Constitutive Representation of Damage Development and Healing in WIPP Salt[#]

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ABSTRACT: A continuum mechanics approach for treating damage healing is formulated as part of a constitutive model for describing coupled creep, fracture, and healing in rock salt. Formulation of the healing term is described and the constitutive model is evaluated against experimental data of rock salt from the Waste Isolation Pilot Plant (WIPP) site. The results indicate that healing anistropy in WIPP salt can be modeled with an appropriate power-conjugate equivalent stress, kinetic equation, and evolution equation for damage healing.

1. INTRODUCTION

There has been considerable interest in characterizing and modeling the constitutive behavior of rock salt with particular reference to long-term creep and creep failure. The interest is motivated by the projected use of excavated rooms in salt rock formations as repositories for nuclear waste. It is presumed that closure of those rooms by creep ultimately would encapsulate the waste material, resulting in its effective isolation.

Complete isolation of waste material can be maintained only when there is an effective sealing system in the shafts. The presence of damage in the form of microcracks in salt, however, can increase the permeability and therefore the potential for fluid flow around the sealing system. Damage development and the healing of damage are therefore important factors affecting the encapsulation process.

The constitutive models for creep deformation in salt are reasonably well developed (Munson and Dawson 1984; Aubertin et al. 1991). Some progress has also been made recently in developing time-dependent, damage-based constitutive models for describing the fracture characteristics of rock salt (Aubertin et al. 1993; Cristescu 1993; Chan et al. 1994a, 1994b). Recent work has shown that damage healing can lead to inelastic flow in rock salt under hydrostatic compression (Brodsky and Munson 1994). None of the constitutive models, however, has incorporated the effects of damage healing on the inelastic and failure responses of rock salt.

In this paper, a continuum mechanics approach for treating damage healing is formulated as an extension of a coupled creep and fracture constitutive model (Chan et al. 1992, 1994a, 1994b) by considering damage healing as a physical mechanism that contributes, together with creep and damage mechanisms, directly to the macroscopic strain rate. Anisotropy of healing by multiple mechanisms is treated in terms of a power-conjugate equivalent stress measure. Appropriate power-conjugate equivalent stress and strain rate measures are developed for the healing term together with the corresponding flow law, kinetic equation, and damage evolution equation. A scalar damage variable in the context of Kachanov (1956) is used to describe both damage growth and healing. A summary of the model is presented with an evaluation of the proposed approach by comparison of model calculation against experimental data of WIPP salt.

2. THE MULTIMECHANISM DEFORMATION COUPLED FRACTURE (MDCF) MODEL

Inelastic flow in rock salt can proceed by dislocation, microfracture, and damage healing mechanisms. Creep due to three different dislocation mechanisms is considered in the constitutive equations formulated by Munson and Dawson (1984), which have been referred to as the Multimechanism Deformation (M-D) model. The M-D constitutive equations have been extended to include continuum, isotropic damage as a fully coupled variable that enhances the stress influence by reduction of the effective area and also contributes directly to the inelastic strain rate. The total inelastic strain rate equation thereby becomes pressure dependant since the subsidiary equations include the effect of pressure to suppress damage development, i.e., the opening of microcracks. The extended model, referred to as the Multimechanism Deformation Coupled Fracture (MDCF) model, has been applied successfully for representing the creep and fracture response of WIPP salt subjected to triaxial compression. (Chan et al. 1994a, 1994b)

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In the MDCF formulation (Chan et al. 1992, 1994a, 1994b), the total strain rate, $\dot{\varepsilon}_{ij}$, for a solid deformed under isothermal conditions is given as the sum of the elastic strain rate, $\dot{\varepsilon}_{ij}^c$, and the inelastic strain rate, $\dot{\varepsilon}_{ij}^I$. The latter is described as a generalized kinetic equation that contains both creep and damage terms. To incorporate damage healing, a healing term is added to the generalized kinetic equation, leading to

