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How close are hyperdeformed states to the scission point?

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Abstract

The HFB method with the Gogny force is used to study the effects of reflection asymmetry at $I = 0\hbar$ on the barriers separating superdeformed and hyperdeformed minima from fission in the ^{176} W and ^{168} Yb nuclei. The fission barrier for the HD minimum is reduced by 5 MeV in ^{176} W when reflection asymmetry is taken into account.

Since the discovery in 1986 of the first superdeformed band in ^{152} Dy [1], more than a hundred and fifty superdeformed bands have been observed [2]. The reason for the existence of superdeformed bands are the strong shell effects showing up for prolate shapes with axis ratios near 2:1. One can understand this very easily by looking at the behavior of the energy levels of a deformed harmonic oscillator as a function of deformation (see for instance [3]): they bunch together when the ratio of the harmonic oscillator frecuencies, $\omega_\perp$:ω_z is a rational number giving rise to new energy gaps that favor deformation. For instance, the ratio 2:1 favors superdeformed (SD) prolate shapes. A 3:1 ratio also yields a bunching of levels that favors very extended shapes, the so-called hyperdeformed (HD) shapes. Therefore, one would expect that if many SD bands have been observed there would be also experimental evidence for HD bands but unfortunately such is not the case up to now. One may argue that the preceding argument is based on a pure harmonic oscillator and the spin-orbit coupling might wash out the shell effects giving rise to hyperdeformation. However, there are calculations with the Woods-Saxon potential [4] indicating that the strong shell effects giving rise to the HD states still remain. Realistic Woods-Saxon plus Strutinsky calculations predict HD minima in several rare earth nuclei that usually become yrast at $I \sim 70 - 90\hbar$. The question now is whether such minima could be populated in heavy-ion induced reactions, since the fission barriers are estimated [5] to vanish at spins around $75 - 85\hbar$ for rare earth nuclei. In the work of ref. [4] HD minima are obtained for several Yb, Er and Hf isotopes. Among them, the best candidates are ^{168} Yb, ^{166} Er and ^{170} Hf. In these nuclei the HD minimum ($\beta_2 \sim 0.9$) becomes yrast at spin $80\hbar$, a value which is compatible with the presence of a fission barrier thus favoring the population of such states. More recently, using a different

parameterization of the shape of the nucleus that includes a necking degree of freedom, extensive studies have been carried out in the $A \sim 180$ region with the Strutinsky method [6] looking for very extended shapes beyond SD. In this calculation several HD minima with axis ratios $(\langle z^2 \rangle / \langle x^2 \rangle)^{1/2}$ of the order of 2.2 and higher are found at high spins. For some isotopes they become yrast already at $I = 62\hbar$ as is the case for ^{182} Os. The reliability of the shape parametrization used in [6] has been assessed by more fundamental Cranked HFB calculations with the Gogny force for the test case of ^{182} Os [7]. In these studies the octupole degree of freedom, i.e. reflection asymmetric shapes, were not taken into account. However, there are studies in the $A \sim 180$[8] at I=0, indicating that octupole deformation plays an important role in stabilizing hyperdeformed minima found in this region. When we extended our high spin studies [6] to include the octupole degree of freedom, we found [9] a substantial lowering of the energy of the nuclear surface in the vicinity of the fission barrier. The most significant one is that of ^{176}W at $I = 70\hbar$ where a lowering of ~ 7 MeV is found for shapes with axis ratios of 3.7:1. The significance of this results becomes clear if one takes into account that in the absence of reflection asymmetry ^{176} W had a HD minimum (axis ratio 2.2:1) which became yrast at $I = 70\hbar$ and had an outer barrier (i.e. the barrier to fission) 6.3 MeV high, making it a possible candidate for an experimental search [6]. The lowering in energy due to the octupole degree of freedom means that the outer barrier may be substantially lowered in all these nuclei making it less likely that HD states will be observed. Our [9] study did not indicate that this was a serious problem in ^{182} Os.

The most important shortcoming of the Strutinsky method is that the shape of the nucleus has to be characterized in terms of a few parameters. This is not a serious drawback for moderately deformed nuclei, but in the region of very extended shapes in the vicinity of fission, it can be a problem. Therefore, it is particularly desirable here to compare the Strutinsky calculations with HFB studies, in which such shape parameterization restrictions are not present. With this in mind we decided to carry out HFB calculations for ^{176} W using the Gogny force. Our objectives are to determine if this large lowering in energy is an artifact of the Strutinsky method and also to consider the impact of the onset of fission on the possibility of populating very extended shapes. It is important to consider fission, as the shapes in which this effect is manifest have surface areas that are close to those of two separated spherical fragments.

