Modeling the Hall Thruster

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Abstract

The acceleration of the plasma in the Hall thruster to supersonic velocities is examined by the use of a steady state model. Flows that are smooth across the sonic transition plane are found. The possibility of generating flows in which the acceleration across the sonic plane is abrupt, is also studied.

I. Introduction

Since the introduction of the Hall thruster concept,[1–6] there has been a continuous effort to explore the dependence of the thrust, the specific impulse, and the efficiency on various parameters, such as thruster dimensions, magnetic field, applied voltage, ion mass, electron conductivity, and mass flow rate.[7–33] Recently, we have introduced a simple theoretical model,[34] that enabled us to identify dimensionless parameters that govern the thruster performance and to perform a fast parametric study. A good agreement was found between the predictions of the model and the experimental measurements.[23] We later extended this comparison to measurements at various channel lengths.[35]

In our theoretical analysis[34] we have noted a singularity in the equations due to the presence of the sonic transition. Our purpose in the present paper is to further examine the acceleration of the plasma in the Hall thruster to supersonic velocities. We discuss the conditions needed for the flow to be smooth at the sonic transition plane and compare them to those at other physical systems in which there is a smooth acceleration.[36–40] We explain why the geometry effect that plays a crucial role in the generation of a smooth flow through the sonic transition in those other devices is not essential in the Hall thruster.

An additional purpose of this paper is to explore the possibility of an abrupt acceleration at the sonic transition plane. In accelerators the ability to generate a localized electric field and the ability to control the location could be a great advantage. A non neutral region with a strong electric field is known to be formed at a plasma-surface boundary.[41,37] Non-neutral regions, usually called double layers, are also known to be formed in a plasma far from its boundary in space and laboratory plasmas.[42,43,37] Anode layers [44,45] are known to possess such a double layer. There are no systematic and clear studies of the anode layer, and the configuration used seems quite rigid. We present here a configuration where a large electric field is formed inside the plasma. The localization of the electric field does not rely on a lo-

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calization of the magnetic field, and it occurs even when the magnetic field is uniform. The reason for the occurence of this large electric field is an imposed discontinuity in the thruster parameters at the sonic transition plane. In the future we intend to investigate the usefulness of this effect for the thruster.

In Sec. II. we briefly describe the model, that is similar to the model we introduced previously.[34] In the present paper we add to our model an energy equation for the electrons, rather than specifying a uniform electron temperature, as we have done before. In addition to being a more realistic description of the processes in the Hall thruster (as also indicated by the comparison mentioned above[35]), this more general treatment allows us to demonstrate the possibility of an abrupt plasma acceleration across the sonic plane. In Sec. III we examine the sonic transition in the Hall thruster and in other devices and discuss smooth and abrupt accelerations. In Sec. IV we show two examples of flows in the Hall thruster, one with a conventional magnetic field configuration and one with an abrupt acceleration through the sonic plane in a uniform magnetic field. We conclude in Sec. V.

II. The governing equations

In this section we present the governing equations. We assume that all quantities depend mainly on x, the coordinate along the thruster axis, and we approximate the equations by a one-dimensional (1D) geometry. We first write the continuity equation for the ions

$$\frac{d}{dx}(n_i v_i A) = SA \ , \tag{1}$$

where the source function due to ionization is

$$S = \beta n_e n_a \ . \tag{2}$$

Here $\beta \equiv < \sigma v_t >$, σ being the ionization cross section and v_t the electron thermal velocity, n_i, n_e and n_a the densities of the ions, the electrons, and the neutral atoms, v_i is the ion flow velocity, and A is the cross section of the thruster channel. The sign $<>$ denotes averaging over the electron distribution function.

