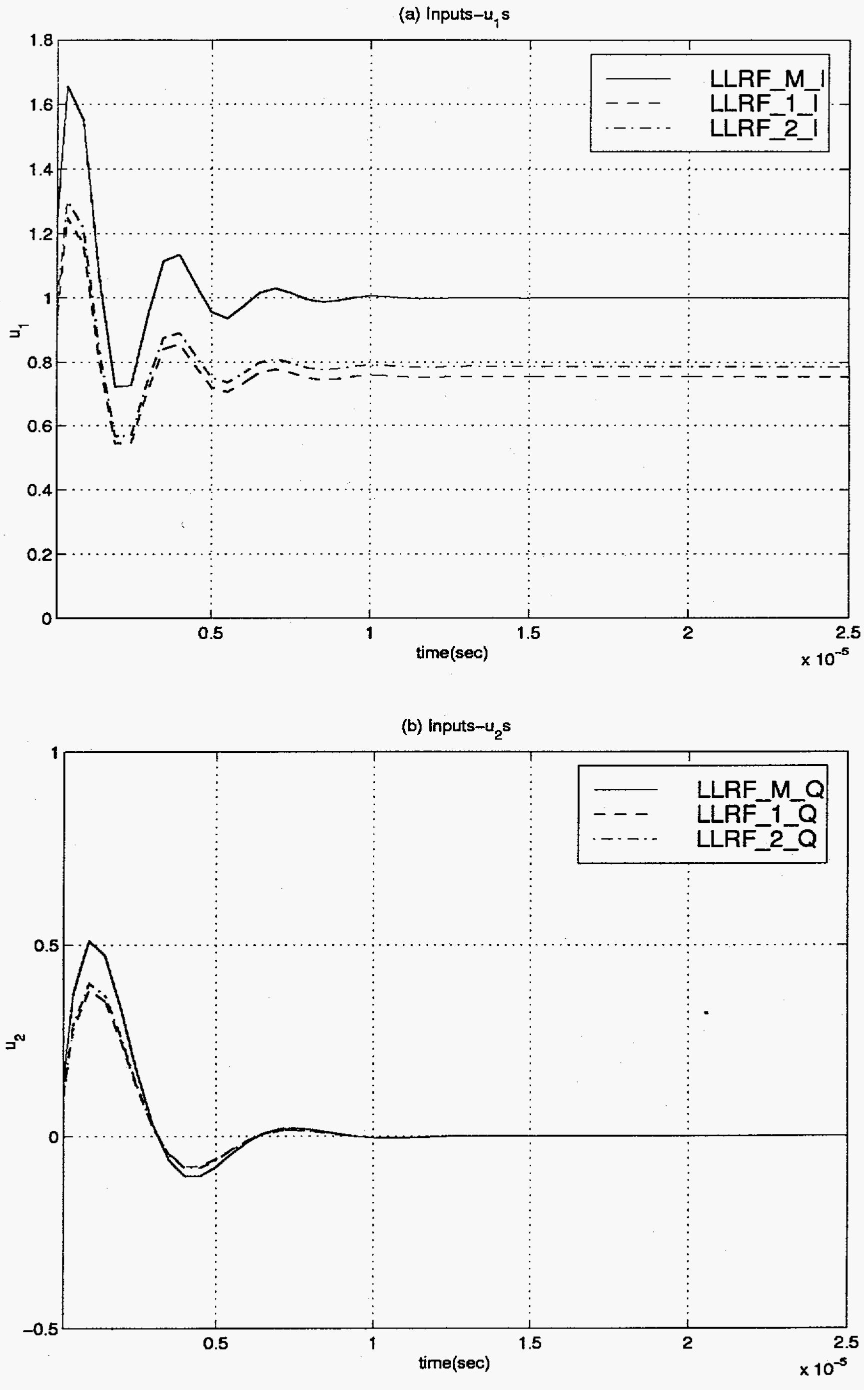

Figure 7: Inputs for three klystrons.