Because we wish to study shapes that are extremely elongated, shapes extended to the point that fission occurs, we must use a very large oscillator basis in our HFB calculation (with shells in the z-direction of the order of 30). In principle, one would like to study hyperdeformation at high spins allowing for the possibility of asymmetric shapes. The large basis space needed in this case as well as the number of operators involved in the relevant constraints make such calculations extremely time-consuming, even with present day computers [10]. Therefore we restrict ourselves to *axially symmetric*, i.e. $I = 0\hbar$, calculations. In order to perform calculations in a large basis space a new code has been written. The need for the new code arose from instabilities in the standard calculation of the matrix elements of the force for large values of the harmonic oscillator quantum numbers of the basis - see [13] for details. This code fully implements the HFB method with the Gogny force for axially symmetric systems; including shapes that are reflection asymmetric. The flexibility of the gradient method used in the solution of the HFB equations allows us to handle many constraints such as any multipole operator $\hat{Q}_{\lambda 0}$ or the asymmetric necking [11] operator

$\hat{Q}_{nk} = \exp(-(\hat{z} - z_0)^2/a_0^2)$. In the calculations the DS1 parameter set of the Gogny force has been used. This set of parameters was fitted to yield a lower surface coefficient a_s in seminfinite nuclear matter; giving theoretical fission barriers for ^{240} Pu and other actinides in very good agreement with the experimental data [12]. Therefore we believe that this set is well suited for the study of very extended shapes.

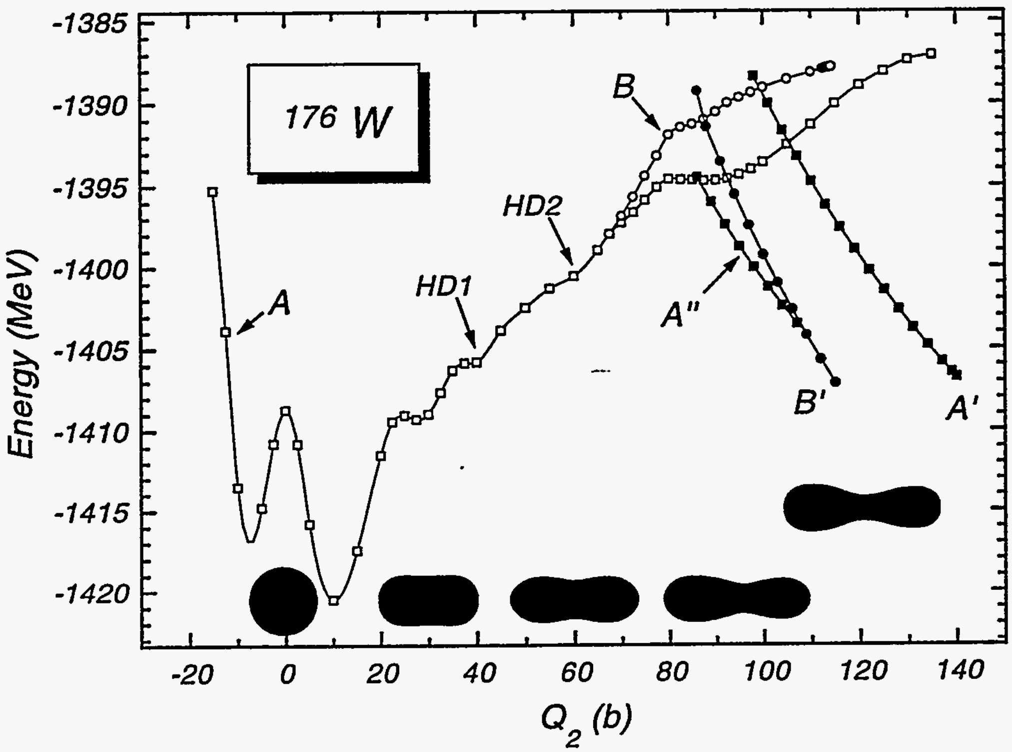

Figure 1: The HFB energy in MeV versus the mass quadrupole moment Q_2 given in barns. The open squares curve (A) stands for the unconstrained calculation. The open circles curve (B) correspond to the reflection symmetric ($Q_3 = 0$) calculation. The two-fragment solutions are represented by full squares (A' and A") and full circles (B') corresponding to reflection asymmetric and reflection symmetric solutions respectively. In the bottom part of the figure the shape of the nucleus for the unconstrained calculation (defined as the isosurface at $\rho_0 = 0.08\text{fm}^{-3}$) is depicted for several values of Q_2 (from left to right $Q_2 = 0b$, $30b$, $60b$, $95b$ and $120b$).

