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Existence and Consequences of Coulomb Pairing of Electrons in a Solid

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Existence and consequences of Coulomb pairing of electrons in a solid

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Abstract

It is shown from first principles that, in the periodic potential of a crystalline solid, short-range (i.e., screened) binary Coulomb interactions can lead to a two-electron bound state. It is further suggested that these composite bosonic states (charge $-2e$, and typically spin zero) could mediate an effectively **attractive** interaction between pairs of conduction electrons close to the Fermi level.This necessarily short range attractive interaction, which is crucially dependent on the band structure of the solid, and is complementary to the phonon-mediated one, may provide a source for the existence and properties of short correlation-length electron pairs (analogous to but distinct from Cooper pairs) needed to understand high temperature superconductivity. Several distinctive and observable characteristics of the proposed pairing scheme are discussed.

I. INTRODUCTION AND MOTIVATION

The energy spectrum of free electrons is a continuous function of their momentum ('free electron parabola'). It is obvious and well-known that interaction via their mutual Coulomb repulsion, leads merely to scattering states; bound pair states are never formed. It is neither obvious nor well-known that, under certain well-defined conditions, a radically different conclusion holds for electrons moving in a periodic external potential.

The energy spectrum of free electrons in a periodic potential is disconnected, and consists of allowed and forbidden bands; the band gaps arise due to Bragg reflections. Slater *et al.* and Hubbard [1,2] have shown that two interacting electrons moving in a periodic external potential can form bound states with spin zero or one and charge $-2e$, even if the effective interaction is repulsive! This conclusion is a characteristic result of the wave nature of matter and does not apply to classical particles. It can be qualitatively explained by the fact that the Bloch spectrum implies negative effective masses for certain wave numbers (close to zone boundaries) and particles with negative effective masses subject to repulsive forces can form bound states. A deeper explanation may, however, lie in the fact that Bragg reflection actually constrains two electrons to be close even though they repel each other, provided they both have energies close to a zone boundary.

The purpose of this paper is to first provide a simple quantum mechanical calculation demonstrating that two electrons interacting through a short-range binary repulsion can indeed form a bound state in the periodic potential of a crystalline solid. The calculation is then extended to show that this interesting and relatively obscure phenomenon may persist in a many-electron system (a metallic solid, for example) where the Pauli principle places essential and nontrivial constraints on the bound state formation. We further suggest that a virtual exchange of these bound states (to be termed gap bosons, since they form in the Bloch band gaps, and are spin singlets in this work) could be instrumental in mediating a

new attractive effective interaction between real pairs of conduction electrons close to the Fermi level in much the same way that phonon or exciton exchange leads to Cooper pairing, as described in standard BCS theory.

II. A TWO-PARTICLE MODEL

We first exhibit a solution of the *two-electron* problem in one dimensional quantum mechanics which illustrates the fact that bound electron pairs can be formed in the presence of a mutual *repulsive* interaction when the constituents move in a periodic external potential. We adopt a simple and direct approach which is quite distinct from those of earlier workers [1,2]. This problem, while exactly soluble in principle, is of course not strictly relevant to a real solid which has a very large number of electrons. We shall also discuss the latter case and show that under certain well-defined conditions, a very similar result applies.

Consider two electrons a, b interacting with each other via a repulsive potential $V_r(|x_a - x_b|)$ in the presence of an external periodic potential $V_p(x+d) = V_p(x)$. The time-independent Schrödinger equation, describing their motion is,

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x_a^2} - \frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x_b^2} + [V_p(x_a) + V_p(x_b) + V_r(|\, x_a - x_b\,|)]\,\Psi = E\Psi(x_a, x_b). \qquad (1)$$

In the absence of either V_p or V_r, this equation may be solved to yield non-square integrable solutions corresponding respectively to 'Bloch states' ($V_r = 0$) or 'scattering states' ($V_p = 0$). For a short-ranged (e.g., screened Coulomb) repulsive potential V_r, we find that Eq. (1) admits solutions that are Bloch-like $\simeq e^{iKx}\,W(x,y)$ in the center of mass coordinate $x = (x_a + x_b)/2$ [$W(x,y)$ is periodic in x with the lattice period d], and localized in the relative coordinate $y = x_a - x_b$. These eigensolutions are nondegenerate and are labelled by K, the center-of-mass 'wavenumber'. Thus the wave function $\Psi(x_a, x_b, K)$ has a characteristic energy $E(K)$. The proof applies *mutatis mutandis* to any reasonable periodic potential and to any purely repulsive potential which is sufficiently short-ranged.

