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A Method for Generating Skewed Random Numbers Using Two Overlapping Uniform Distributions

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ABSTRACT

The objective of this work was to implement and evaluate a method for generating skewed random numbers using a combination of uniform random numbers. The method provides a simple and accurate way of generating skewed random numbers from the specified first three moments without an *a priori* specification of the probability density function. We describe the procedure for generating skewed random numbers from uniform random numbers, and show that it accurately produces random numbers with the desired first three moments over a range of skewness values. We also show that in the limit of zero skewness, the distribution of random numbers is an accurate approximation to the Gaussian probability density function. Future work will use this method to provide skewed random numbers for a Langevin equation model for diffusion in skewed turbulence.

CONTENTS

1. INTRODUCTION

The motivation for this work on skewed random number generation is the study of the dispersion of pollutants in the convective boundary layer, CBL. The CBL is the daytime, unstably-stratified atmospheric boundary layer, and typically extends from the ground surface to a height of 1 km. Pollutant dispersion within the CBL is qualitatively different from that within a stably- or neutrally-stratified boundary layer, where dispersion can be described by standard Gaussian (unskewed) turbulence models.

During the last two decades, a clear understanding has emerged of the complex processes involved in the time-averaged vertical dispersion of a pollutant in the CBL. For an elevated source, the height of maximum concentration of pollutant exhibits the behavior of approaching the ground surface with increasing time and downwind distance. For a source at the ground surface, the height of maximum concentration increases with downwind distance. The initial understanding of these behaviors emerged from a combination of laboratory (Willis and Deardorff; 1976, 1978, 1981) and numerical studies (Lamb; 1978a, 1978b, 1982). Confirmation of these phenomena in the atmosphere has been found more recently in a field study by Briggs (1993).

These behaviors cannot be explained by the first moment (mean) of the vertical wind velocity, $\overline{w}$, which is typically zero, nor by the second moment, $\overline{w^2}$. However, they can be explained by the third moment, $\overline{w^3}$. In the CBL, $\overline{w^3} > 0$ and correspondingly the probability distribution of vertical velocity, $P(w)$, is positively skewed, where skewness is defined as $S = \overline{w^3}/\left(\overline{w^2}\right)^{3/2}$. Positive skewness in the vertical wind velocity is the result of strong solar heating of the ground surface, e.g., during cloudless midday conditions. This heating generates strong updrafts or thermals ($w > 0$) over approximately 40% of the horizontal area, on average, and weak downdrafts ($w < 0$) over the remaining 60%. Consequently, a plume from an elevated source has a higher probability of encountering a downdraft, so the locus of maximum concentration decreases in height with downwind distance. For near-surface releases, plumes travel horizontally near the ground surface until they become incorporated in an updraft, causing an increase in height of the maximum concentration.

The impact of these phenomena on the ground level concentration is of great importance, especially for elevated sources. Significant underprediction, by as much as a factor of 2.9 (Briggs, 1993), can occur if these processes are not taken into account in atmospheric dispersion models. Since accurate prediction of ground-level concentrations are critical to atmospheric dispersion modeling applications such as environmental impact assessment, safety analysis, and emergency response, these effects need to be included.

Atmospheric dispersion models based on the Langevin equation have been developed which attempt to simulate turbulent dispersion in the skewed turbulence of the CBL. The most successful of these models have used a Gaussian (unskewed) random force (e.g., Thomson, 1987; Luhar & Britter, 1989; and Weil, 1990). We are developing a Langevin equation model that uses a skewed random force, but which overcomes limitations of existing skewed-random-force models. To implement such a model, a method of generating skewed random numbers from the specified first three moments is required. Such a method is described and tested below.

2. THE DOUBLE-BLOCK DISTRIBUTION

We have developed a simple method which can be used to generate skewed random numbers. It uses a combination of two overlapping uniform probability distributions, which shall henceforth be referred to as a "double-block" distribution. Uniform distributions are the simplest probability distributions, and uniformly-distributed random numbers are easily generated using computer pseudo-random number generators.

Figs. 1a-b show an example of the combination of two overlapping uniform probability density functions. Fig. 1a shows the two individual probability density functions, $P_1(b)$ and $P_2(b)$, defined by six parameters: the means m_1 and m_2, half-widths Δ_1 and Δ_2, and probability densities p_1 and p_2. These parameters have the following properties:

$$\Delta_1 > |m_1| \text{ and}$$

$$\Delta_2 > |m_2|.$$

Therefore, the two distributions overlap each other and both cover $b = 0$.

Fig. 1b shows the double-block probability density function, $P(b)$, which is the sum of these two uniform distributions, i.e.,

$$P(b) = P_1(b) + P_2(b), \tag{1}$$

where

$$P_1(b) = \begin{cases} p_1, \text{ if } (m_1 - \Delta_1) < b < (m_1 + \Delta_1) \\ 0 \text{ , elsewhere} \end{cases} \tag{2}$$

and

$$P_2(b) = \begin{cases} p_2, \text{ if } (m_2 - \Delta_2) < b < (m_2 + \Delta_2) \\ 0 \text{ , elsewhere} \end{cases} \tag{3}$$

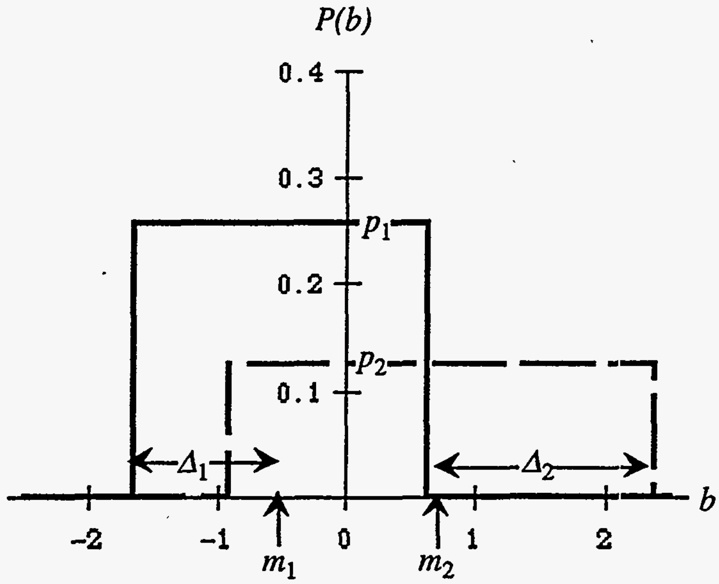


Figure 1a. Example of two overlapping uniform probability density functions, $P_1(b)$ and $P_2(b)$ with means $m_1 = -0.51$ and $m_2 = 0.73$; half-widths Δ_1=1.1 and Δ_2 =1.6; and probability densities $p_1 = 0.26$ and $p_2 = 0.13$, respectively.