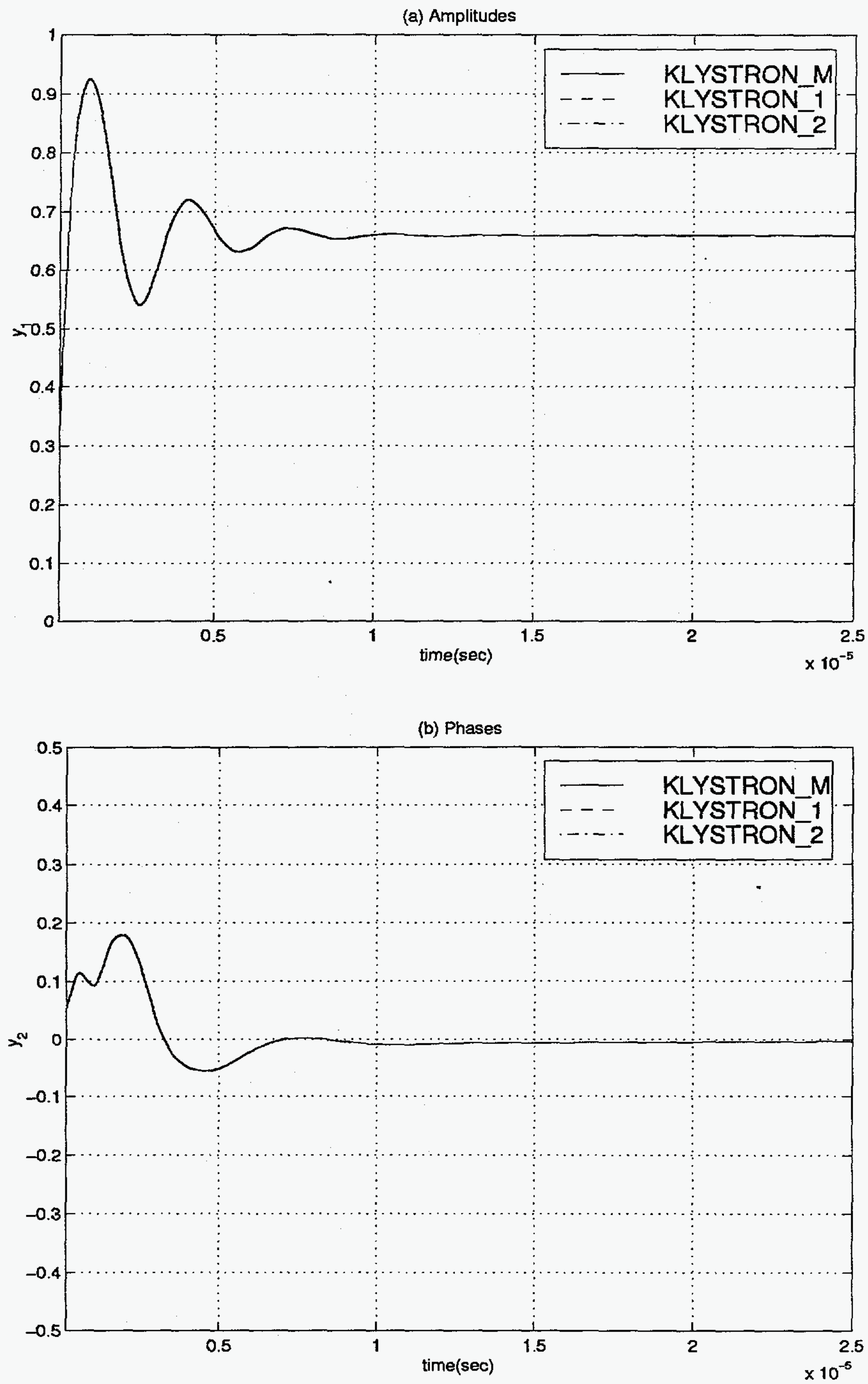

Figure 8: Amplitudes y_1 and Phases y_2 for three klystrons.

