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RELEASE OF WATER TRAPPED IN DAMAGED FUEL SUBSURFACE VOIDS

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Key Words: *8"* Canister pressurization, K Basin, sludge, water, hydrate

Abstract: Flow rate calculations were undertaken to examine the problem of the release of water trapped in subsurface voids during the vacuum
drying for damaged fuel. To calculate the flow rates of the water vapor, five flow models are developed based on the kinetic theory of gases. The difference between the vapor pressure and the operating pressure provides the driving force for crevice water removal. Gas flow is divided into three types: viscous flow, molecular flow, and slip flow in the transition range. These calculations were focused on assessing that measured moisture release from the whole element drying studies currently in progress includes any water that may be trapped in subsurface voids.

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RELEASE OF WATER TRAPPED IN DAMAGED FUEL SUBSURFACE VOIDS

F. **F. Huang Fluor Daniel Northwest, Inc. Rich1 and, Washington**

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Richland, Washington

EXECUTIVE SUNNARY

Uranium metal reacts actively with water to produce uranium oxide and hydrogen gas, and radiolysis of water also produces hydrogen in fuel containers. These fuel reactivities increase the potential for overpressurization and pyrophoric events during storage. For these reasons, the water content in spent nuclear fuel will be reduced by heat and vacuum. In the conditioning process, free water is expected to be removed by vacuum drying at 50 ' to **75** 'C, while chemically bound water can be released during storage at temperatures calculated to be as high as 150 **'C.**

The following calculations were undertaken to examine the problem of the release of water trapped in subsurface voids during the vacuum drying cycles. Measured moisture release is being studied by performing the whole element drying testing currently in progress; these calculations were focused **on** assessing that release of moisture includes any water trapped in subsurface voids. These calculations also add insight into the moisture release mechanism during the post Cold Vacuum Drying (CVD) pressure rise test for product acceptance.

Free water will boil as the pressure in the vacuum system decreases, it then cools rapidly and is removed from the system. However, water trapped in damaged fuel subsurface voids has to evaporate and then flow out through cracks and small openings to the surface. Water filling the pores or cracks in fuel will evaporate from the open end; the water interface will recede inward at a rather high rate. To calculate the flow rates of the vapor, five flow models are developed based on the kinetic theory of gases. The long tube

model assumes that water is contained in a pocket of a diameter much larger than the diameter of the long tube that connects the pocket to the surface. The orifice model is similar to the long tube model except for the very short tube. The filled pore model has water in the tube without a pocket. The porous bed flow model has water and a bed of granular particles filled in the tube without a pocket. Calculations show that water can evaporate easily in the pocket. The difference between the vapor pressure and the operating pressure provides the driving force for crevice water removal. Analysis of gas flow considers the flow behavior from comparisons of the molecular mean free path and a tube dimension, from which gas flow is divided into three types: viscous flow, molecular flow, and slip flow in the transition range.

The flow rates of the five models are calculated for each of the three flow types. The rate of orifice flow is much higher than that of long tube flow, and water filled in pores of length 5 cm can be cleared in minutes during the **CVD** process. Because most of the defects in N Reactor fuel have a radius ranging from 10 μ m to 100 μ m, and the drying temperature is around 50 **'C,** the flow of water vapor through defects is slip flow in the transition range. Results show that the long tube flow rate'of water vapor decreases significantly with defect radius below 100 μ m and that the pocket and defect sizes may be estimated by performing whole element tests. The method to determine the values of pocket and defect sizes by performing pressure rise tests on damaged fuel elements is discussed.

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RELEASE OF WATER TRAPPED IN DAMAGED FUEL SUBSURFACE VOIDS

1.0 INTRODUCTION

The purpose of removing uranium fuel from the K Basin wet storage to the 200 Area dry storage is to achieve a non-corrosive and stable condition for storing the fuel over an extended period of time. Water in damaged fuel will be subjected to chemical reactions and radiolysis, causing over-pressure and corrosion problems in storage containers. Free water and chemically bound water can be removed by applying heat and vacuum. Sufficient energy will be provided to remove adsorbed and free water during the process of Cold Vacuum Drying (CVD). Hydrates are expected to decompose during staging at moderate temperatures.

remaining in the element following CVD is therefore of concern for long term are in progress to measure the water content after CVD (Ritter 1997; Marschman 1998). Calculations are performed based on existing theories to predict the pressure increase resulting from release of water by heat and vacuum. The pressure profile in single element canisters during staging following the CVD has been studied in a separate report (Huang 1997). In this report, the flow behavior of water vapor trapped in subsurface voids through various defect shapes is investigated. Wet fuel will be vacuum dried to remove free water. The amount of water

During the CVD process the dryness of spent nuclear fuel will be evaluated by a moisture monitor. As the pressure in the vacuum system decreases to the partial pressure of water at the container temperature, the free water begins to boil and cools rapidly. It is then removed until equilibrium pressure and temperature in the vacuum system are reached. While the free water on the fuel surface boils and is removed, the water trapped in subsurface voids in the fuel will take a longer time to escape. The flow rate from the subsurface voids varies significantly depending on the size of defect. Some of the water may take a long time to flow out from subsurface voids so that the moisture monitor may not be capable of detecting whether the trapped water has been removed in a short test duration. Therefore, the transport behavior of trapped water is analyzed to determine if it presents problems for the CVD of fuel.

Measured moisture release is being studied by performing the whole element drying testing currently in progress (Marschman 1998); these calculations were focused on assessing that release of moisture includes any water trapped in subsurface voids. These calculations also add insight into the moisture release mechanism during the post CVD pressure rise test for product acceptance.