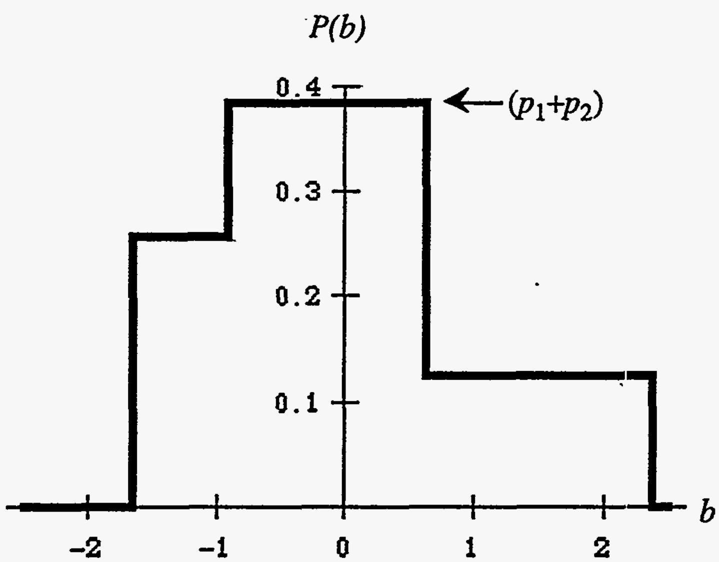


Figure 1b. Example of double-block probability density function, $P(b)$, which is the sum of the two overlapping uniform distributions in Fig. 1a. This distribution has a mean of zero, a variance of 1 and a skewness of 0.5.

In general, only the first three moments of the vertical wind velocity are known. Consequently, we will derive the six parameters of this distribution so that the desired first moment (the mean, assumed to be zero), second moment (σ^2), and third moment (ζ^3) result. Moments zero through three provide us with four equations:

$$\overline{b^0} = \int_{-\infty}^{\infty} P(b)db = 1, \tag{4a}$$

$$\overline{b} = \int_{-\infty}^{\infty} bP(b)db = 0, \tag{4b}$$

$$\overline{b^2} = \int_{-\infty}^{\infty} b^2 P(b)db = \sigma^2, \text{ and} \tag{4c}$$

$$\overline{b^3} = \int_{-\infty}^{\infty} b^3 P(b)db = \zeta^3. \tag{4d}$$

Using Eqs. (1 -3), Eqs. (4a-d) become

$$2p_1\Delta_1 + 2p_2\Delta_2 = 1, \tag{5a}$$

$$m_1p_1\Delta_1 + m_2p_2\Delta_2 = 0, \tag{5b}$$

$$\tfrac{2}{3}p_1\Delta_1^{\,3} + 2\Delta_1 m_1^{\,2}p_1 + \tfrac{2}{3}p_2\Delta_2^{\,3} + 2\Delta_2 m_2^{\,2}p_2 = \sigma^2, \text{ and} \tag{5c}$$

$$2\left[\Delta_1^{\,3}m_1p_1 + \Delta_1 m_1^{\,3}p_1 + \Delta_2^{\,3}m_2p_2 + \Delta_2 m_2^{\,3}p_2\right] = \zeta^3. \tag{5d}$$

Since there are four and equations and six unknowns, two more equations are required for closure. We use the following two equations:

$$\Delta_1 = -am_1 \text{ and} \tag{5e}$$

$$\Delta_2 = am_2. \tag{5f}$$

where a is a positive constant (assuming $m_1 < 0$ and $m_2 > 0$), which must be specified. For a greater than one, the two distributions overlap at $b = 0$. We have chosen $a = \sqrt{5}$, which gives the best fit to a Gaussian distribution (when ζ^3 is zero).

Assuming $a = \sqrt{5}$ and solving Eqs. (5a-f) yields

$$m_1 = \frac{2}{9\sigma^2}\left[\zeta^3 - \left(\zeta^6 + \tfrac{243}{32}\sigma^6\right)^{1/2}\right], \tag{6a}$$

$$m_2 = \frac{2}{9\sigma^2}\left[\zeta^3 + \left(\zeta^6 + \tfrac{243}{32}\sigma^6\right)^{1/2}\right], \tag{6b}$$

$$p_1 = \frac{m_2}{2am_1(m_1 - m_2)}, \text{ and} \tag{6c}$$

$$p_2 = \frac{m_1}{2am_2(m_1 - m_2)}. \tag{6d}$$

Parameters Δ_1 and Δ_2 are then defined by eqs. (5e- f).

These equations (5e,f; 6a-d) completely define the double-block probability function $P(b)$, given the desired moments, $\overline{b} = 0$, $\overline{b^2} = \sigma^2$, and $\overline{b^3} = \zeta^3$. A set of random numbers, $\{b_i\}$, can then be generated with these desired first three moments. An algorithm for doing this (which was used for the sample calculations shown below in section 4) is as follows:

Double-block random number algorithm

(1) For the desired values of σ^2 and ζ^3, calculate m_1, m_2, Δ_1, Δ_2, and p_1 using Eqs. (6a), (6b), (5e), (5f), and (6c), respectively.