In the following, $\epsilon(k,n)$ is the eigenenergy. For transparency of notation we use the reduced zone scheme in which k is the reduced zone wavenumber $-\pi/d \leq k \leq \pi/d$, and n is the band index. In the rest of this paper we will use ξ as a composite symbol for k and n with $\int_\xi \equiv \sum_{n=1}^{\infty} \int_{-\pi/d}^{\pi/d}(dk/2\pi)$ denoting integration over the allowed bands. The total (spin plus orbital) wavefunction must be antisymmetric. Since the potentials (by assumption) are independent of spin, we may look for spin zero (singlets) or spin one (triplet) solutions. For simplicity, we discuss the singlet case, corresponding to spatially *symmetric* wave functions. Unless otherwise stated, in the following the label Ψ refers to the symmetric spatial orbital. This orbital wave function is expanded in terms of symmetrized products of one-electron Bloch states, $e^{ikx}U_k(x)$ and satisfies, when substituted in Eq. (1), an integral equation for the amplitude $\widehat{\Psi}$.

The solution is of the form $\Psi(\widehat{\xi_a,\xi_b}) = \delta(K-k_a-k_b)F(k_a,K-k_a,n_a,n_b)$ with F satisfying

$$[E-\epsilon(\xi_a)-\epsilon(\xi_b)]\,F(k_a,n_a,n_b) = (2\pi d)^{-1}\sum_{n_a'n_b'}\int \widehat{P}\,F(k_a',n_a',n_b')dk_a'. \tag{2}$$

These spatially symmetric solutions are even in $y \equiv x_a - x_b$. The kernel $\widehat{P}$ of the equation is obtained straightforwardly by transforming to the center of mass x and the relative coordinate y [$x_a = x+y/2, x_b = x-y/2$, $dx_a\,dx_b \equiv dx\,dy$], exploiting the periodicity of the Bloch functions, and using the standard identity $\sum_{n=-\infty}^{n=+\infty}\exp[in\theta] = 2\pi\delta(\theta)$ for $-\pi \leq \theta \leq \pi$:

$$P = (2\pi/d)\delta(k_a'+k_b'-k_a-k_b)\int dy V_r(|y|)\exp[-iy(k_a'-k_a)]\int_{-d/2}^{d/2} dx\,\Delta(x+y/2)\Delta(x-y/2) \tag{3}$$

where $\Delta_{a(b)} \equiv [U_k'(x)\,U_k^*(x)]_{a(b)}$ are periodic in $x_{a(b)}$, and the double integral is $\widehat{P}$.

We simplify Equation (3) [essentially equivalent to Eq. (1)] to elucidate the basic physics of Coulomb pair-formation by putting Bloch functions equal to unity, by modelling the repulsive potential by $V_r = V_0 d\delta(y)$ leading to $\widehat{P} = V_0 d^2$, and by restricting the k integration

only to the two relevant bands $n = 1, 2$. Defining new dimensionless variables $k = kd/2\pi$, $K = Kd/2\pi(-1/2 \leq k, K < 1/2)$, $u = k - K/2$, and remembering the constraints on k'_a integration, Eq. (3) leads to the integral equation/dispersion relation,

$$\frac{1}{2V_0} = \sum_{i,j} \int_0^{(1-K)/2} \frac{du}{E - \epsilon_i(u - K/2) - \epsilon_j(u + K/2)}. \tag{4}$$

This equation is clearly a generalisation of Hubbard's [2]. Given the single-particle energy spectrum (i.e., $\epsilon(k)$) and the interaction potential V_0, this equation is readily solved numerically to give the energy $E(K)$ and the two-particle wave function in terms of the relative coordinate y. It admits of course various 'scattering states' which are not square integrable in y. We shall be more interested in the bound states. Even without solving Eq. (4), it is evident from the nature of the energy denominators that when E lies in the two-particle gaps caused by non overlapping Bloch bandenergies, the equation has nontrivial solutions which correspond to bound states of the two electrons. It can be seen from elementary arguments that the pair wave function in this case is localized in y (i.e., $\int_{-\infty}^{+\infty} |\Psi(x, y)|^2 dy < |C|^2$).