The ion dynamics is governed by the ion momentum equation

$$\frac{d}{dx}(M n_i v_i^2) = -e n_i \frac{d\phi}{dx} \ , \tag{3}$$

where the ions are assumed collisionless and unmagnetized. Here M is the ion mass, and ϕ is the electric potential. As in our previous papers,[34,35] we make the simplifying approximation of neglecting the ion pressure in the momentum equation for the ions. Since ions are born through ionization along the thruster the ion pressure is not negligible. Ions are almost collisionless and there is no simple equation of state that relates their pressure to the lower moments. The theory can be generalized to describe the ions kinetically. Although here we treat the ions as a cold fluid, we do retain nevertheless the ion production term through ionization, which, as will be shortly shown, appears as an effective drag term. The neglect of loss terms at the walls also simplifies our analysis.

The electron dynamics is governed by the electron momentum equation

$$-v_{ex} = \mu \left[-\frac{d\phi}{dx} + \frac{1}{e n_e}\frac{d}{dx}(n_e T_e) \right] . \tag{4}$$

Here

$$\mu \equiv e\nu / m\omega_c^2 \tag{5}$$

is the mobility of the electrons across the magnetic field, where e, m, ω_c and ν are the electron charge, mass, cyclotron frequency and collision frequency. In writing Eq. (5) we used the assumption that $\nu \ll \omega_c$. Also, v_{ex} and T_e are the electron axial velocity and temperature. The electron temperature is governed by the energy equation.

$$\frac{d}{dx}\left(\frac{5}{2} j_e T_e A\right) = e j_e \frac{d\phi}{dx} A + e\epsilon_i S A + e\beta' n_e c_s T_e d \ . \tag{6}$$

The electrons are heated by Joule heating, expressed by the first term on the right hand side (RHS) of the equation. Also included are two energy loss mechanisms. The first loss mechanism is due to ionization, described by the second term on the RHS, ϵ_i is the ionization energy. The second loss mechanism is due to collisions with the walls followed by electron cooling due to secondary electron emission. The electron flux density to the wall is assumed to be $n_e c_s$ and β' is the fraction of energy loss per one electron hitting the wall due to secondary electron emission. Here $c_s \equiv \left(2T_e/M\right)^{1/2}$ is the ion acoustic velocity, and d is the length of the wall in the direction perpendicular to the x direction.

We now write the equations for the four unknowns j_i, v_i, T_e and ϕ. They are

$$\frac{dj_i}{dx} + j_i \frac{d \ln A}{dx} = eS \ , \tag{7}$$

$$M v_i \frac{dv_i}{dx} + \frac{MSv_i}{n_i} = -e\frac{d\phi}{dx} \ , \tag{8}$$

$$\begin{aligned} \frac{(j_T - j_i)}{n_e \mu} =& - e\frac{d\phi}{dx} \\ &+ \frac{dT_e}{dx} + \frac{T_e}{n_e}\left[\frac{1}{ev_i}\frac{dj_i}{dx} - \frac{j_i}{ev_i^2}\frac{dv_i}{dx} - \frac{d\Delta n}{dx}\right] \ , \end{aligned} \tag{9}$$

and

$$\frac{dT_e}{dx} = \frac{2}{5}e\frac{d\phi}{dx} + \frac{eS}{j_e}(\frac{2}{5}\epsilon_i + T_e) + \frac{2}{5}\frac{d}{A}\frac{e\beta' n_e c_s T_e}{j_e} \ . \tag{10}$$

We expressed the electron velocity by $v_{ex} = j_e/(en_e)$, and then the electron current by $j_e = j_T - j_i$, where j_T is the total current density, $j_T = I_d/A$, and I_d is the discharge current. Also, $\Delta \equiv n_i - n_e$. Note that in Eq. (8) the ion production term appears as an effective drag term, as we mentioned above.