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2.0 WATER EVAPORATION

Water tends to evaporate when exposed to a gaseous atmosphere; the rate of evaporation is dependent on its molecule energy, mainly its temperature.
At 50 °C, some of the water molecules have sufficient energy to break away At 50 °C, some of the water molecules have sufficient energy to break away
from the water surface and enter the air in the vapor state. Eventually s from the water surface and enter the air in the vapor state. Eventually some
of the water molecules will re-enter the water surface when an equilibrium condition is reach in which the rate of evaporation is equal to the rate of condensation. In this condition, the air above the water is saturated with water vapor molecules, and the pressure on the water surface is called the vapor pressure. The rate of the vapor mass striking the surface is given by (see Appendix A for derivation of the equation).

$$
G = P\sqrt{(M/2\pi RT)}
$$
 (1)

(2)

At 50 **'C,** the vapor pressure given in the Steam Table (ASME 1983) is

 $P = 1.7891$ psi = 0.1217 atm $R = 0.08206$ atm-l/mol-K ^M= 18 g/mole; T = **323** ^K

 $G = 1.265$ g/cm² per second

The rate of evaporation shown in Equation (1) predicts that the water in a void of 1 cm3 will evaporate in about ¹second from a free surface at 50 **'C.** The water molecules are released from the surface quite rapidly if the pressure of the surrounding air is lower than the vapor pressure of water.

3.0 FLOW OF TRAPPED VAPOR

Although water evaporates rapidly under vacuum it may take some time for the water vapor to flow through a long tube with a small diameter before it is vacuum system is the mean free path, which is the average distance traveled before collision with another molecule. The flow rate of gas depends on how large the mean free path of a molecule as compared to the characteristic dimension of the tube through which the gas is flowing. Because the higher the pressure the smaller the mean free path, collisions between molecules are more frequent than those between molecules and the tube wall under high pressures; the flow of an individual molecule is determined by the neighboring molecules and their net motion. In the high-pressure condition of vacuum system, the gas is considered to be viscous, the flow is analyzed hydrodynamically and is called viscous flow. The flow at high pressures can be either laminar or turbulent which generally occurs at very high gas velocities. molecules and their net motion. In the high-pressure condition of vacuum

On the other hand, at low pressures the flow is called molecular flow because the mean free path is large compared to the characteristic dimension and the flow is predominated by molecular collisions with the walls of the tube; thus, molecular flow is dependent on the geometry of the tube and density differentials. When both types of collisions are important the flow
is in the transition range. Therefore, the flow rates are formulated based on the ratio of mean free path (L_a) and characteristic dimension (a) (Knudsen number) as follows.

> Viscous Flow: $L_a/a < 0.01$ Molecular Flow: L,/a > 1.00 **(3)**

Slip Flow in Transition Range: $0.01 < L/a < 1.00$

A flow rate is defined as the product of volumetric flow rate and pressure as foll ows:

$$
Q = P dV/dt = \kappa T dN/dt \tag{4}
$$

using the ideal gas law. T in Equation **(4)** is in K, *^K*is Boltzmann constant, and dN/dt is the number of molecules per unit time; the Q is directly proportional to a molecule current. The flow rate is sometimes written as the product of pressure difference and conductance which is

$$
F = Q/(P_2 - P_1)
$$

Since the mass of one molecule of water vapor is 2.99 *x* **g,** the mass flow rate, w, is calculated from the following equation:

$$
w = 2.99 \times 10^{-23} \text{ Q/} \times T = \text{MQ/} \text{R} \text{T}
$$

= 2.1656 \times 10^{-6} \text{ F(P₂ - P₁)/T g/s} (5)

where 0 is in Pa.cm³.s⁻¹ and T is in K. In estimating the mass flow rate of water vapor, the flow type is first determined by checking the criteria established in Knudsen number **(3)** with defect size and average pressure, the flow conductance (F) for the flow type determined is then computed using the flow rates for viscous, molecular, and slip flow to be presented in the following sections. With P and **5** known under the operating conditions the mass flow rate can be evaluated from Equation (5).

3.1 **MEAN FREE PATH**

The mean free path (L,) is defined as the average distance traveled by a
molecule. For an equilibrium simple gas of hard sphere molecules the value of L_s is related to the molecular diameter of δ by (Bird 1976)

$$
L_{\rm a} = \frac{1}{\sqrt{2}\pi M\delta^2}
$$

$$
\delta = \left(\frac{5}{16\eta}\right)^{1/2} \left(\kappa \frac{m}{\pi}\right)^{1/4}
$$

where **n** is the number of molecules per cubic centimeter and **n** is the viscosity of the gas.

As shown in Equation (6), both mean free path and viscosity are dependent on temperature and pressure. If the average pressure is in micron *of* Hg the following simple relationship between mean free path and pressure is widely used for approximation (Weissler 1979)

$$
L_a \approx 5/P_g
$$
 P_g in micron of Hg
or

$$
a/L_a \approx 2aP_g
$$
 P_g in Pa (7)

 (6)

where $P_g = (P_2 + P_1)/2$. Substituting the mean free path into the division given **ii** the criteria in **(3)** leads to

$$
Viscous Flow: aPg > 50
$$
\n
$$
Molecular Flow: aPg < 0.5
$$
\n
$$
Slip Flow in Transition Range: 0.5 < aPg < 50
$$
\n
$$
(8)
$$

For example, the flow of gas in a long hole with a diameter of 0.1 cm under an
average pressure of 800 Pa (6 Torr) in vacuum system is viscous flow when $aP_a = 0.1$ (cm) x 800 (Pa) = 80 > 50.

3.2 **VISCOUS FLOW**

The rate of flow through a straight tube of circular cross section is calculated using the Poiseuille equation:

$$
Q = \frac{\pi a^4}{8n\ell} P_g (P_2 - P_1)
$$

 (9)

where:

a = Tube radius

 $l =$ Tube length

ⁿ= Viscosity of gas

The equation is applicable with assumptions that the gas is incompressible, the flow-velocity is constant and zero at the tube walls, and there is no turbulent motion of gas.