(2) For each random number b_i:

(a) Obtain two uniformly-distributed random numbers r_i' and r_i'' on (0,1)

(b) If $r_i' < 2\Delta_1 p_1$ (total probability of first uniform distribution), then

$$b_i = m_1 + \left[2\Delta_1(r_i'' - 0.5)\right]$$

else

$$b_i = m_2 + \left[2\Delta_2(r_i'' - 0.5)\right]$$

An alternate method which requires only one uniform random number is as follows:

<u>Alternate double-block random number algorithm</u>

(1) For the desired values of σ^2 and ζ^3, calculate m_1, m_2, Δ_1, Δ_2, and p_1 using Eqs. (6a), (6b), (5e), (5f), and (6c), respectively.

(2) For each random number b_i:

(a) Obtain one uniformly-distributed random numbers r_i on (0,1)

(b) If $r_i < 2\Delta_l p_l \equiv \mathbb{P}_1$ (total probability of first uniform distribution), then

$$r_i' = r_i / \mathbb{P}_1$$

$$b_i = m_1 + \left[2\Delta_1(r_i' - 0.5)\right]$$

else

$$r_i' = (r_i - \mathbb{P}_1)/(1 - \mathbb{P}_1)$$

$$b_i = m_2 + \left[2\Delta_2(r_i' - 0.5)\right]$$

3. COMBINATIONS OF DOUBLE-BLOCK DISTRIBUTIONS

While double-block distributions as in Fig. 1b, have the desired first three moments, they are not smooth and continuous. However, skewed random numbers with continuous distributions, and with the desired first three moments, can be generated from combinations of double-block random variables. Such a random variable, B, can be defined as follows

$$B = \frac{1}{N^{1/2}} \sum_{i=1}^{N} b_i, \tag{7}$$

where b_i is a double-block random number and N is the number of double-block random numbers used to generate B. The moments of B are related to the moments of b, as follows:

$$\overline{B} = \frac{1}{N^{1/2}} \sum_{i=1}^{N} \overline{b_i} = 0 , \tag{8a}$$

$$\overline{B^2} = \overline{\left(\frac{1}{N^{1/2}}\sum_{i=1}^{N} b_i\right)^2} = \frac{1}{N}\sum_{i=1}^{N}\overline{b_i^2} = \overline{b^2}\ , \tag{8b}$$

$$\overline{B^3} = \overline{\left(\frac{1}{N^{1/2}}\sum_{i=1}^{N} b_i\right)^3} = \frac{1}{N^{3/2}}\sum_{i=1}^{N}\overline{b_i^3} = N^{-1/2}\overline{b^3}. \tag{8c}$$

Therefore, random numbers, B, can be generated with desired moments $\overline{B}$ (= 0), $\overline{B^2}$, and $\overline{B^3}$ by using Eq. (7) with double-block random numbers, b_i, which have the following moments:

$$\overline{b} = 0, \tag{9a}$$

$$\overline{b^2} = \sigma^2 = \overline{B^2} \tag{9b}$$

$$\overline{b^3} = \zeta^3 = N^{1/2}\overline{B^3} \tag{9c}$$

Note that the third moment of b, $\overline{b^3} = \zeta^3$, increases in a manner that is proportional to $N^{1/2}$, where N is the number of double-block random numbers, b_i, per random number B. This is necessary in order to maintain the desired skewness of B regardless of the number of double-block random numbers, b_i, that are combined to form B. If $\overline{b^3}$ did not depend on N, then, for large N, the Central Limit Theorem states that the distribution of B as N becomes large will approach a Gaussian distribution, which has zero skewness.

This method was tested for several values of skewness,

$$S = \frac{\overline{B^3}}{\left(\overline{B^2}\right)^{3/2}},$$

and for several values of N. A FORTRAN computer code which generates a sample of random numbers using this method, and calculates the corresponding histogram and probability density function is listed in the Appendix. Sample calculations using this method are given in the next section.

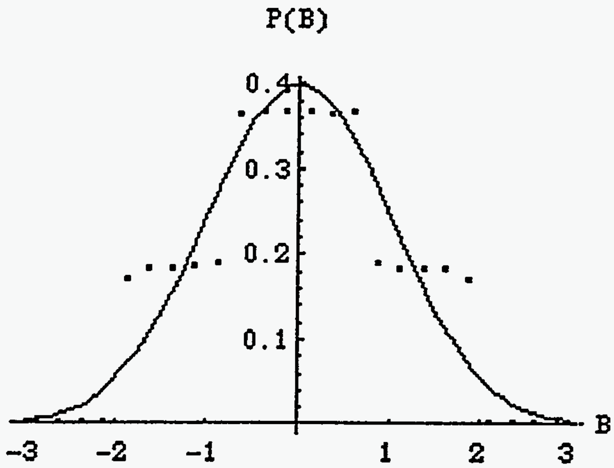
4. SAMPLE CALCULATIONS

Figs. 2-4 show calculated probability density values for B using the above method for three values of skewness (S = 0.0, 0.5, and 1.5) and for five values of N (N = 1, 2, 3, 5, and 10). All calculations are for a desired mean of 0, desired variance (σ^2) of 1, and a sample of 10^6 values of B. The number of occurrences of B values in evenly-spaced bins from $B = -3\sigma$ to $+3\sigma$ were counted. For Figs. 2a-e, twenty four bins were used. For Figs. 3a-e and 4a-e, forty eight bins were used. Probability density values for each bin were then calculated and plotted as dots.

Figs. 2a-e show the calculated probability density for zero skewness for values of $N = 1$, 2, 3, 5, and 10. Also plotted (as a solid line) in these figures is the corresponding Gaussian probability density function with the same desired moments, i.e., mean of zero and variance of one. In this case, the Central Limit Theorem predicts that B should approach a Gaussian distribution. From Figs. 2a-e, it can be seen that this occurs very quickly with increasing N. For $N = 1$ (Fig. 2a), the first three moments are in excellent agreement with the specified values ($\overline{B} = 0.0$, $\overline{B^2} = 1.0$, and $\overline{B^3} = 0.0$). With increasing N, the fourth and higher moments approach the corresponding values for a Gaussian distribution. For a Gaussian distribution with zero mean, the odd moments are zero and the fourth moment is $\overline{B^4} = 3\sigma^4 = 3.0$.