The equations are now simplified by assuming quasi-neutrality, $\Delta n = 0$, and therefore we define $n \equiv n_i = n_e = j_i/ev_i$. While the continuity equation is not changed as a result of this assumption, the other three equations become

$$M v_i \frac{dv_i}{dx} + \beta n_a v_i = -e\frac{d\phi}{dx} \ , \tag{11}$$

$$\begin{aligned} \frac{(j_T - j_i)ev_i}{j_i \mu} =& -e\frac{d\phi}{dx} + \frac{dT_e}{dx} \\ &+ T_e\left[\frac{1}{j_i}\frac{dj_i}{dx} - \frac{1}{v_i}\frac{dv_i}{dx}\right] \ , \end{aligned} \tag{12}$$

and

$$\begin{aligned} \frac{dT_e}{dx} =& \frac{2}{5}e\frac{d\phi}{dx} + \frac{e\beta n_a}{v_i}\frac{j_i}{j_T - j_i}(\frac{2}{5}\epsilon_i + T_e) \\ &+ \frac{2}{5}\frac{d}{A}\frac{\beta' j_i c_s T_e}{v_i(j_T - j_i)} \ . \end{aligned} \tag{13}$$

We now combine the equations to derive an equation for v_i:

$$\begin{aligned} (\frac{6}{5}v_i^2 - c_s^2)\frac{dv_i}{dx} =& \frac{2ev_i^2}{M\mu}\left(\frac{j_T - j_i}{j_i}\right) \\ &- \beta n_a\left[\frac{6}{5}v_i^2 + c_s^2 + \frac{j_i}{(j_T - j_i)}(c_s^2 + \frac{2}{5}\frac{2\epsilon_i}{M})\right] \\ &+ c_s^2 v_i \frac{d\ln A}{dx} - \frac{2}{5}\frac{d}{A}(\beta' c_s)c_s^2\frac{j_i}{(j_T - j_i)} \ . \end{aligned} \tag{14}$$

This form of the equation exhibits the singularity at the sonic transition plane. With this expression for the derivative of the flow velocity we can now write the equations for the other two derivatives. The derivative of the potential is

$$e\frac{d\phi}{dx} = -Mv_i\frac{dv_i}{dx} - M\beta n_a v_i \ , \tag{15}$$

and that of the electron temperature

$$\begin{aligned} \frac{dT_e}{dx} =& \frac{2}{5}e\frac{d\phi}{dx} \\ &+ \left[\beta n_a(\frac{2}{5}\epsilon_i + T_e) + \frac{2}{5}(\beta' c_s)T_e\right]\frac{1}{v_i}\frac{j_i}{(j_T - j_i)} \ . \end{aligned} \tag{16}$$

We express the neutral density as

$$n_a = \frac{\dot{m}}{MAv_a} - \frac{j_i}{ev_a} \ . \tag{17}$$

Here $\dot{m}$ is the mass flow rate. In writing Eq. (17) we used the fact that the sum of the ion flux and the neutral flux is constant along the thruster. We also assumed that the neutrals move balistically while their density varies along the channel due to ionization and due to variations in the cross section along the channel.

Equations (7) and (14)-(16) are the governing equations. We write the equations also in dimensionless form.

The dimensionless continuity equation is

$$\frac{dJ}{d\xi} = -J\frac{d\ln A}{d\xi} + p\frac{J}{V}(1-J) \ . \qquad (18)$$

Similarly, the ion momentum equation becomes

$$\begin{aligned}(\frac{6}{5}V^2 - C_s^2)\frac{dV}{d\xi} &= \frac{V^2}{\mu_N}\left(\frac{J_T-J}{J}\right) \\ &- p(1-J)\left[\frac{6}{5}V^2 + C_s^2 + \frac{J}{(J_T-J)}(C_s^2 + \frac{2}{5}\epsilon_I)\right] \\ &+ C_s^2 V\frac{d\ln A}{d\xi} - \frac{2}{5}(\beta'_N C_s)C_s^2\frac{J}{(J_T-J)} \ . \end{aligned} \qquad (19)$$