The time required for water vapor to flow from a pocket inside a damaged fuel element to the low pressure area can be estimated by using Equation (9). Figure 1a depicts a void of $\pi D^3/6$ with a long hole leading to the surface of fuel element. The pocket is half full of one gram of water. As estimated in Section 2, it takes only 1 second to evaporate one gram of water at 50 $^{\circ}$ C. The steam pressure is given in ASME Steam Table (ASME 1983):

 $a = 0.1$ cm; $l = 5$ cm; T = 323 K; P₂ = 1.7891 psi = 12334 Pa

 $P_1 = 3$ Torr = 400 Pa; $P_n = 47.76$ Torr = 6367 Pa

 $n = 10.6 \times 10^{-6}$ Pa s; $K = 1.38 \times 10^{-23}$ J/K

The flow is viscous flow as $aP_a = 636 > 50$ (see Criteria 8). The flow is viscous flow as aP_g = 636 > 50 (see Criteria 8). Thus, from
Equation (9) we have Q = 2.63 x 10⁵ Pa·cm^{3.}s". Using Equation (5) the number of molecules that cross the plane per unit time can be estimated. The mass of
water crossing the plane per second (w) is equal 3.52 x 10⁻³ g. Therefore, it
takes about 5 minutes for the water vapor of 1 g to flow from t the opening of the long tube with a radius of 0.1 cm.

If water fills up the 5-cm channel the water in the pocket may not evaporate before the channel is cleared. In this case the water at the end of channel subjected to the low pressure will evaporate first with a rate of 1.265 \times π (0.1 \times 0.1) = 0.04 g/s. However, as the water interface in the tube recedes, it will take a longer time for the water vapor to flow out of the tube. This subject will be discussed in more detail in Section 4.2.

3.3 MOLECULAR FLOW

For conditions of aP, < 0.5 where the mean free path is greater than the characteristic dimension, the molecular flow rates for flow through long tubes and orifices are given below (Knudsen 1911).

Flow Through Long Tubes

The conductance of a long tube of length ℓ , variable cross section A, and perimeter H calculated by Knudsen is

$$
Q = \frac{4}{3} \frac{v_a}{\int \frac{H}{A^2} dl} (P_2 - P_1)
$$
 (10)

where v_a is given in Equation (A6). For a cylindrical tube of radius a and
length of *l*, the flow rate is

$$
Q = F_{t}(P_{2} - P_{1})
$$

\n
$$
F_{t} = 2\pi/3 (a^{3}/\ell) v_{a} = \sqrt{(32\pi/9) (a^{3}/\ell) (RT/M)^{1/2} s^{-1}}
$$
\n(11)

where

$$
R = 8.3133 \times 10^6 \text{ Pa} \cdot \text{cm}^3 \cdot \text{deg}^{-1} \text{K} \cdot \text{g} - \text{mole}^{-1}
$$

Flow Throuqh Orifices or Short Tubes (Dushman 1922)

$$
Q = F_o(P_2 - P_1)
$$

\n
$$
F_o = \frac{1}{4}K(\pi a^2)v_a = \sqrt{(\pi/2)Ka^2(RT/M)^{1/2} s^{-1}}
$$
\n(12)

where:

$$
K = (1 + 3\ell/8a)^{-1}
$$

and

$$
R = 8.3133 \times 10^6 \text{ Pa} \cdot \text{cm}^3 \cdot \text{deg}^{-1} \text{K} \cdot \text{g} \text{--mole}^{-1}
$$

As :

$$
F_o/F_t = 3\ell/8a
$$
, $F_o = F_t$ when $\ell/a = 8/3$.

3.4 SLIP FLOW IN TRANSITION RANGE

As the pressure decreases to a pressure below the viscous limit, the flow velocity at the tube walls becomes significant and contributes an additional conductance as "slip" correction. In this transition range, both viscous flow
and molecular flow are influential in evaluating the rate of slip flow. With an operating pressure of 400 Pa and the water vapor pressure of 12300 Pa at 50 ^oC, slip flow is important if the radius of pore is in the range of 10 μ m to 100 μ m, according to the criteria in (8) (0.5<aP $_{\rm g}$ <50). As shown in Figure 2, the crack size in fuel elements is about **30** pm; the flow rate *of* water vapor in defects can be estimated more accurately by considering viscous flow corrected for slip.

The flow rate for slip flow in the transition range is given by (see Appendix B for details)

$$
F = F_{t}(F_{v}/F_{t} + Z)
$$

= $F_{t}(0.1472(a/L_{a}) + Z)$ (13)

$$
Z = (1 + 2.507(a/L_{a}))/(1 + 3.095(a/L_{a}))
$$

Equation (13) also can be written in terms of viscous conductance F,:

$$
F = F_v (1 + 7.793(L_a/a)Z)
$$
 (14)

The mean free path L_a is dependent on the average pressure P_a . At high pressures, a/La is large, and Z approaches a small value *of* 0.81 compared to the value of F_v/F_e in Equation (13), thus, the flow is predominated by viscous
flow. On the other hand, when a/L, is small at low pressures, Z approaches 1
and F_v/F_e becomes small, the conductance F approaches molecular by so in the ratio to defect radius: a/L_a = 2 aP_a (cm·Pa)⁻¹. This is used to evaluate its ratio to defect radius: a/L_a = 2 aP_a (cm·Pa)⁻¹. This calculate the rate of slip flow from Equations (13) or (14).

4.0 CALCULATION

Water trapped in a pocket with a short tube and large orifice can evaporate easily at 50 'C under vacuum as shown in Equation (2). For water pockets with a long and small cross section tube, water may partially fill the fuel defects water may fill the pores without subsurface pockets as shown in Figure 1b. The water-air interface may be anywhere in the pocket or the tube.
From the free surface of the interface water molecules will break away as discussed in Section 2.0 and provide pressure difference as a driving force for water removal. To estimate the flow rate of water vapor through defects, long tube or orifice, or they have only pores filled with water. The flow rates for water interface in a pocket or a pore are estimated based on the flow rate equations given above.