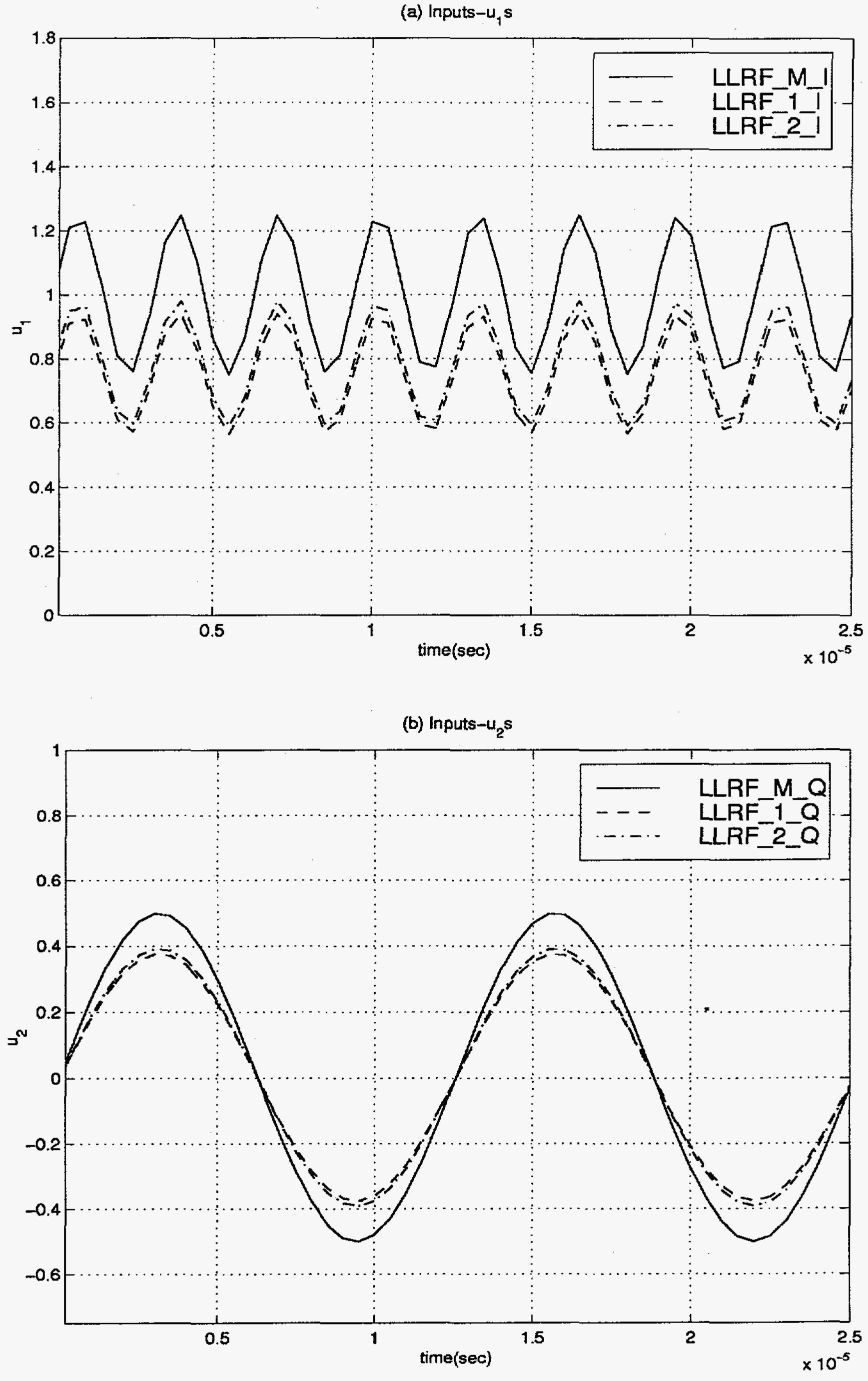

Figure 9: Inputs for three klystrons.