We present as an explicit example, numerical solutions based on the one-particle dispersion relation $\cos 2\pi\epsilon^{1/2} + (\sin 2\pi\epsilon^{1/2}/2\pi\epsilon^{1/2})Q_0 = \cos 2\pi k$ corresponding to the well-known Kronig-Penney potential $V_p(x) = Q_0(\hbar^2/md) \sum_{n=-\infty}^{n=\infty} \delta(x - nd)$, $Q_0 = (md/\hbar^2)Qb$, where $Q(b)$ is the strength (range) of the potential. Here we show a typical strong potential ($Q_0 = 5$) example for which the gap size is comparable to the band width. In Fig. 1, we plot the functions $\epsilon_1(k)$ [single-particle 'valence' band], $\epsilon_2(k)$ [single-particle 'conduction' band] and $E(K)$ for $V_0 = 0.5$ (a typical value when the screened Coulomb potential is approximated by a delta function). The precise location of the energy eigenvalue $E(K)$ will depend on the strength of the repulsive potential V_0. The plot of $E(K)$ as a function of K shows the minimum pair energy occurs at $K = 1/2$ and the slow K variation (high effective mass) of the pair energy for the present conditions. In Fig. 2 we display a plot of the relative probability density $|\Psi(x, y)|^2/|\Psi(x, 0)|^2 = \rho(y)$ as a function of y (measured in d) for the

most tightly bound $K = 1/2$ state. The probability density falls off rapidly implying a short coherence length $\xi \simeq d$, the lattice period. The pair size remains between $1 - 10d$ for all reasonable values of Q_0 and V_0.

The crucial results of the preceding calculation can now be summarized:

1. In a periodic external potential, two electrons interacting through a short-ranged repulsive potential can be 'bound' to form a spin zero compound boson with charge $-2e$ and a spatial extent of the order of a lattice length. This result is remarkable in that each of the two potentials by themselves cannot lead to states localized in the *relative* coordinate (but Bloch-like in the centre of mass coordinate).

2. An essential requirement for solutions of this type to exist is the disconnected nature of the single-particle energy spectrum (i.e, the existence of distinct band gaps).

3. Typically, the two-particle energy $E(K)$ is a continuous function of the crystal momentum K of the pair, and forms a band with higher energies than would be the case if both constituents had energies in the valence (i.e., lower) band. Thus these pair states possess higher energies and are 'excited' relative to the ground state of the two-particle system. This is perfectly understandable since the repulsive interaction can only ever increase the energy of a pair relative to the unperturbed system.

4. The simplified analysis presented did not take account of *umklapp* processes. These can be easily included rigorously (at the expense of some algebraic complexity). The integral equation is replaced by a coupled system of such equations, and a numerical solution yields the same qualitative results.

III. MANY-BODY EXTENSION

We now proceed to investigate if the two-electron bound state survives in a many-body environment. Let us consider a solid in which the filled shells of the periodic array of ion

cores can be treated as a 'rigid' external background. The valence electrons then obey, in the usual Hartree-Fock self-consistent approximation, the standard Bloch-Wilson single-particle equations. As a result we obtain the standard Bloch spectrum and the associated wavefunctions. Measuring energies from the lowest of these Bloch levels, we may form the associated Slater determinant for the ground state from the single particle Bloch states in the usual manner. We shall assume that the lowest band is not totally filled (i.e., the substance is a metal) and that the next higher band is empty at zero temperature. As in the two-electron problem, we shall assume that there is a definite gap between these two bands (i.e., they are non overlapping).

We start with the above system at zero temperature and examine the effect of a short range binary repulsion between the electrons. The repulsive potential is assumed for simplicity to be of the same form as in Section 2. We adopt the standard [3] (in particular, see Section 7-1 of this reference) t-matrix approach based on Green's functions. In the ladder approximation, two-particle bound states are predicted in this approximation when the following integral equation is satisfied:

$$\frac{1}{2V_0} = \sum_{i,j} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{[1 - f(\epsilon_i(\mathbf{k})) - f(\epsilon_j(\mathbf{K} - k))]}{[E - \epsilon_i(\mathbf{k}) - \epsilon_j(\mathbf{K} - \mathbf{k})]}. \tag{5}$$

Here the summation is over the two lowest bands considered. $\mathbf{K}$ is the centre-of-mass crystal wave vector. The Fermi occupation factor, $f(x) = 1$ for $x < \epsilon_{\text{Fermi}}$, and zero otherwise. By assumption, $\epsilon_{\text{Fermi}} < \epsilon_1^{\text{Max}} < \epsilon_2^{\text{Min}}$, where, $\epsilon_1^{\text{Max}}, \epsilon_2^{\text{Min}}$ are the maximum and minimum energies of the lower and upper bands respectively.