4.1 LONG TUBE MODEL

The long tube model is depicted in Figure la for a fuel crevice. Water is deposited in the pocket ready for evaporation at temperatures maintained at 40 **OC** or higher during the CVD. The long tube leading to the surface of fuel is empty for vapor to flow toward low pressure space. **As** an example, the flow rate of water vapor at 50 °C through a tube with a length of 5 cm and a
diameter of 100 μ m is computed as follows. The test conditions are the same
as those listed in Section 3.2. Firstly, the criteria in Equation (8) P_{g} = 6367 Pa, and 0.5 < a P_{g} = 32 < 50, the flow is found to be slip flow in transition range according to the criteria in (8). Using Equations (11) and (13) we found that $F_t = 1.020 \times 10^{-3}$ cm³/s and $F = 9.545 \times 10^{-3}$ cm³/s. For long tube flow, the mass flow rate in terms of a is estimated cm, cm²/s and $F = 9.545 \times 10^{-3}$ cm²/s. For

$$
w_1 = 0.00492 \times (P_2 - P_1) \times (0.1472a(P_2 + P_1) + Z)a^3/(\ell T^{1/2}) g/s
$$

\n
$$
Z = (1 + 2.507a(P_2 + P_1))/(1 + 3.095a(P_2 + P_1))
$$
\n(15)

where a and l are in cm, P is in Pa, and T is in K. The value of a/L_a is estimated from Equation (7) to be 63.67, the following results are obtained:

 $aP_n = 32 < 50$ (slip flow) $Z = 0.811$; $F_v/F_t = 9.387$ $w_1 = 8.319 \times 10^{-7}$ g/s $= 2.995 \times 10^{-3}$ g/h

The mass flow rate is computed from flow conductance with Equation (5). Figure 4 shows the mass flow rate in terms of defect radius up to 500 μ m for temperatures of 40 °C, 50 °C, and 75 °C. The time, $t = m/w_1 = \pi \rho D^3/6w_1$,

required to remove the water in a pocket of 1 cm diameter (about 0.53 g)
through a 5-cm long tube with a radius of 50 μ m is about 180 hours. For tubes of much smaller cross sections, as the one shown in [Figure 2](#page-36-0) where the diameter of a crack is about 30 μ m, it may take months for the water to flow diameter of a crack is about 30 pm, it may take months for the water to flow out of the channel. **As** shown in Figure 5, the time required for water vapor to flow out from a long tube increases rapidly with tube radius smaller than $100 \cdot \mu m$.

4.2 **FILLED PORE MODEL**

channel, water vapor will flow through the orifice, and the water interface is lowered until y = 8a/3 when orifice flow conductance (F,) is equal to molecular flow conductance (F,) (Figure lb); below that height the flow conductance is calculated using the rate equation of the long tube model. Upon evaporation the interface recedes with time as expressed by (see Appendix C): If water is filled in a pore with interface close to the opening of the

$$
y = (20.68at - 0.0053)^{1/2}
$$

t = $(y^2 + 0.0053)/20.68a$ (C3)

At $y = 5$ cm and $a = 50 \mu m$, $t = 241.8$ s. Neglecting the small value of y for orifice flow, the mass flow rate of a filled pore, which is time dependent, is given by

$$
w_2 = 0.00492x(P_2 - P_1)x(0.1472a(P_2 + P_1) + Z)a^3/((20.68at - 0.0053)T)^{1/2} g/s
$$

\n
$$
Z = (1 + 2.507a(P_2 + P_1))/(1 + 3.095a(P_2 + P_1))
$$
\n(16)

for

0.053 < t **(e2** + 0.0053)/20.68a

The mass flow rate decreases with increasing time or the distance between the opening and the interface. The rate continuously declines until the interface reaches the end of the pore where the mass flow rate equals the rate for the long tube model with the same tube length (Figure 6). Because of higher flow rates it does not take long to remove the water in the filled tube (241.8 seconds for a defect of the size given above).

4.3 ORIFICE MODEL

A water pocket may be located beneath a cladding defect (Figure IC). The' mass flow through an orifice is computed by Equations (12) and (13). For cladding with a thickness of 0.07 cm, the mass flow rate is given by

$$
w_o = 1.845 \times 10^{-5} \times (1 + 0.02625/a)^{-1} (0.1472a(P_2 + P_1) + Z)(P_2 - P_1)a^2T^{-1/2} g/s
$$

\n
$$
Z = (1 + 2.507a(P_2 + P_1))/(1 + 3.095a(P_2 + P_1))
$$
\n(17)

The mass flow rate for orifice model as a function of defect size is plotted in Figure 7. With residual water of 1 g, the time to remove the free water through orifice is shown in [Figure 8.](#page-42-0) Å comparison between Figures 4 and 7
indicates that the orifice flow rate is 60 times higher than that from the indicates that the orifice flow rate is 60 times higher than that from the long tube model. At 50 °C and a = 50 μm, the mass flow rate of the orifice
model is found to be w_o = 4.86 x 10⁻⁵ g/s, which is lower than the flow rate
(1.3 x 10⁻⁴ g/s) under the same conditions computed from an ori (1.3 x 10 ' g/s) under the same conditions computed from an orifice model
proposed by Pajunen. However, the rate of the former orifice model increases
faster than that of the latter as the defect size increases into the v flow range as seen in Figure 9.