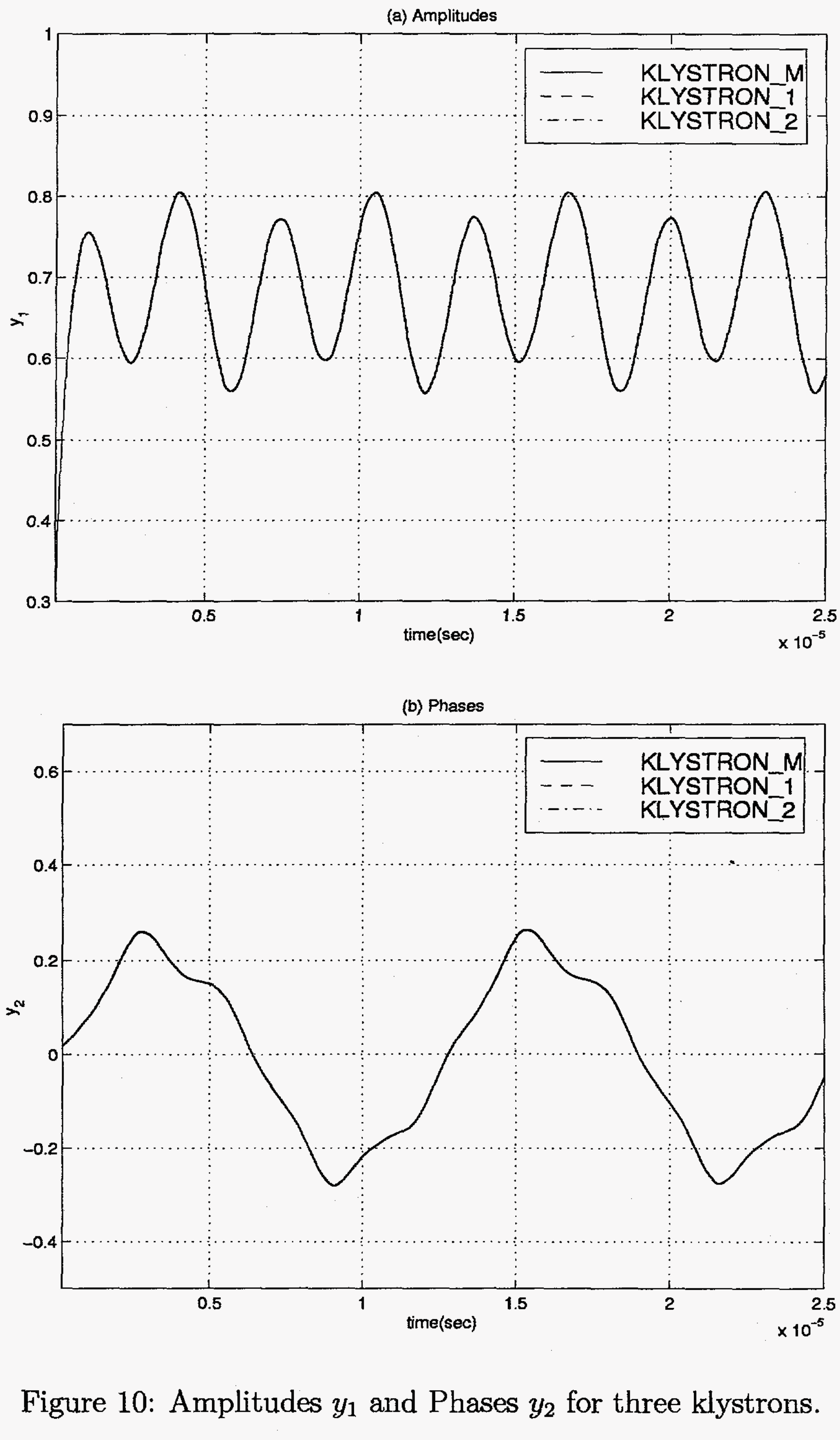

Figure 10: Amplitudes y_1 and Phases y_2 for three klystrons.