The integral in Eq. (5) arises from $\int \frac{d\mathbf{k}}{(2\pi)^3} G_{0i}(\mathbf{k}) G_{0j}(\mathbf{K} - \mathbf{k})$, where $G_{0i}(\mathbf{k})$ is the Bloch electron propagator:

$$G_{0i}(\mathbf{k}) = \frac{1}{k_0 - \epsilon_i(\mathbf{k}) + i\eta}. \tag{6}$$

The infinitesimal η is positive if $k > k_{\text{Fermi}}$ and negative otherwise [3]. For factorizable potentials the t-matrix equation can be solved exactly by evaluating the integral over the

product of the Bloch propagators. This is done by using residue calculus (as in [3]) making use of the pole prescription. The Fermi occupation factors in Eq. (5) arise essentially as a result of this evaluation.

Indeed, Eq. (5) is exactly equivalent to Eq. (7-6) of [3], upon making the appropriate changes of notation. Taking account of the fact that the single particle energies exhibit band-gap discontinuities, it is easily seen that two electron bound states *can* be found in the present case even though the potential is repulsive. This is in sharp contrast to the situation discussed in [3]. In addition, there will also be a contiuum of two-particle scattering states. These are are not localized in any sense and simply represent the ordinary Coulomb scattering of the two electrons.

The nature of the two-particle spectrum predicted by Eq. (5) is best understood by considering $\mathbf{K} = 0$ and using the fact that $\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k})$. The dispersion equation becomes,

$$\frac{1}{2V_0} = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{(1 - 2f\left[\epsilon_1(\mathbf{k})\right])}{\left[E - 2\epsilon_1(\mathbf{k})\right]} + I_{12} + I_{22} \tag{7}$$

where I_{12}, I_{22} are corresponding integrals involving energies from the upper band. The integral I_{11} can obviously be written in the form,

$$I_{11} = -\int_0^{\epsilon_{\text{Fermi}}} N^*(\epsilon) \frac{d\epsilon}{E - 2\epsilon} + \int_{\epsilon_{\text{Fermi}}}^{\epsilon_1^{\text{Max}}} N^*(\epsilon) \frac{d\epsilon}{E - 2\epsilon}. \tag{8}$$

The first, negative term (remembering that $E > 2\epsilon_1^{\text{Max}}$) arises from the occupied states in the lower band, whereas the second term is the positive contribution of the unoccupied states between the assumed Fermi level and the band maximum. This latter term, the only one responsible for giving a bound-state solution for positive V_0, can be made arbitrarily large and positive by choosing the pair energy sufficiently close to $2\epsilon_1^{\text{Max}}$. The integral I_{22} also has a simple expression:

$$I_{22} = -\int_{\epsilon_2^{\text{Min}}}^{\epsilon_2^{\text{Max}}} N^*(\epsilon) \frac{d\epsilon}{2\epsilon - E}. \tag{9}$$

It is seen that I_{22} is always negative and bounded, since $E < 2\epsilon_2^{\rm Min}$ for regular solutions of Eq. (7). In order to write the integral I_{12} in its most transparent form, we introduce the function, $\nu(\epsilon)$ defined by,

$$\nu(\epsilon) = \int \frac{d\mathbf{k}}{(2\pi)^3} \left[1 - f(\epsilon_1(\mathbf{k}))\right] \delta \left[2\epsilon - \epsilon_1(\mathbf{k}) - \epsilon_2(-\mathbf{k})\right]. \tag{10}$$