4.4 EXHAUST OF WATER VAPOR

The flow rates evaluated above are based on the constant vapor pressure maintained by the water in a pocket at the operating temperature. The water continues to evaporate as the vapor flows out through a tube or orifice. In the case where only water vapor is present, the pressure in the void of volume V changes with time and the conductance F varies with pressure, the rate of exhaust is given by -VdP/dt = FP. Let P, designate the pressure in the vacuum system during the exhaust. Using Equations (7) and (13), we obtain

$$
-Vd(P - P_o)/dt = F_t(0.1472a(P + P_o) + Z)(P - P_o)
$$
 (18)

Solving the differential Equation (18) gives

$$
\ln\left(\frac{P_1 - P_o}{P_2 - P_o}\right) - \ln\left(\frac{P_1 - P_o + \frac{Z}{O_o} + \frac{Z}{O_o} - \frac{Z}{P_o}}{P_2 - P_o + \frac{Z}{O_o} + \frac{Z}{O_o} - \frac{Z}{P_o}}\right) = \frac{Z / F_t}{V} (t_2 - t_1)
$$
 (19)

where P₁ is the pressure at time t₁, P₂ is the pressure at time t_{2,3} and
t₂ > t₁. For a tube of length 5 cm, radius 50 μ m, F_t = 1.020 x 10⁻³ cm³/s at
50°C and P_o = 400 Pa (3 Torr), and Z' = Z + 0.29 are obtained:

 $P_1 = 12334$ Pa (92.53 Torr), $P_2 = 1593.3$ Pa, $t_2 - t_1 = 799.5$ V sec/cm³; P_1 = 1593.3 Pa (11.95 Torr), P_2 = 519.3 Pa, t₂ - t₁ = 1438.3V sec/cm³; $P_1 = 519.3$ Pa (3.90 Torr), $P_2 = 411.9$ Pa, $t_2 - t_1 = 1591.0$ V sec/cm³.

It takes a total of 64 minutes to exhaust the vapor from a 1 –cm³ pocket
through a long tube to an vacuum environment of 400 Pa at 50 °C. Results also show that the time required to reduce the pressure 10% increases with decreasing pressure. Compared to the long tube flow rate (w_1) , the rate of vapor exhaust is about 35 times slower because the flow conductance decreases with time at the stated pressure.

4.5 POROUS BED FLOW MODEL

The defects shown in [Figure](#page-36-0) **2** suggest that the fuel is likely to have pores or cracks packed with a bed of dense swarm of particles. The driving force from steam pressure for flow to the fuel element surface is similar to that for the filled pore model described in Section 4.2. Considering the bed of granular particles as a porous mass, the flow rate of water vapor through a porous mass is calculated using an empirical relation of Darcy's equation:

where:
$$
v = -(k/n)grad P
$$
 (20)

 $v =$ The rate of flow through a surface element of unit area

 k = The permeability of the porous mass

 $P =$ The pressure

Equation (20) is modified to maintain the equilibrium between the forces acting on a volume element of fluid - the pressure gradient, the divergence of the viscous stress tensor and the damping force caused by the porous mass (Brinkman 1947). The modified equation is in the form of

$$
\text{grad } P = - \left(\frac{n}{k} \right) v + \frac{n}{\Delta v} \tag{21}
$$

This equation.is good for a wide range of permeability of the porous mass. For low values of k, Equation (21) is approximated to be Equation (20); if k is large, the first term on the right hand side of Equation (21) is neglected. A solution can be obtained by imposing Equation **(21)** with the condition of incompressibility: div v = 0. Let the total volume of **^N** particles of radius R and the volume of pore be V_0 and V, respectively,

 $V_a = (4\pi/3)R^3N$, $V = \ell \pi a^2$

Assuming **n'** = **n,** the rate of flow is

$$
v = \frac{\pi D_p^2 a^2}{18 \eta V_o} \left[1 + \frac{3V_o}{4V} 1 - \sqrt{\left(\frac{8V}{V_o} - 3\right)} \right] (P_2 - P_1)
$$
 (22)

 (23)

Using the bed voidage and the diameter of particle given in (Pajunen 1997), the N, **D,, V,,** and V can be estimated as follows:

 $\epsilon = 0.6 = 1 - V_o/V$ or $V_o = 0.4 V$

 $D_n = 15 \mu m = 1.5 \times 10^{-3} \text{ cm}$

Substituting these data into Equation (22) gives

$$
v = 1.969 \times 10^{-8} (P_2 - P_1)/n\ell.
$$

Thus, the mass flow rate of porous bed flow model is given by

$$
w_3 = 1.34 \times 10^{-13} a^2 P_g (P_2 - P_1) / (n \ell T) g/s
$$
 (24)

The mass flow rate of a pore (a radius of 50 μ m and a length of 5 cm) with a bed of porous mass at 50 **'C** can be calculated from Equation (24).

 $w_3 = 1.49 \times 10^{6}$ g/s
= 8.94 x 10⁻⁷ g/min

Therefore, the mass flow rate in a 5-cm-long pore with a swarm of particles is about 165 times slower than the one without particles. However, because the rate is dependent on pore length and time, the rate of flow the opening of the channel as discussed in Section 4.2 for the filled pore model.

As derived in Appendix C, the length of bed dried in a particular time (t) can be found as follows:

$$
y = \frac{D_p}{9} \left\{ \frac{Mt}{\eta (1-\epsilon) RT} \left[1 + \frac{3(1-\epsilon)}{4} (1 - \sqrt{\left(\frac{8}{1-\epsilon} - 3\right)} \right] P_g (P_2 - P_1) \right\}^{\gamma_2} \tag{25}
$$

where:

 $P_n = (P_2 + P_1)/2$

This equation is similar to Equation 9.2-10 in (Pajunen 1997) except that the latter is multiplied by a shape factor. For comparisons the lengths of dried bed are calculated for the two porous bed flow models with the same constants given in Figure 9.2-4 in the Reference. After drying at 50 °C for 1 hour, the lengths of dried bed are calculated to be:

 $y = 2.65$ cm from Equation (25), this work

L, = 1.78 cm from Equation 9.2-10, HNF-1851

The present porous bed model predicts that the cracks filled with porous (Pajunen 1997). If the present model considers a shape factor (ϕ_s) of 0.6 for mass in the damaged fuel is dried faster by 33% than the one proposed in the length of dried bed in the form of

$$
L_1 = y \times \phi_s \tag{26}
$$

L, is found to be 1.59 cm from the present flow model, which is 11% shorter than L_2 . Therefore, they appear to agree each other if both are assumed to have a same shape factor.