Figs. 3a-e and 4a-e show calculations for non-zero skewness, $S = 0.5$ and 1.5, respectively. (Skewness of 0.5 is typical for vertical velocities in the CBL.) These figures show that the desired skewness is achieved quite accurately (to two decimal places) for all values of N. For increasing N, smooth distributions are quickly obtained.

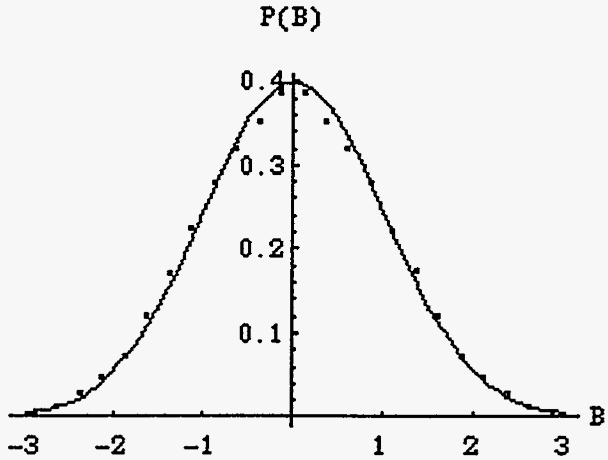
<u>**Figure 2a**</u>



Desired Skewness = 0.
Desired Mean = 0.
Desired Variance = 1.0
N = 1
Number of final ran. num.= 1000000
Calculated Skewness of B = 0.00225
Calculated Mean of B = 0.00105
Calculated Variance of B = 0.99829
Calculated 3rd moment of B = 0.00224
Calculated 4th moment of B = 2.24393
Calculated 5th moment of B = 0.00533

(Note: N = double-block random numbers, b_i, per final random number, B)

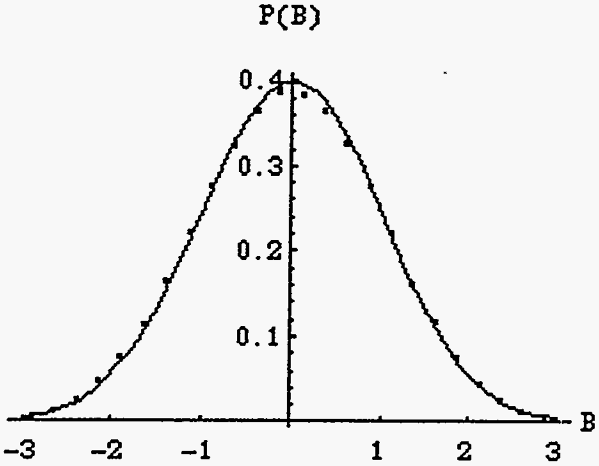
Figure 2b



Desired Skewness = 0.
Desired Mean = 0.
Desired Variance = 1.0
N = 2
Number of final ran. num.= 1000000
Calculated Skewness of B = -0.0035
Calculated Mean of B = -0.0005
Calculated Variance of B = 0.99948
Calculated 3rd moment of B = -0.0035
Calculated 4th moment of B = 2.62087
Calculated 5th moment of B = -0.0190

(Note: N = double-block random numbers, b_i, per final random number, B)

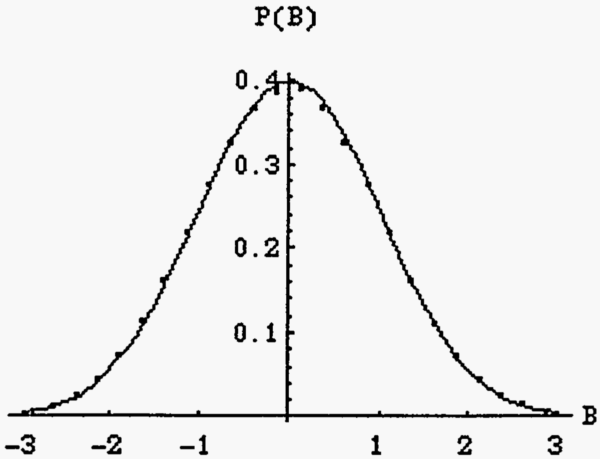
Figure 2c



Desired Skewness = 0.
Desired Mean = 0.
Desired Variance = 1.0
N = 3
Number of final ran. num.= 1000000
Calculated Skewness of B = -0.0042
Calculated Mean of B = -0.0012
Calculated Variance of B = 1.00090
Calculated 3rd moment of B = -0.0042
Calculated 4th moment of B = 2.75655
Calculated 5th moment of B = -0.0247

(Note: N = double-block random numbers, b_i, per final random number, B)

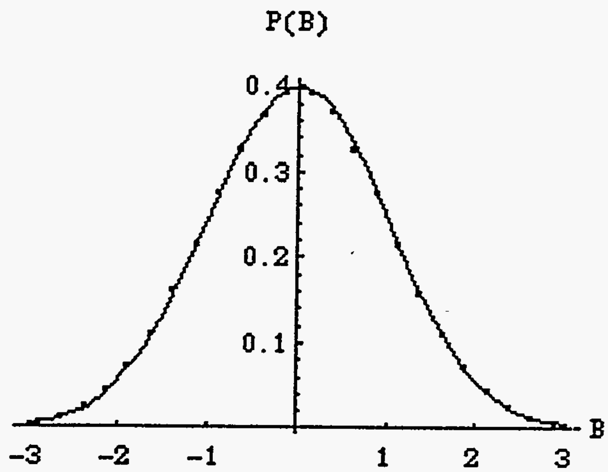
Figure 2d



Desired Skewness = 0.
Desired Mean = 0.
Desired Variance = 1.0
N = 5
Number of final ran. num.= 1000000
Calculated Skewness of B = -0.0028
Calculated Mean of B = -0.0008
Calculated Variance of B = 1.00209
Calculated 3rd moment of B = -0.0029
Calculated 4th moment of B = 2.86242
Calculated 5th moment of B = -0.0143