The energy equation is

$$\begin{aligned}\frac{dC_s^2}{d\xi} &= \frac{2}{5}\frac{d\psi}{d\xi} \\ &+ \left[p(1-J)(\frac{2}{5}\epsilon_I + C_s^2) + \frac{2}{5}(\beta'_N C_s)C_s^2\right]\frac{1}{V}\frac{J}{(J_T-J)} \ , \end{aligned} \qquad (20)$$

where $\epsilon_I \equiv \epsilon_i/(e\phi_A)$. Finally, the equation for the electric potential is

$$\frac{d\psi}{d\xi} = -2V\frac{dV}{d\xi} - 2pV(1-J) \ . \qquad (21)$$

Equations (18)-(21) are the dimensionless governing equations. In these equations the dimensionless unknowns are the ion current $J \equiv j_i MA/e\dot{m}$, the ion flow velocity $V \equiv v_i/v_0$, the electron temperature $C_s^2 \equiv T_e/e\phi_A$ and the electric potential $\psi \equiv \phi/\phi_A$. Here ϕ_A is the applied voltage. The normalized coordinate is $\xi \equiv x/L$. The dimensionless parameters that appear in the equations are

$$\mu_N \equiv \frac{\mu\phi_A}{Lv_0} \ , \qquad (22)$$

$$\beta'_N \equiv \beta'\frac{Ld}{A} \ , \qquad (23)$$

and

$$p \equiv \frac{L\beta\dot{m}}{v_a v_0 MA} \ . \qquad (24)$$

We note that we can characterize the thruster performance by our dimensionless variables. The propellant utilization, $\eta_m \equiv (MAj_i/e\dot{m})$, is J, the current utilization $\eta_C \equiv j_i/j_T$, is J/J_T, the energy utilization $\eta_E \equiv (v_i/v_0)^2$, is V^2, and the total efficiency, the product of these three efficiencies, is $\eta_T = J^2V^2/J_T$. We can also express the specific impulse as $I_{sp} = JVv_0/g$ and the thrust as $T = JV\dot{m}v_0$. In these expressions J and V are calculated at $\xi = 1$.

To Equations (18)–(21) for J, V, ψ and C_s^2 we add the boundary conditions: $J(0) = 0$, $V(0) = 0$, $\psi(0) = 1$, $\psi(1) = 0$ and $C_s^2(1) = 0$. The boundary conditions for the ion current and velocity at the anode mean that a monotonically decreasing potential from the anode towards the cathode is assumed and the possibility of a backwards ion flow towards the anode is excluded.

III. Smooth and abrupt sonic transitions

The singularity at the sonic velocity is well known in plasmas in relation to the sheath at a plasma-surface boundary. A sheath is formed at such a boundary if the Bohm criterion is met,[41] that the ion flow into the sheath is supersonic. The quasineutral equations yield an infinite electric field, and the sigularity is resolved by resorting to Poisson's equation, which governs the non-neutral sheath. In that case Δn is not zero. The singularity at the sonic transition in the non-neutral region has been studied in detail.[41,37]

Smooth acceleration of a gas to supersonic velocities occurs at the neck of the Laval nozzle where the cross section of the channel is at its minimum.[36] Other cases in which a smooth acceleration occurs also involve geometry effects. In a widening geometry the geometry effect might be cancelled by Joule heating, as happens in the expansion of the cathode spot plasma in vacuum arc discharges.[37,38] Similarly, the geometry effect may be balanced by both the ablative and the dissipative terms in ablative discharge capillaries.[40] Smooth acceleration can occur in the solar wind where the geometry effect is balanced by gravitation.[39] As can be seen in Eq. (19), in our case the source term and the collisional term are of opposite signs. This happens since the electrons and the ions flow in opposite directions in the Hall thruster, in contrast to the other systems we mentioned. Therefore, the source term and the collisional term may balance each other so that a smooth acceleration occurs in the Hall thruster without a geometry effect.

We find solutions that are smooth by requiring that the RHS of Eq. (19) is zero at the sonic plane. We point out that by looking for smooth solutions and by requiring that the RHS of Eq. (19) vanish at the sonic plane, we specify the value of the total current (or equivalently of the electron current emitted from the cathode). We calculate the finite derivative of the flow velocity there by employing L'Hôpital rule and calculating the ratio of the derivatives of the RHS and of the coefficient in front of the derivative on the left hand side (LHS) of the equation. Then, by expressing the derivatives of J and C_s^2 by Eqs. (18) and (20) we find an algebraic second order equation for the derivative of the flow velocity at the sonic plane. With this expression we also find the finite values of the derivatives of the electric potential and the electron temperature at the sonic transition plane. The location of the transition point is found by a shooting method, which assures also that the boundary conditions are satisfied. We integrate the equations from the sonic plane in both directions, towards the cathode and towards the anode. We adjust the location of the sonic tansition plane and the values of the ion current and velocity and the electron temperature there until the solution satisfies the boundary conditions at the anode and at the cathode.