We have performed HFB calculations in the ^{176} W and ^{168} Yb nuclei using the mass quadrupole moment $Q_2 = z^2 - \frac{1}{2}(x^2 + y^2)$ as the main constraint. In Fig. 1 we present the major results for the nuclide ^{176} W. Five different curves are displayed that correspond to different physical situations: The open squares curve labeled A is obtained by constraining the quadrupole moment of the nucleus. The open circles curve (labeled B) is obtained making the additional constraint of having reflection symmetric (i.e. $Q_3 = 0$) shapes. The curves labeled A' and A" (full squares) correspond to reflection asymmetric *two-fragment* solutions while the one labeled B' is for the reflection symmetric one. Curve A present three minima at $Q_2 \sim -7.5b$, $Q_2 \sim 10b$ and $Q_2 \sim 28b$ corresponding to an oblate solution, the ground state ($\beta_2 = 0.31$ [1]) and a superdeformed state ($\beta_2 = 0.73$), respectively. The three minima are reflection symmetric. In addition to these minima there are also two shoulders in the energy curve for Q_2 values of $40b$ and $60b$. The one at $Q_2 = 40b$ (labeled as HD1 in Fig. 1) has an axis ratio of 2.2 that roughly correspond to the one of the HD solution becoming yrast at $I = 70\hbar$ in the Strutinsky calculation of ref. [6]. The shoulder at $Q_2 = 60b$ (labeled as HD2) has an axis ratio of 2.8 that could be associated to the second HD minimum seeing in the calculation of ref. [6], the one that becomes yrast at $I = 76\hbar$. For Q_2 values higher than $70b$ the energy levels off and is lower than the one of curve B indicating that for this range of Q_2 the system becomes reflection asymmetric (with β_3 values in the range of 0 to 0.3). Comparing curves A and B we observe that the maximum energy gain due to reflection asymmetry is 4.66 MeV and correspond to $Q_2 = 95b$. This result is in good agreement with the lowering of 6.2 MeV obtained for this nucleus at $I = 0\hbar$ [9]. To better compare our results with the ones of [9] one first has to define the concepts of elongation η_2 and necking in η_{nk} shape parameters for the HFB results where the density is not a sharp one. To this end, we define the HFB shape as the isosurface corresponding to roughly half density, i.e. $\rho = 0.08 fm^{-3}$. The region with Q_2 larger than $70b$ (where reflection asymmetry is important) has η_2 values in the range 1.0 to 1.3 and η_{nk} ones in the range -0.14 to -0.3 in good agreement with the Strutinsky results [9]. The agreement of our results with those obtained with the Strutinsky method [9] for ^{176}W indicate that the strong octupole effects seen in the $A \sim 176$ region for very extended shapes are genuine effects and not artifacts of the Strutinsky method. The implication is then that the outer barriers observed in that region are going to be strongly suppressed, making it less likely that very extended shapes in those nuclides can be populated.

The end points of curves A and B correspond to configurations which are no longer stable against fission: increasing the quadrupole moment slightly results in solutions with two fragments lying on curves A' and B' respectively. The curve B' corresponds to symmetric fission (two ^{88}Rb nuclei) while the curve A' is for a mass asymmetric split (^{66}Ni and ^{110}Pd). These two end points are saddle points of the corresponding fission paths. The energy differences between the HD minimum HD1 and these saddle points are 19 MeV and 21 MeV, respectively.

The octupole moments of the constrained solutions of curve A' are very similar to the ones of curve A for the same values of the quadrupole moment but these solutions differ in their hexadecapole moments. The ones of curve A' are typically about $20b^2$ lower than the ones of curve A. Therefore it is possible to reach the two-fragment curve A' from the one-fragment one (A) by constraining the hexadecapole moment. The corresponding energy curve shows a maximum that corresponds to another saddle point before fission. This saddle point is 20 MeV above the HD1 minimum. On the other hand, curve A" correspond to a very asymmetric mass split of ^{50}Ti and ^{126}Te. The octupole deformation of the two fragments as a whole is rather high with β_3 values of the order of 0.7. Therefore, we can reach curve A" from curve A by constraining the octupole moment. The energy difference between the maximum of this octupole constrained curve (that corresponds to the scission point) and

[1]The definition is $\beta_2 = \sqrt{20\pi} Q_2 / (5\langle r^2 \rangle)$

the HD1 minimum (i.e. the fission barrier of HD1 along the very mass asymmetric fission chanel) is only 15 MeV, that is, around 5 MeV lower than the other fission barriers mentioned before. The effect of reflection asymmetry is, therefore, to reduce the fission barrier as was previously suggested. A more detailed analysis of these solutions and their properties can be found in [13].