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5.0 **RESULTS AND DISCUSSION**

The mass flow rate of saturated steam flowing through a long or short tube of defects in uranium fuel has been estimated based on the gas flow models. The rate of vapor exhaust is also estimated for fuel defects full of water vapor without liquid. All of the flow rates are found to depend on the water vapor pressure and the size of defect. Without knowledge of the type and size of defect, estimates of residual water inventory and time required to release trapped water from fuel subsurface voids during the CVD are difficult to make. However, rough estimates can be made by assuming the sizes of all fuel defects are the same.

5.1 **RELEASE OF TRAPPED WATER**

In damaged fuel, water may be trapped in the following one or more types of fuel subsurface voids evaluated (Figure 1):

- 1. A long tube of radius a and length *e* constrained by a water pocket of volume V
- An orifice of radius a with a water pocket of volume V 2.
- A pore (a and *e)* without pocket is filled with water **3.**
- A long tube and a pocket like Type 1 defect but full of vapor only 4.
- *5.* A pore is filled with water and a bed of granular particles.

for purposes of comparisons, the flow rates and thus the time spent to remove the water vapor from the four types of defect are calculated using the above rate equations with the following assumptions. The ratio of defect length to diameter is 100.

 $a = 50 \mu m$, $\ell = 5 \text{ cm}$, $V = 1 \text{ cm}^3$, m (mass of water) = 0.53 g,

 $T = 50 °C$, $P = 12334 Pa$, and $P_a = 400 Pa$.

Results are listed in Table 1. Because defect Type 1 contains much more water than Type 4, it takes a longer time to reduce the pressure in Type 1 pocket to the system pressure. The water trapped in other types of defect generally does not take long to flow out of voids during the CVD process.

the latter flows at a much higher rate. The water in defect Type 1 will continue to flow out at a constant rate until no liquid is left, the flow rate then slows down to equal the rate of Type **4.** Clearly, the fewer the Type 1 defects in fuel the easier the trapped water will be released. Since the damaged fuel is likely to have more defect Types 2, **3,** 4, or **5** most of free water should be removed during the 10-hour CVD process. Defect Types 1 and 2 contain the same amount of water, but the vapor in

Table 1. Mass Flow Rate and Time Required to Release Trapped Water. (1.50 m) , $\ell = 5 \text{ cm}$, $V = 1 \text{ cm}^3$, $m = 0.53 \text{ g}$, $T = 50^{\circ}$ °C)

5.2 PRESSURE RISE TESTS

The results of similar flow rate evaluations have been used to define pressure rise testing following the cold vacuum drying to demonstrate the residual free water inventory of an Multi-Canister Overpack (MCO) (Pajunen 1997). At a constant temperature of 50 °C and the fuel container maintained at a pressure less than 30 Torr, the pressure increase and test time will be recorded. The measured increase in pressure is obtained by dividing the pressure increase with the test time. This pressure increase is converted to the mass flow rate using the following equation:

$$
W_r = \text{MV}(P_2 - P_1)/\text{RTt}
$$

A measured rate of pressure rise in Torr/hr can be converted to a measured mass rate in g/s through Equation (27). In pressure rise testing, the fuel sample was isolated from flowing gas and vacuum pumping, the pressure increase was monitored and the total water release was measured with a moisture monitor at the CVD conditions. The results of a specimen of undamaged fuel element 1990 is shown in Figure 10 where moisture partial pressure was recorded in terms of time at 53 °C (Marschman 1998). Results show that the moisture partial pressure increases linearly with time during CVD for 1 hour. Because the sample was cut from an undamaged fuel element, there was no water trapped in subsurface voids; the sources of water vapor that contributed the pressure rise was the surface film of the sample.

If the value of measured pressure rise rate (w_+) is higher than that calculated w₁ from Equation (15) or w₂ from Equation (17), more than one
defect should exist in damaged fuel. The magnitude of w₁ is dependent on the
type and size of fuel crevice and is equal to the sum of flow rate types of crevices present in the sample. The flow mass in defect type 4 is considered to be a small part of defect type 1. To estimate the maximum residual water (M,) for one type of defect with each defect assumed to be constrained to a spherical water pocket with a diameter of D, the residual water is given by (Pajunen 1997)

$$
M_u = w_t \rho(\pi/6) D^3/w_t
$$

 (28)

 (27)

where w_i is w₁, w₂, or w₀ given in Equations (15), (16), or (17) which is a function of defect size. As indicated by Equation (28), for a value of w_t selected from pressure rise testing, the residual water can be estimated only if the ratio of water pocket and defect size (diameter ratio D/d) is known because wi is dependent upon the parameter d. [Figures](#page-45-0) **11** and **12** show the dependence of D/d on w, and temperature for the long tube model and orifice models for residual waier content of **1** gram, respectively. As [Figure](#page-45-0) **¹¹** implies, the ratio D/d is no more than **120** for a pressure rise rate of **3 Torr/h.** Converting the pressure rise to w_t using Equation (27) and residual water should be less than **1** gram to meet the selected test criterion. From Figure **4** a single water pocket must be constrained by a defect of radius **50 pm** for the selected w_r; thus, the value of $D=120 \times 100 \mu = 1.2$ cm confirms 1 g water in the pocket. The result also proves that $w_r = w_1$, and $M_w = \pi D^3/6$, as shown in Equation (28), which indicates that the value of D is obtained only if the amount of residual water is given, and that the value of D/d is unknown unless the entire system has only one single defect and water pocket.

Since the fuel elements contain more than one defect, the relationship between water pocket size (0) and the size (d) of its constrained defect has to be established theoretically or empirically before the residual water in damaged fuel can be estimated.