We also look for flows with an abrupt acceleration at the sonic transition plane. We assume that the wall changes at some location along the thruster channel, so that β', the coefficient of secondary electron emission, has a discontinuity. We then adjust parameters, such as the intensity of the magnetic field, until the sonic transition occurs at the location of the discontinuity. While the denominator of Eq. (19) is small in the neighborhood of the sonic transition plane, the numerator is large and changes sign at the plane of discontinuity. The acceleration there is thus very large. We find the solutions by assuming that β' is continuous, though varrying rapidly at that point. Equivalently we assume a jump in β', and find by asymptotic expansion the large value of $dV/d\xi$ near the discontinuity. As expected, the results obtained by both techniques are similar.

In the next Section we present examples of both a smooth and an abrupt acceleration across the transition plane.

IV. Two examples

In this Section we employ the model presented in Section II to calculate the steady-state electric plotential and plasma flow profiles for two configurations. Figure 1 shows the profiles of the magnetic field for the two configurations. The normalized magnetic field in the figure is $b = 1/\mu_N^{1/2}$. The profile of the varying magnetic field (case 1) is similar to the profile of the magnetic field in the Soreq Hall thruster.[19] The second profile (case 2) is of a uniform magnetic field. In case 1 β'_N is assumed zero, while in case 2 the profile of β'_N is as shown in Fig. 1. This varying emission coefficient is due to a change in the wall material along the channel.