(Note: N = double-block random numbers, b_i, per final random number, B)

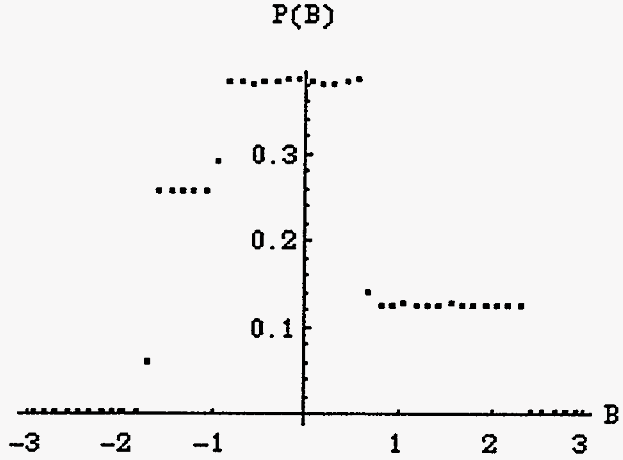
Figure 2e



Desired Skewness = 0.
Desired Mean = 0.
Desired Variance = 1.0
N = 10
Number of final ran. num.= 1000000
Calculated Skewness of B = -0.0021
Calculated Mean of B = -0.0008
Calculated Variance of B = 1.00205
Calculated 3rd moment of B = -0.0021
Calculated 4th moment of B = 2.93680
Calculated 5th moment of B = -0.0083

(Note: N = double-block random numbers, b_i, per final random number, B)

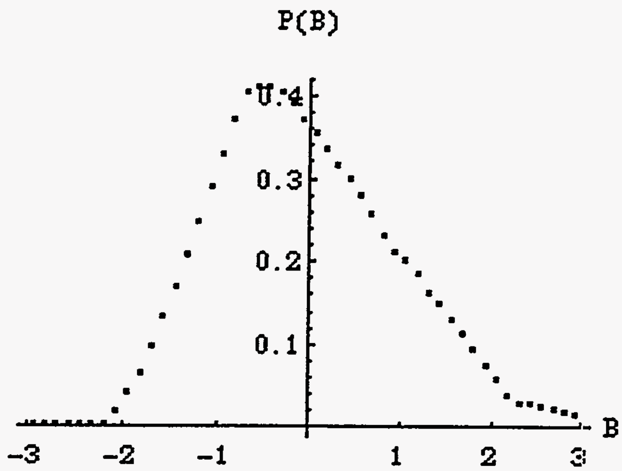
Figure 3a



Desired Skewness = 0.5
Desired Mean = 0.
Desired Variance = 1.0
N = 1
Number of final ran. num.= 1000000
Calculated Skewness of B = 0.50456
Calculated Mean of B = 0.00118
Calculated Variance of B = 0.99906
Calculated 3rd moment of B = 0.50385
Calculated 4th moment of B = 2.54383
Calculated 5th moment of B = 2.85461

(Note: N = double-block random numbers, b_i, per final random number, B)

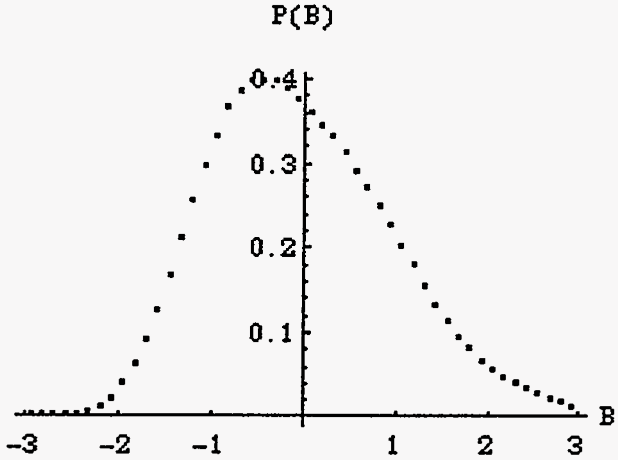
Figure 3b



Desired Skewness = 0.5
Desired Mean = 0.
Desired Variance = 1.0
N = 2
Number of final ran. num.= 1000000
Calculated Skewness of B = 0.49820
Calculated Mean of B = -0.0003
Calculated Variance of B = 0.99951
Calculated 3rd moment of B = 0.49784
Calculated 4th moment of B = 2.91312
Calculated 5th moment of B = 3.98529

(Note: N = double-block random numbers, b_i, per final random number, B)

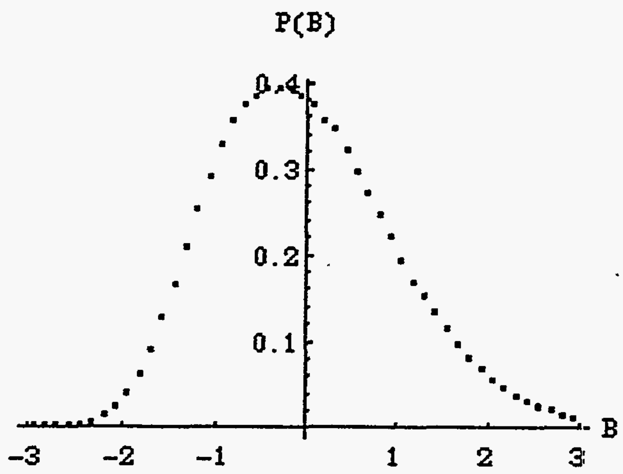
Figure 3c



Desired Skewness = 0.5
Desired Mean = 0.
Desired Variance = 1.0
N = 3
Number of final ran. num.= 1000000
Calculated Skewness of B = 0.49600
Calculated Mean of B = -0.0016
Calculated Variance of B = 0.99882
Calculated 3rd moment of B = 0.49513
Calculated 4th moment of B = 3.04044
Calculated 5th moment of B = 4.37807