5.3 ESTIMATE OF RESIDUAL WATER

Assuming the measured increases in pressure are 1 x 10 $^{\circ}$ or 1 x 10 $^{\circ}$ g/s
at 50 °C and 1 x 10 $^{\circ}$ g/s at 75 °C for the orifice model, the values of D and a
(radius =d/2) can be computed from Equations (15) and of residual water [\(Figure](#page-47-0) **13).** Because there are more than one defect, the pocket diameter can be any value other than **1.2** cm for **1** g of residual water depending on the measured increase in pressure (see curves **1, 3, 5** in [Figure](#page-47-0) 13). With the measured pressure rise (w_t) in terms of mass and the known values of D and *a,* the amount of residual water can be estimated from similar plots of D versus a constructed for various amounts of residual water.
For example, a measured w_t of 1 x 10 g/s predicts 1 g of residual water if the values of D/a is either 0.75 cm/50 μ (= 75, point x in [Figure](#page-47-0) 13) or **1.17** cm/100 μ (= 58, point y); the residual water content would be 2 g if D is larger **(1** cm) with a same value of a **(100** *p)* as designated by point z in [Figure](#page-47-0) **13.** Although the curve of pocket diameter versus defect radius is nearly linear, the ratio *D/a* is not unique in relating to residual water close to 74 but predict different amounts of residual water (1 g and 2 g).

The residual water (M_w) is described by the values of geometric features, D and *a*, for each measured increase in pressure (w_r) at a given temperature (T) as shown in Equation **(28).** Using Equation **(28** the size of defect can be estimated from two pressure rise tests at two different temperatures with two sets of w_t , P, and T. The value of defect radius a is calculated from the following equation:

$$
f_1(a, P)/f_2(a, P) = (w_{1t}/w_{2t})(P_2^2 - P_1)/(P_2^1 - P_1)(T_1/T_2)^h
$$
 (29)

$$
^{25}
$$

where:

$f(a, P) = 0.1472a(P_2+P_1) + (1+2.507a(P_2+P_1))/(1+3.095a(P_2+P_1))$

With the value of *a* determined, the mass flow rate can be evaluated and Equation (28) is pocket diameter D. For pressure rise tests performed at 50 $\,$ the only unknown variable for estimating the residual water inventory in and 75 °C, the defect radius of the damaged fuel element is evaluated as a
function of w_{it}/w_{2t} ratio as shown in Figure 14. The defect size is small for
w_{it}/w_{2t} < 8, but becomes sensitive to the rate ratio as it a plot shown in Figure 14 indicates that there is a large uncertainty in determining defect size using Equation (28) if the pressure rise rate ratio (75 "C to 50 "C) is larger than 9.3. Since the value of M, is linearly dependent on D which cannot be evaluated from pressure rise tests, the present analysis is capable of determining the residual water inventory of damaged fuel only if the pocket size of voids can be measured.

On the other hand, the value of D can be calculated from Equation (28) if and heating at 50 to 75 °C for a long period of time and summing up the total amount of moisture monitored throughout the entire test. Thus, the geometric features of subsurface voids in damaged fuel can be characterized by performing two pressure rise tests at different temperatures and heating the fuel element to a high temperature at the end of the test. The test data will be used to evaluate the pocket and defect sizes for each damaged fuel elements. The following is sample calculation made from results of damaged fuel element 0309M (Oliver 1998).

in Figure 15. The rate of water vapor pressure increase was 0.28 Torr/hr, the
mass flow rate was calculated to be w_{zt} = 2.25 x 10 ⁴ g/s using Equation (27) with $V = 10,000 \, \text{cm}^3$. temperatures, w_{it} = 1.98 x 10 ° g/s at 75 °C is assumed giving w_{it}/w_{2t}
Thus, the defect radius and mass flow rate are found to be 40 μ m and
1.97 x 10 ⁵ g/s according to Figures 14 and 7, respectively. Because hydrates will be decomposed to release hydrated water at temperatures higher than 75 "C, only the water released during the entire CVD test is used as the amount of residual water to estimate the pocket size. Results of drying tests on element 0309M show 5 g of water released during the CVD process; therefore, the value of **D** is calculated to be 0.94 cm. The above data are summarized as follows: Results of pressure rise test performed after the CVD at 50 **'C** are shown There is no pressure rise test performed at other
98 x 10⁻³ g/s at 75 °C is assumed giving w_{1t}/w_{2t} = 8.8. g/s using Equation (27) Because more

Sample: damaged fuel element 0309M Test temperature = 50° C, 75 $^{\circ}$ C Pressure rise rate wZt = 2.25 *x* g/s at 5OoC Assumed pressure rise rate w_{1t} = 1.98 x 10⁻³ g/s at 75 °C HNF-2191, Rev. 0

Defect mass flow rate w_o = 1.9/ x 10 $^{\circ}$ [g/s at 50](#page-52-0) $^{\circ}$ C Defect radius $a = 40 \mu m$ Residual water $M_{w} = 5$ g Pocket diameter D = 0.94 cm

6.0 CONCLUSION

Water trapped in fuel subsurface voids will evaporate easily during the cold vacuum drying at 50 **'C.** The difference between the water vapor pressure and the operating pressure provides the driving force for the vapor to flow out from water pockets. Five flow models are developed to evaluate the flow rates based **on** the kinetic theory of gases. Depending on defect size the flow rates of long tube and orifice flow models vary significantly; the rates drop rapidly as the defect radius decreases to less than 100 μ m. Water trapped in a pore without a pocket evaporates from the open end with the water interface receding at fairly high rates. The flow rate of water vapor through a bed of granular particles is found to be much slower than the rate through a pore filled of only water. **In** general, most of the water trapped in damaged fuel drying. The flow behavior of vapor through crevices is analyzed: results of pressure rise tests performed during the CVD process may be used to estimate the initial residual water in the damaged fuel elements provided the geometric features of the voids (D,d) are known.

7 .O REFERENCES

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Figure **3.** Ratio of Total Conductance to Molecular **Flow** Conductance (F/F,) Versus a/L, for Cy1 indrical Tube.

Figure **4.** Variation of **Mass** Flow Rate with Defect Radius for Long Tube Model.

DEFECT RADIUS, micron

Figure 6. Mass Flow Rate Versus Test Duration for Filled Pore Model.