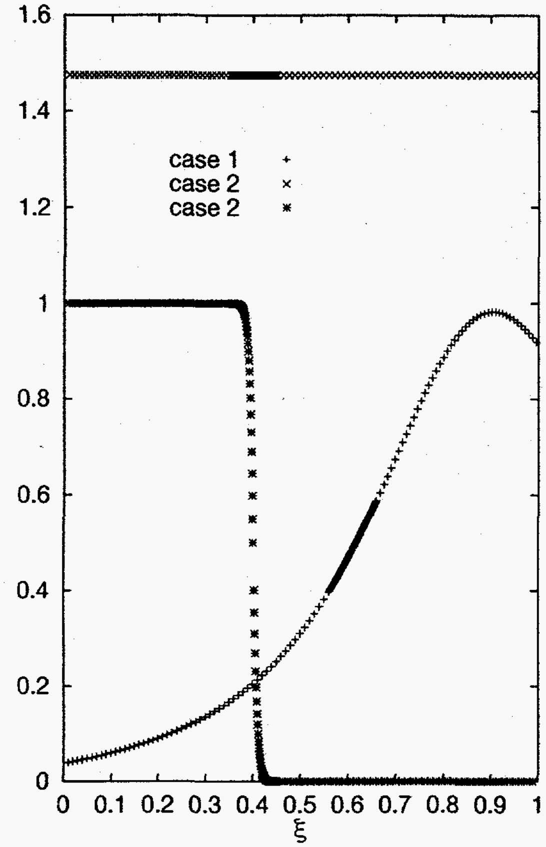

Fig. 1 The magnetic field and the emission coefficient

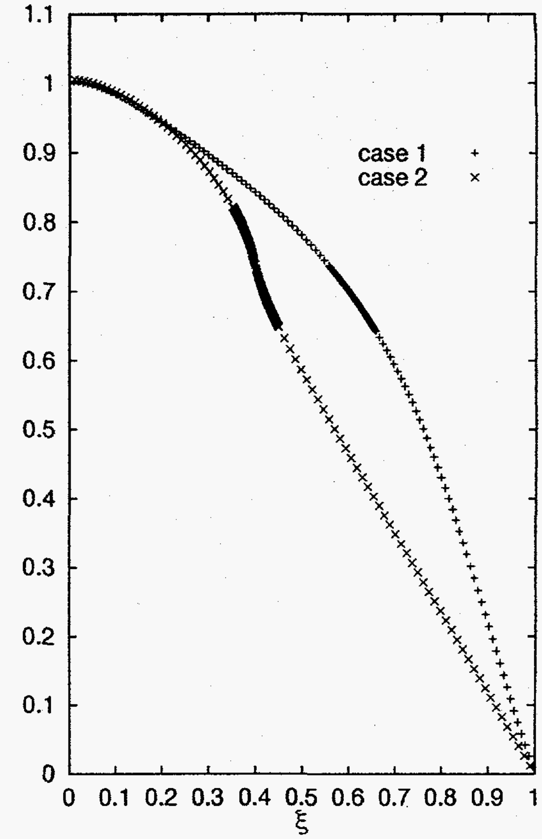

Fig. 2 The potential distribution

We present steady state solutions for the two cases. While in case 1 the acceleration is smooth across the sonic transition plane, in case 2 the acceleration is abrupt. The electric potential, the electric field and the flow profiles for the two cases are compared in the figures 2-6.

Figure 2 shows the distribution of the electric potential ψ. In case 1, the regular Hall thruster configuration, the magnetic field profile determines the distribution of the electric potential along the current channel. The potential drop is large in the area in which the magnetic field is large. In case 2, however, the potential drop is maximal at the region where β'_N changes abruptly, and there the sonic transition plane is located.

The associated electric field profiles are shown in Fig. 3. Again, while in case 1 the electric field is large in the area in which the magnetic field is large, in case 2 the electric field is maximal at the region where β'_N changes abruptly.

The region of the large electric field results in a large acceleration and a large electron temperature gradient. The large gradients in the ion velocity and in the electron temperature are noticed for case 2 in Figs. 4 and 5.

The ion current for the two cases is shown in Fig. 6. As expected, there is no large gradient of the current in any of the two cases.

We note that the total efficiency in case 1 is 32% only, while in case 2 the total efficiency is 66%. The higher efficiency in case 2 is a result of the larger magnetic field in case 2, as shown in Fig. 1.