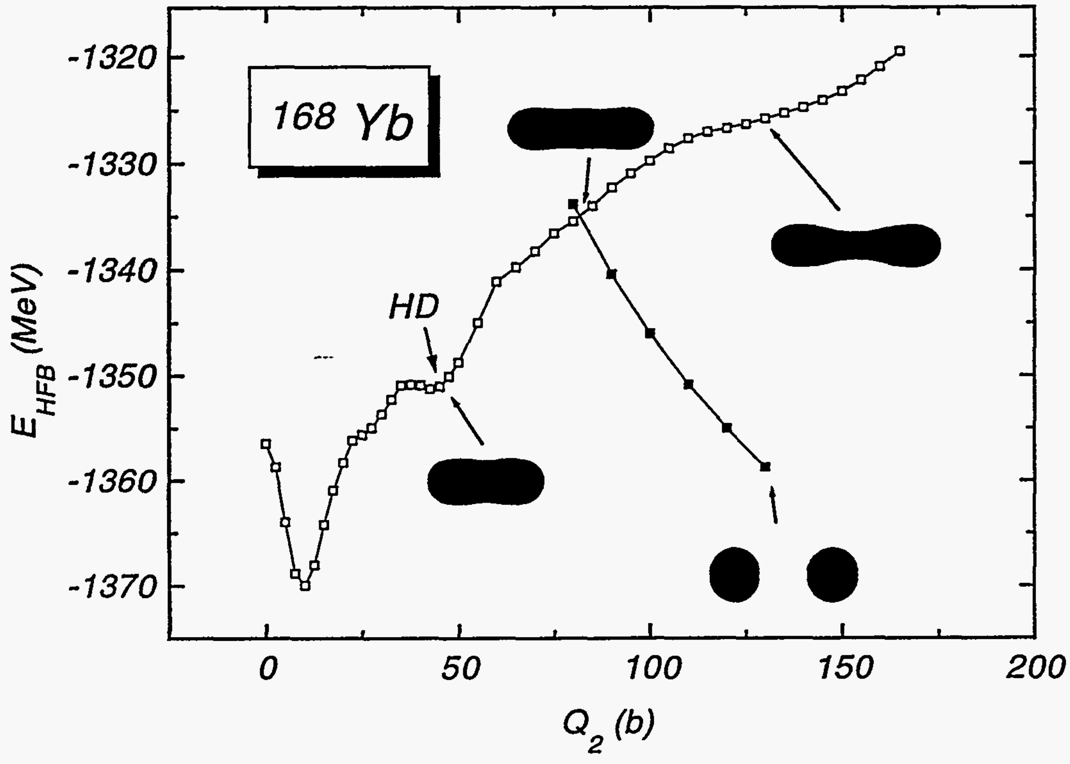

Figure 2: The HFB energy in MeV versus the mass quadrupole moment Q_2 given in barns for ^{168} Yb. The open squares curve correspond to the one-fragment solutions while the filled squares curve is for the reflection symmetric two-fragment solution. The shape of the nucleus as defined in Fig. 1 is also depicted for some values of the quadrupole moment.

In Fig.2 we have plotted the energy as a function of the mass quadrupole moment for ^{168} Yb. The most important result of this calculation is that in the whole range of Q_2 considered the nucleus remains reflection symmetric. In addition to the prolate ground state minimum located at $Q_2 = 10b$ there is another, very shallow, minimum at $Q_2 = 45b$ with $\beta_2 = 1.01$, $\beta_4 = 0.61$ and an axis ratio of 2.5. The shape parameters of this minimum agree quite nicely with the ones of the HD minimum found by Dudek et al. [4] at high spins. In the same plot we also represent the reflection symmetric two-fragment solution (full squares). This solution has Q_4 values much lowers than the one fragment solution in contrast to ^{176} W. For instance, in the crossing point at $Q_2 = 80b$ the values for Q_4 are $62b^2$ and $34b^2$ for the one-fragment and two-fragment solutions, respectively. This means that we can force symmetric fission by constraining in Q_4. The maximum of this energy curve corresponds to the scission point and is located $\sim 25 MeV$ higher in energy than the HD minimum found in our calculation. This barrier energy is 10 MeV larger than in the ^{176} W case. If the HD minima were to become yrast at the same spins, we would estimate that it is more likely to populate the HD minimum of ^{168} Yb than that of ^{176} W .

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