It then follows that we may write I_{12} in the form of a single integral over ϵ:

$$I_{12} = -\int \nu(\epsilon) \frac{d\epsilon}{2\epsilon - E}. \tag{11}$$

Note that ν is nonzero only when its argument varies between the range of variation of $(\epsilon_1(\mathbf{k}) + \epsilon_2(-\mathbf{k}))/2$. Naturally E has to be chosen below this range to keep the integral finite. Thus I_{12} is always negative and can be made arbitrarily large and negative. It is therefore evident that as E is varied over the allowed range (that is, the range over which all the I's remain finite) the right hand side of Eq. (5) varies from an arbitrarily large positive value to an arbitratrily large negative value. The positive values correspond to a bound state for a repulsive potential. Thus for a given positive (repulsive) V_0, Eq. (5) has a unique regular solution provided the band structure of the single particle problem allows the necessary gap or the forbidden energy range. This always happens in one dimension but need not for all(indeed any) wave vectors $\mathbf{K}, \mathbf{k}$ in higher dimensions. When the solution exists, it is clearly a two-particle bound state with localized wave function in the relative coordinate. These states will be referred to as 'gap bosons'.

It is therefore clear that gap bosons are not forbidden by the Pauli principle in the many-electron problem, provided the band structure has the required characteristics. However, in contrast to the two electron problem solved in the preceding section, the Pauli principle does 'block' two electrons from interaction if they are in the Fermi sea. This then implies that the lowest pair energy occurs for $\mathbf{K} = 0$ unlike the result for the two-electron problem. Indeed, Eq. (5) makes a straightforward experimental prediction: under suitable conditions, one

ought to be able to observe these pairs as *real* states by photo excitation of the ground state (Fermi sea). At finite temperatures, there will be a small but finite probability of thermally excited gap bosons, given by the Boltzmann factor, as in semi-conductors. It is perfectly clear however, that these excitations by themselves cannot form a Bose condensate and lead to macroscopically coherent behaviour. Like phonons, these composite bosons are not conserved in number and are in any event, excitations of the Fermi sea. For this reason, we believe that these gap boson excitations do not correspond to the real space pairs envisaged by Schafroth and others as capable of leading to superconductivity via the Bose-Einstein condensation mechanism [4].

IV. ELECTRON PAIRING AT THE FERMI LEVEL: THE GAP BOSON MECHANISM

We now propose a model of electron pairing at the Fermi level mediated by the gap bosons. The mechanism is quite analogous to the phonon pairing of BCS theory [3] but with the essential difference that the correlation length associated with this mechanism is likely to be short. Let us consider a material in which gap bosons can exist and have some significance. If one starts with an insulator at $T = 0$ and dopes it with electron accepting impurities (as is the case in La_2CuO_4 doped with Sr), the Fermi level moves down slightly into the lower (i.e., 'valence' band), the gap boson states can be regarded as virtual bosons which can affect the interaction of the electrons at the Fermi level. We consider a model Hamiltonian analogous to the Fröhlich Hamiltonian:

$$H = \sum_{s,\mathbf{k}} \epsilon(\mathbf{k}) a_s^+(\mathbf{k}) a_s(\mathbf{k}) + \sum E(\mathbf{q}) b^+(\mathbf{q}) b(\mathbf{q}) + V_{int} \tag{12}$$

where a's are electron operators including spin The b's represent the bosonic excitations. The interaction term $V_{int} \equiv \sum W_{\mathbf{km}} a_{\sigma+}(\mathbf{k}) a_{\sigma-}(\mathbf{m}) b^+(\mathbf{k}+\mathbf{m})$+h.c., involves the matrix element for the basic electron-gap boson interaction. It gives the amplitude for two electrons at or

near the Fermi level to be scattered in to the gap boson state with energy E; this must be a virtual state since the gap boson energy $E > 2\epsilon_{\text{Fermi}}$. The model Hamiltonian does not involve the two-electron scattering states. These are already accounted for in the standard Hartree-Fock theory to some extent. In principle, they could contribute a repulsive Coulomb correlation energy (over and above the Hartree-Fock smoothed out potential) that which must be taken into account in the same way as is done in BCS theory.