(Note: N = double-block random numbers, b_i, per final random number, B)

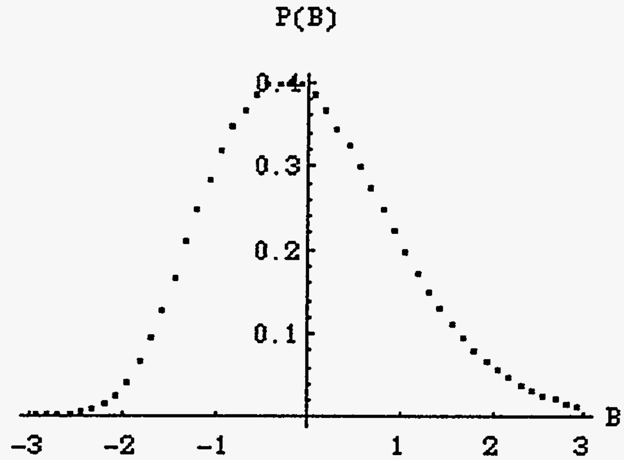
Figure 3d



Desired Skewness = 0.5
Desired Mean = 0.
Desired Variance = 1.0
N = 5
Number of final ran. num.= 1000000
Calculated Skewness of B = 0.49894
Calculated Mean of B = -0.0011
Calculated Variance of B = 1.00133
Calculated 3rd moment of B = 0.49995
Calculated 4th moment of B = 3.15260
Calculated 5th moment of B = 4.71858

(Note: N = double-block random numbers, b_i, per final random number, B)

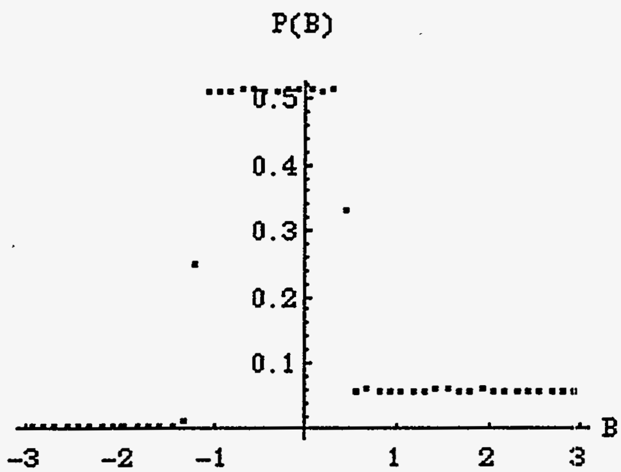
Figure 3e



Desired Skewness = 0.5
Desired Mean = 0.
Desired Variance = 1.0
N = 10
Number of final ran. num.= 1000000
Calculated Skewness of B = 0.49844
Calculated Mean of B = -0.0010
Calculated Variance of B = 1.00032
Calculated 3rd moment of B = 0.49868
Calculated 4th moment of B = 3.22919
Calculated 5th moment of B = 4.97588

(Note: N = double-block random numbers, b_i, per final random number, B)

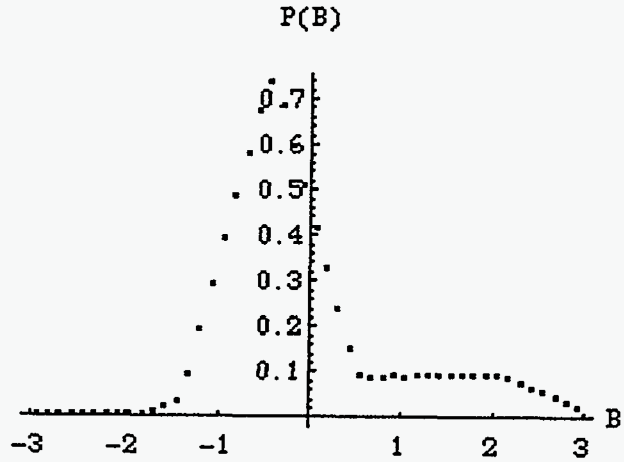
Figure 4a



Desired Skewness = 1.5
Desired Mean = 0.
Desired Variance = 1.0
N = 1
Number of final ran. num.= 1000000
Calculated Skewness of B = 1.50333
Calculated Mean of B = 0.00091
Calculated Variance of B = 0.99837
Calculated 3rd moment of B = 1.49967
Calculated 4th moment of B = 4.90601
Calculated 5th moment of B = 12.7426

(Note: N = double-block random numbers, b_i, per final random number, B)

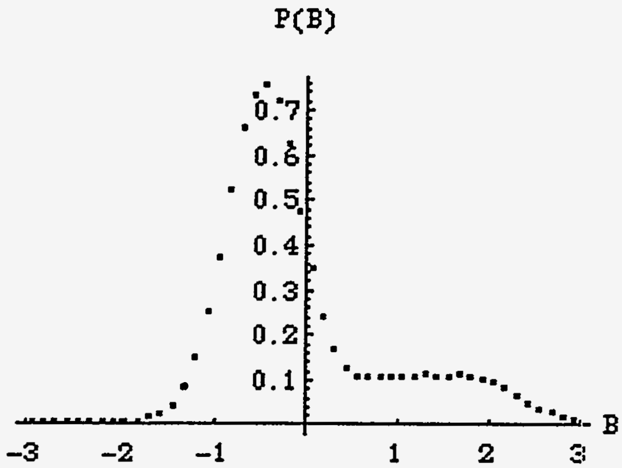
Figure 4b



Desired Skewness = 1.5
Desired Mean = 0.
Desired Variance = 1.0
N = 2
Number of final ran. num.= 1000000
Calculated Skewness of B = 1.49890
Calculated Mean of B = 0.00006
Calculated Variance of B = 0.99955
Calculated 3rd moment of B = 1.49791
Calculated 4th moment of B = 5.26957
Calculated 5th moment of B = 16.1751