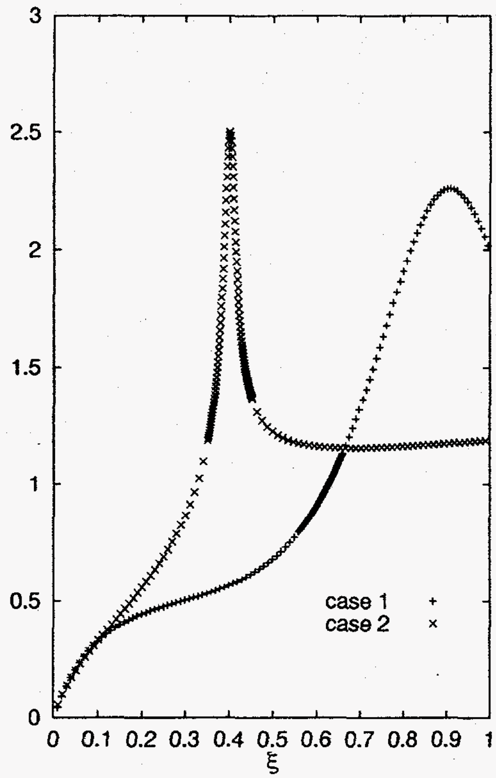

Fig. 3 The electric field distribution

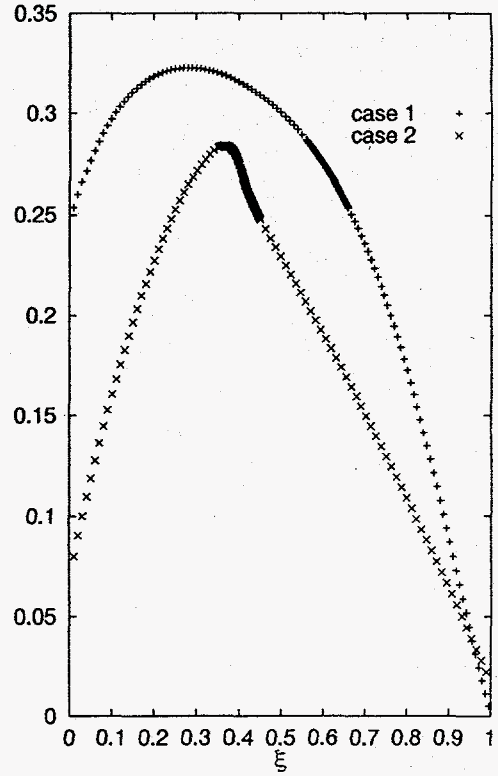

Fig. 5 The electron temperature

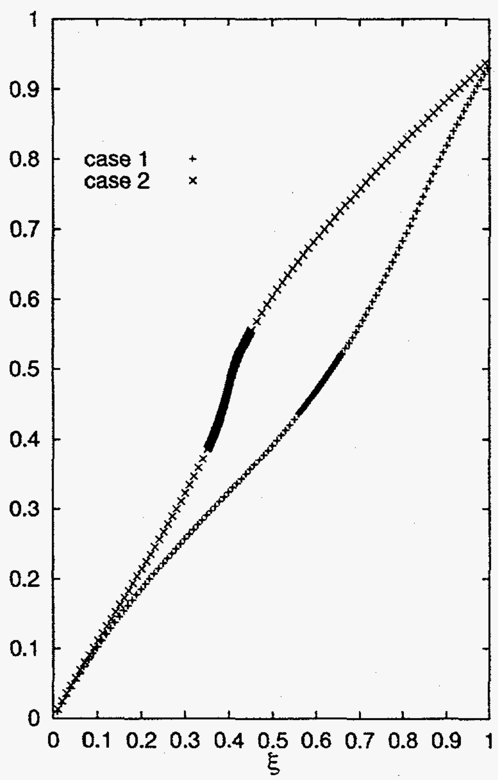

Fig. 4 The ion flow velocity

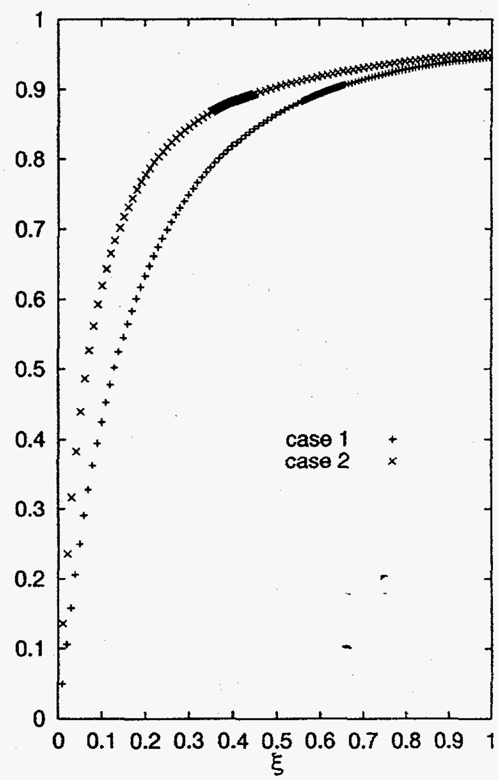

Fig. 6 The ion current J

For this set of parameters ($p = 1.2$, $\epsilon_I = 0.05$), steady state solutions do not exist for a smaller value of μ_N (larger value of the magnetic field).

V. Summary

We have examined the acceleration of the plasma in the Hall thruster to supersonic velocities. We have found the conditions for a smooth acceleration and have explained why a geometry effect is not necessary. Of paticular importance is the possibility we have demonstrated of controlling the location of the maximal potential drop by controlling the location of the sonic transition. This additional "knob" can be used instead of (or in addition to) tailoring the profile of the magnetic field to improve the thruster performance. Configurations that exploit this resulting large electric field are the subject of our ongoing research.

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