One can now either use standard second order perturbation theory or a canonical transformation method [6] to write the effective (or reduced) Hamiltonian by eliminating the gap bosons. Since $E > 2\epsilon$, irrespective of the sign and the size of the interaction elements W, the effective interaction is attractive. The standard second-order perturbation theory [5] gives the reduced interaction for electrons with oppositely directed spins and momenta:

$$V_{\text{red}} = \sum \frac{W_{\mathbf{k},-\mathbf{k}} W^*_{\mathbf{l},-\mathbf{l}}}{2\epsilon_{\mathbf{k}} - E(0)} a^+(\mathbf{l}) a^+(-\mathbf{l}) a(\mathbf{k}) a(-\mathbf{k}) \tag{13}$$

where the spin indices are suppressed for clarity. Note that the initial and final energies are assumed same and only the lowest energy gap boson state is taken into account. This reduced interaction between electrons is always attractive (assuming, as in the phonon case [6] that the matrix elements are nearly equal). Indeed, the form is closely analogous to that obtained for the usual BCS Hamiltonian [5,6]. If one takes into account the phonons as well, the total interaction Hamiltonian would have the same form. The two interactions contribute to the total attractive (effective) interaction. The direct Coulomb repulsion between the electrons should not be included since the gap bosons are precisely the result of such forces. However, in principle, the two-electron scattering states could make a repulsive contribution (as in BCS theory). They are also reponsible for screening the effective Coulomb repulsive potential strength V_0 and the gap states themselves. It thus appears that phonons and gap bosons (if they exist) can provide a net effective attraction between electrons at the Fermi level and hence result in superconductivity.

Just as in the phonon case, the excitations responsible for pairing are *virtual* states. Unlike the phonons, however, the gap boson mediated pairing does not involve ionic motion (absence or weakened isotope effect) but does involve very short range forces, leading to extreme type II superconductivity behaviour. The gap boson mediated pairs are called 'Coulomb pairs'. They are analogous to Cooper pairs in the sense that an effective attraction is reponsible for the pairing mechanism. They must be distinguished from the gap bosons themselves. The latter result from the combined effects of band structure and binary Coulomb repulsion.

For the Hamiltonian proposed above, the calculation of the transition temperature and other thermodynamic properties are done exactly as in BCS theory. Thus, the energy gap $\Delta(0)$ at zero temperature can be estimated to be,

$$\Delta(0) \simeq (E(0) - 2\mu) \exp -1/N(0)V_{\text{eff}} \tag{14}$$

where, $N(0)$ is the density of states at the Fermi level μ at zero temperature and V_{eff} is the effective (attractive) matrix element tending to cause the gap boson mediated pairing. We note that from the general perturbation theory result that V_{eff} decreases is magnitude as the energy difference, $E(0) - 2\mu$ increases for a given interaction amplitude W. This energy replaces the Debye energy of the standard phonon-mediated BCS interaction. Its relatively large size may explain why gap boson mediated superconductivity, relying as it does on the strong Coulomb force, could be reponsible for higher transition temperatures than would be the case for purely phonon mediated superconductivity. Since the formation of gap bosons is essentially controlled by band gaps, it is expected that the energy gap Δ estimated above should be lattice dependent and anisotropic in general.

The structure of the effective (attractive) matrix element qualitatively explains also why there should be an optimal doping since the difference between the Fermi level and $E/2$ is controlled by the number of electron states available for pairing. The weak isotope effect and the relatively short coherence length as well as the sensitive dependence of the whole

mechanism to band structure (unlike phonon mediated pairing which can occur in principle in 'jellium') likewise seem natural to the mechanism suggested. A test of the model would be that superconductivity mediated by gap bosons should be lost whenever the external conditions are such as to lead to band overlap (eg. due to pressure). Materials close to the metal insulator transition would be expected to be ideal for observing gap bosons and their interactions.

In summary, the proposed model of gap boson mediated pairing seems to have qualitatively the sort of mechanism needed to account for certain types of superconductivity. Among many current theories [7-11] currently under study, there are very few which can account for the simple fact that high temperature superconductivity involves very short coherence lengths. From the Uncertainty Principle, this means that the forces involved must be relatively strong and must certainly involve the Coulomb interaction and exchange interaction of paired spins in an essential way. However, any such model must avoid the paradox that pure repulsion leads to pairing in real space and not merely in momentum space, as in the phonon case. The gap boson pairing mechanism seems capable of resolving this issue as well as others, such as the relatively weak or nondetectable isotope effect in high T_c materials. The analysis presented makes straightforward predictions (albeit qualitative at this point) which can be tested by experiment(e.g., as in [12]). The model presented is a possible 'electronic' mechanism [13] which can coexist with, and in principle enhance a phonon mechanism, although the latter is not strictly required. The dependence of the gap on the details of the band structure may account for the observed symmetries of the wave function, although this is an open question at this time.