(Note: N = double-block random numbers, b_i, per final random number, B)

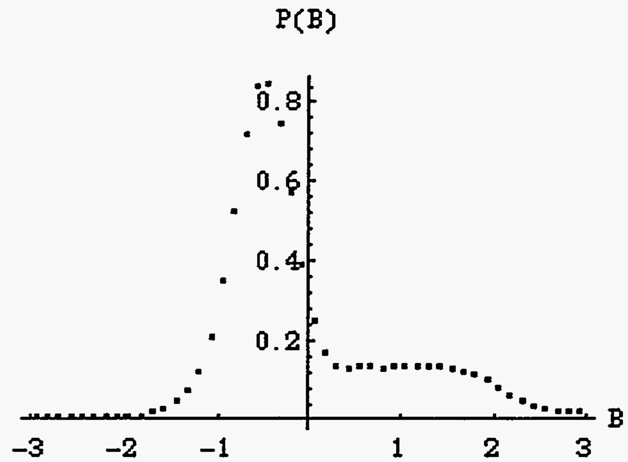
Figure 4c



Desired Skewness = 1.5
Desired Mean = 0.
Desired Variance = 1.0
N = 3
Number of final ran. num.= 1000000
Calculated Skewness of B = 1.49754
Calculated Mean of B = -0.0011
Calculated Variance of B = 0.99859
Calculated 3rd moment of B = 1.49437
Calculated 4th moment of B = 5.38421
Calculated 5th moment of B = 17.2935

(Note: N = double-block random numbers, b_i, per final random number, B)

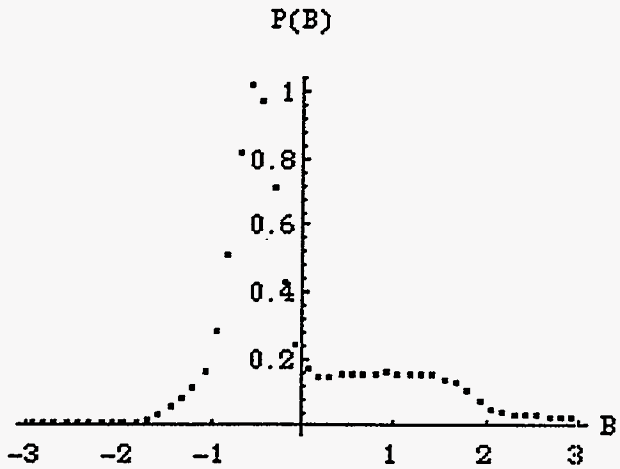
Figure 4d



Desired Skewness = 1.5
Desired Mean = 0.
Desired Variance = 1.0
N = 5
Number of final ran. num.= 1000000
Calculated Skewness of B = 1.50270
Calculated Mean of B = -0.0005
Calculated Variance of B = 1.00105
Calculated 3rd moment of B = 1.50508
Calculated 4th moment of B = 5.52842
Calculated 5th moment of B = 18.3748

(Note: N = double-block random numbers, b_i, per final random number, B)

Figure 4e



Desired Skewness = 1.5
Desired Mean = 0.
Desired Variance = 1.0
N = 10
Number of final ran. num.= 1000000
Calculated Skewness of B = 1.50213
Calculated Mean of B = -0.0006
Calculated Variance of B = 0.99922
Calculated 3rd moment of B = 1.50039
Calculated 4th moment of B = 5.61930
Calculated 5th moment of B = 19.4374

(Note: N = double-block random numbers, b_i, per final random number, B)

5. SUMMARY

The initial testing of probability distributions from combinations of double-block random numbers has shown that this is a promising technique for generating skewed random numbers. The method can generate random numbers with the desired first, second and third moments. For calculations with zero skewness, the higher moments of a Gaussian distribution were accurately reproduced when multiple double-block random numbers were combined.

Ongoing and future work will use this skewed random number generator for the random forcing term in a Langevin model for skewed turbulence in the convective boundary layer. The results will be compared with analytical results, laboratory and field experiments, and other Langevin models.