Figure **7.** Variation of Mass Flow Rate with Defect Radius for Orifice Flow Model.

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Figure 10. Plots of Pressure Rise Versus Time
for Undamaged Spent Nuclear Fuel Element 1990.
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Figure 11. Ratio Pocket Diameter/Defect Diameter Versus Pressure Rise Mass Rate for Long Tube Model.

PRESSURE RISE MASS FLOW RATE, gm/s

Figure 12. Ratio Pocket Diameter/Defect Diameter Versus Pressure Rise Mass Rate at Various Temperatures for Orifice Flow **Model.**

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[Figure 14](#page-16-0). Defect Radius Determined from Pressure Rise **Rate Ratio at 50 OC and 75 OC for Orifice Model.**

Moisture Partial Pressure (mTorr)

APPENDIX A

EVAPORATION RATE

 \sim $^{-1}$

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APPENDIX A

EVAPORATION RATE

According to the kinetic theory of gases the number of the vapor molecules striking unit area of a surface per unit time is (Bird 1976)

$$
V = Nv_a/4 \tag{A1}
$$

where N it the number of molecules per cm^y and v_a is the mean velocity of the
molecules. The mass of the vapor molecule striking the unit free surface per unit time is

$$
G = \rho v_a / 4 \tag{A2}
$$

where ρ is the density of the vapor. For an ideal gas

$$
PV = nRT = GRT/M
$$

\n
$$
\rho = G/V = PM/RT
$$
\n(A3)

where M is the molecular mass in grams. Combining Equations (A2) and (A3) gives

$$
G = PMv_a/4RT
$$
 (A4)

Gas dynamic theory also gives the pressure in terms of root-mean-square molecule speed, V_{rms}

$$
P = \rho v_{\rm rms}^2 / 3 \tag{A5}
$$

The mean molecular velocity is related to the root-mean-square velocity through Maxwell-Boltzmann distribution of gas molecular velocity (Bird 1976) as follows.

> $v_a = 2\sqrt{(2\kappa T/\pi m)} = 2\sqrt{(2RT/\pi N_a m)}$ $(A6)$ $= 2\sqrt{(2RT/\pi M)}$

$$
V_{rms} = \sqrt{3 \times 1/m}
$$
 (A/1)

where $\kappa = 1.38054 \times 10^{-23}$ J/K is Boltzmann constant and m is mass of particle.

velocities is From Equations (A6) and (A7) the relationship between these two molecular

$$
v_{a} = v_{rms} \sqrt{(8/3\pi)} \tag{A8}
$$

Combining Equations (A4), (A5), and (A8) gives the rate of the vapor mass striking the surface

$$
G = P\sqrt{(M/2\pi RT)}
$$
 (A9)

APPEND I X -B

SLIP FLOW RATE

APPENDIX B

SLIP FLOW RATE

The viscous conductance corrected for slip is (Kennard 1938)

$$
F = \frac{\pi a^4}{8\eta l} P_g (1 + \frac{4\eta (2 - f)}{f a P_g} \left(\frac{\pi T R_o}{2M}\right)^{1/2})
$$
 (B1)

Equation (BI) can be rewritten to be

$$
F = F_v + \frac{3\pi}{16} \frac{2 - F}{F} F_t
$$
 (B2)

 $F_{V} = \pi a^4 P_{g}/8n \ell$

where F, is the Viscous conductance from Equation (9) and F, the molecular
flow conductance from Equation (10). The value of f in this equation is the
whole conductance from Equation (10). The value of f in the sequation i transfer ratio *of* momentum ranging between 0.8 and 1.0. Equation (13) clearly combines the characteristics of both viscous and molecular ranges. An empirical relation with'pressure dependence was formulated by Knudsen for slip flow in the transition range:

> $F = F_{+}(F_{y}/F_{+} + Z)$ $(B3)$

where

 $Z = (1 + 2.507(a/L_a))/(1 + 3.095(a/L_a))$ and F_v/F_v can be expressed in terms of a/L_a using $n = 0.499 \rho v_a L_a$ obtained from Equation (6), Equations (A3) and (A6).

$$
F_v/F_t = 0.1472(a/L_a)
$$
 (B4)

Equation (B3) also can be written in terms of viscous conductance F_{ν} :

$$
F = F_v (1 + 7.793(L_a/a)Z)
$$
 (B5)

APPENDIX C

INTERFACE IN FILLED PORE MODEL

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APPENDIX C

INTERFACE IN FILLED PORE MODEL

Assuming a 5-cm long pore with a radius of 50 **pm** is filled with water and the distance from the interface to the opening is y, an increment of y due to water vapor removal **is**

 $\Delta y = w \Delta t / \pi a^2 \rho$

Thus,

$$
dy/dt = 2.1656 \times 10^{-6} F(P_2 - P_1)/\pi a^2 T
$$
 (C1)

At the orifice $F = F_0$, use of Equation (C1) and Equation (12) gives

$$
dy/dt = 0.3899/(1 + 0.375y/a)
$$

\n
$$
y = ((25.44a2 + 8.32at)% -5.333a)/2
$$
 (C2)

 $(C3)$

for

 $0 < y \le 8a/3$ and $T = 50$ °C

At y = 8a/3 [see Equation (12)], t = 0.053 **s. As** the interface recedes to y = 8a/3 (0.0133 cm), the flow rate is estimated for slip flow and
F = F_t(0.1472(a/L_a) + Z) as given in Equation (13). The change rate of y is computed using Equation $(C1)$ and Equation (11) .

 $dy/dt = 10.34a/y$ for $y > 8a/3$ and $T = 50 °C$

 $y^2/2 = 10.34$ at + C

 $C = -0.00265$ for the initial conditions at $t = 0.053$ s for flow at orifice.

 $y = (20.68at - 0.0053)^{1/2}$ $t = (y^2 + 0.0053)/20.68a$

 \bar{z}

 \bar{z} $\sim 10^{-1}$