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Figure Captions

[1] Single particle energies, $\epsilon_1(k)$ (valence band), $\epsilon_2(k)$ (conduction band) and pair energy (ie solution of the dispersion relation, Eq.(4)) $E(K)$ for $Q_0 = 5, U_0 = 0.5$.

[2] The Coulomb pair coherence function $\rho(y)$ (defined in the text) as a function of the relative distance y for $Q_0 = 5, V_0 = 0.5$, and $K = 0.5$.

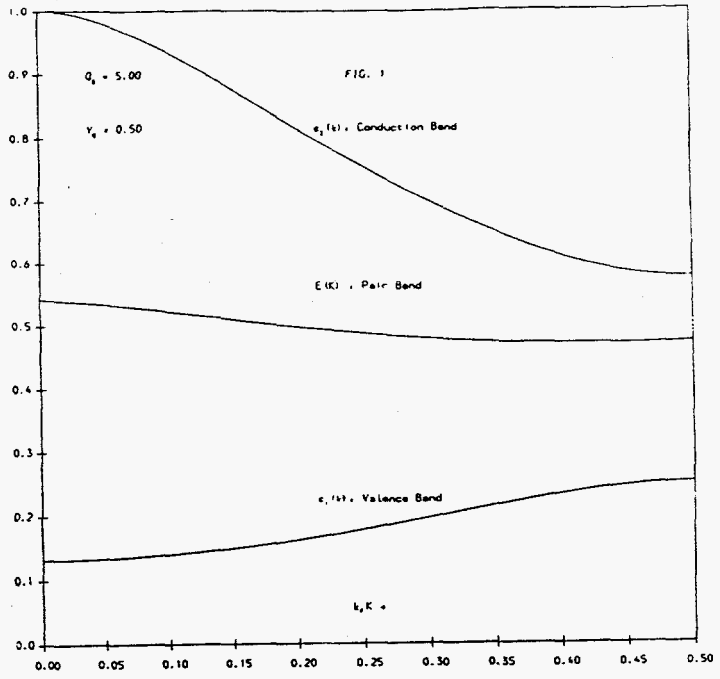
FIG. 1
Q_0 = 5.00
V_0 = 0.50
$\epsilon_1(k)$, Conduction Band
E(K) , Pair Band
$\epsilon_1(k)$, Valence Band
k, K →
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50

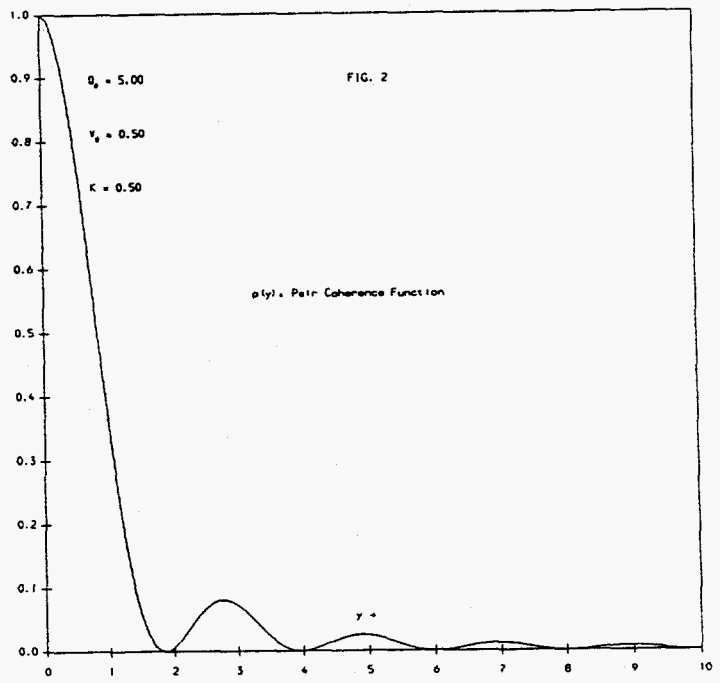
FIG. 2
Q_0 = 5.00
V_0 = 0.50
K = 0.50
ρ(y), Pair Coherence Function
y →
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
9
10