6. ACKNOWLEDGMENTS

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APPENDIX

FORTRAN Double-block random number generator code

```
      program db
c_____________________________________________________________________
c
c
c                        db (Double Block)
c
c
c_____________________________________________________________________
c
c author:  John Nasstrom
c
c date:    April 1994
c
c purpose: Generate random numbers from double, overlapping "blocks"
c          of two uniform probability density functions using method
c          developed by Don Ermak
c
c
c_____________________________________________________________________

      implicit none

      integer ndb, nfinal, idb, ifinal, nbins, ibin

      parameter(nbins=48)

      integer*4 ranseed
      real mom2, skew, dbmom3, dbmean1, dbmean2, dbprob1,
     &     dbdelta1, dbdelta2, rana, ranb,
     &     dbran, sumdbran,
     &     sumdbran2, sumdbran3,
     &     finalran, sumfinalran, sumfinalran2, sumfinalran3,
     &     sumfinalran4, sumfinalran5,
     &     finalmean, finalmom2, finalmom3, finalmom4, finalmom5,
     &     finalskew,
     &     finalpdf(nbins), histbincount(nbins), histbinwidth,
     &     histbinlowval(nbins), histsigmas,
     &     a, b, c, terma, termb
```

```
      parameter(histsigmas=3.)


c--------------- Start execution ---------------

c----- Assign constants

      a = sqrt(5.)
      b = 2./9.
      c = 243./32.
      ranseed = 97531


c---- Read input parameters:
c     - Number of double-block random numbers (ndb) to sum for each
c       random number in final distribution
c     - Number of values of final random number to calculate (nfinal)
c     - Desired skewness (skew) and second moment (mom2) of final
c       distribution (mean is assumed to be zero)

      open(unit=1, name='dbin.dat', err=100, status='OLD')
      go to 120
100   continue
      stop  'Error opening input parameter file: dbin.dat'
120   continue

      read(1,*) ndb, nfinal, skew, mom2
      Close(unit=1)

c------ Open output file
      open(unit=2, name='dbout.dat', err=200, status='NEW')
      go to 220
200   continue
      stop ' Error opening output file: dbout.dat'
220   continue

      write(2,250) ndb, nfinal, skew, mom2
250   format(' ndb=',I8,'; nfinal=',I8, '; skew=', f7.3,
     &       '; mom2=', f7.3,';',/)


c-------- Calculate bins for histogram of final random numbers
      histbinwidth = (2.*histsigmas*sqrt(mom2))/float(nbins)
```

```
      do ibin = 1, nbins
         histbinlowval(ibin) = -histsigmas*sqrt(mom2)
     &                       + float(ibin-1)*histbinwidth
         histbincount(ibin) = 0.
      end do



c-------- Calculate parameters for double block (db) distribution---
      dbmom3 = sqrt(float(ndb)) * skew * (mom2**1.5)
      terma = b/mom2
      termb = sqrt(dbmom3**2 + c*mom2**3)
      dbmean1 = terma*(dbmom3-termb)
      dbmean2 = terma*(dbmom3+termb)
      dbprob1 = dbmean2/(2.*a*dbmean1*(dbmean1-dbmean2))
      dbdelta1 = -a*dbmean1
      dbdelta2 = a*dbmean2

      write(2,300) dbmom3,dbmean1,dbmean2,dbprob1,dbdelta1,dbdelta2
300   format( ' dbmom3=',   F16.10,  ';', /,
     &        ' dbmean1=',  F16.10,  ';', /,
     &        ' dbmean2=',  F16.10,  ';', /,
     &        ' dbprob1=',  F16.10,  ';', /,
     &        ' dbdelta1=', F16.10, ';', /,
     &        ' dbdelta2=', F16.10, ';', /
     & )

c------ Initialize sums for final random number moments ----
      sumfinalran = 0.
      sumfinalran2 = 0.
      sumfinalran3 = 0.
      sumfinalran4 = 0.
      sumfinalran5 = 0.
```

```
c-------------------------------------------------------------
c------ Loop for calculation of final random number -----------
c-------------------------------------------------------------

      Do ifinal = 1, nfinal

c----- Initialize values of final random number
         sumdbran = 0.

c-------------------------------------------------------------
c------ Loop for summation of double block random numbers -----
c------ in each final random number                       -----
c-------------------------------------------------------------
         Do idb = 1, ndb

         rana = ran(ranseed)
         ranb = ran(ranseed)

         if (rana .lt. (2.*dbdelta1*dbprob1) ) then
            dbran = dbmean1 + 2.*dbdelta1*ranb - dbdelta1
         else
            dbran = dbmean2 + 2.*dbdelta2*ranb - dbdelta2
         end if

c--------- add to sums for final random number
         sumdbran = sumdbran + dbran

         end do
c-----------end of loop for double-block random numbers------------


c-------- Calculate final random number
         finalran = sumdbran / sqrt(float(ndb))

c-------- Increment count for appropriate bin of histogram of
c-------- final random number distribution

         do ibin = 1, nbins
            if (finalran.ge.histbinlowval(ibin) .and.
     &      finalran .lt. (histbinlowval(ibin)+histbinwidth) ) then
               histbincount(ibin) = histbincount(ibin) + 1.
               go to 450
            end if
         end do
```

```
450     continue

c-------- Add to sums for moments of final random numbers
         sumfinalran = sumfinalran + finalran
         sumfinalran2 = sumfinalran2 + finalran**2
         sumfinalran3 = sumfinalran3 + finalran**3
         sumfinalran4 = sumfinalran4 + finalran**4
         sumfinalran5 = sumfinalran5 + finalran**5

      end do
c---------end of loop for final random numbers---------------------

c------- Calculate moments of final random number distribution
      finalmean = sumfinalran / nfinal
      finalmom2 = sumfinalran2 / nfinal
      finalmom3 = sumfinalran3 / nfinal
      finalmom4 = sumfinalran4 / nfinal
      finalmom5 = sumfinalran5 / nfinal

      finalskew = finalmom3 / finalmom2**1.5

c------- Calculate final prodability density function (pdf)
      do ibin = 1, nbins
         finalpdf(ibin) = histbincount(ibin) / (nfinal*histbinwidth)
      end do

c------- Write out results in format readable by Mathematica
      write(2,1100) finalmean, finalmom2, finalmom3,
     &    finalmom4, finalmom5, finalskew
1100  format(' finalmean=', F16.10  ,';' ,/,
     &       ' finalmom2=',  F16.10 ,';' ,/,
     &       ' finalmom3=',  F16.10 ,';' ,/,
     &       ' finalmom4=',  F16.10 ,';' ,/,
     &       ' finalmom5=',  F16.10 ,';' ,/,
     &       ' finalskew=',  F16.10 ,';' ,/
     &       )

      write(2, 1200)
1200  format(' finalranhistogram = {' )
      do ibin = 1, nbins
         if(ibin.ne.nbins) then
            write(2, 1210) ibin, histbincount(ibin)
1210        format(' {',i3, ',', f10.0, '},')
         else
```

```
            write(2,1215) ibin, histbincount(ibin)
1215        format(' {',i3, ',', f10.0, '}')
         end if
      end do
      write(2, 1216)
1216  format(' } ;' )


      write(2, 1220)
1220  format(/,' finalranpdf = {' )
      do ibin = 1, nbins
         if(ibin.ne.nbins) then
            write(2, 1230) (histbinlowval(ibin)+histbinwidth/2.),
     &                     finalpdf(ibin)
1230        format(' {', f13.7, ',',  F16.10, '},')
         else
            write(2,1235) (histbinlowval(ibin)+histbinwidth/2.),
     &                     finalpdf(ibin)
1235        format(' {' , f13.7, ',',  F16.10, '} ')
         end if
      end do
      write(2, 1240)
1240  format(' } ;' )

      close(unit=2)

      stop 'normal'
      end
```