$$\dot{\varepsilon}_{ij}^I = \frac{\partial \sigma_{eq}^c}{\partial \sigma_{ij}} \dot{\varepsilon}_{eq}^c + \frac{\partial \sigma_{eq}^{\omega_s}}{\partial \sigma_{ij}} \dot{\varepsilon}_{eq}^{\omega_s} + \frac{\partial \sigma_{eq}^{\omega_t}}{\partial \sigma_{ij}} \dot{\varepsilon}_{eq}^{\omega_t} + \frac{\partial \sigma_{eq}^h}{\partial \sigma_{ij}} \dot{\varepsilon}_{eq}^h \qquad (1)$$

where σ_{eq}^c, $\sigma_{eq}^{\omega_s}$, $\sigma_{eq}^{\omega_t}$, $\dot{\varepsilon}_{eq}^c$, $\dot{\varepsilon}_{eq}^{\omega_s}$, and $\dot{\varepsilon}_{eq}^{\omega_t}$ are power-conjugate equivalent stress measures and equivalent inelastic strain rates for the creep, shear damage, and tensile damage mechanisms, respectively. The parameters represented by σ_{eq}^h and $\dot{\varepsilon}_{eq}^h$ are the conjugate equivalent stress and strain rate measures for damage healing. The formulation of the healing term is described in this paper, while those of the creep, shear damage, and tensile damage are described elsewhere (Munson and Dawson 1984; Munson et al. 1989; Chan et al. 1992, 1994a, 1994b).

3. DEVELOPMENT OF THE DAMAGE HEALING TERM

Healing of damage in rock salt can be considered to proceed by the closure of open microcracks and the removal of microcracks by healing. Both of these processes can be considered to be driven by a similar, if not identical, thermodynamic driving force represented by a pertinent power-conjugate equivalent stress measure. If the healing process is isotropic, the appropriate power-conjugate equivalent stress measure is the first invariant, I_1, of the Cauchy stress. On the other hand, damage healing might be non-isotropic or exhibit induced anisotropy. Under this circumstance, the conjugate equivalent stress measure for healing may be taken as

$$\sigma_{eq}^h = \frac{1}{3}(I_1 - x_{10}\sigma_1) \qquad (2)$$

where x_{10} is a material constant, and σ_1 is the maximum principal stress, with compression being positive. One of the characteristics of this conjugate equivalent stress measure is that when used in conjunction with Eq. (1), the healing term is the only nonzero term under hydrostatic compression, but healing also occurs under nonhydrostatic compression.

Taking the stress derivative of Eq. (2) and substituting the result into Eq. (1) leads to

$$\dot{\varepsilon}_{ij}^I = \frac{\dot{\varepsilon}_{eq}^h}{3}(\delta_{ij} - x_{10}M_{ij}) \qquad (3)$$

for damage healing under hydrostatic compression, since the first three terms in the right-hand-side of Eq. (1) are zero. In the case of nonhydrostatic compression, the inelastic strain rate tension, $\dot{\varepsilon}_{ij}^I$, would include the creep, damage, and healing terms. In Eq. (3), $\dot{\varepsilon}_{eq}^h$ is the equivalent strain rate measure for damage healing — to be determined from the physics of the process, δ_{ij} is the Kronecker delta, and $M_{ij} = \frac{d\sigma_1}{d\sigma_{ij}}$.

Both isotropic and anisotropic healing behaviors can be described in terms of Eq. (3), depending on the value of the material constant, x_{10}. Isotropic damage healing is obtained from Eq. (3) when $x_{10} = 0$, leading to $\dot{\varepsilon}_{ij}^I = \frac{1}{3}\dot{\varepsilon}_{eq}^h \delta_{ij}$ for damage healing under hydrostatic compression. In contrast, damage healing is anisotropic when $x_{10} \neq 0$. As an illustration, healing of damage in a cylinder subjected to hydrostatic compression is considered by taking $x_{10} \neq 0$ and σ_1 being in the axial direction. According to Eq. (3), the axial strain rate, $\dot{\varepsilon}_{11}$, is then given by

$$\dot{\varepsilon}_{11} = \frac{\dot{\varepsilon}_{eq}^h}{3}(1 - x_{10}) \qquad (4)$$

since $\delta_{11} = M_{11} = 1$. Thus, the axial strain rate, $\dot{\varepsilon}_{11}$, is zero when $x_{10} = 1$, but is negative (extensional) when $x_{10} > 1$. Similarly, the lateral strain rate, $\dot{\varepsilon}_{22}$, is given by

$$\dot{\varepsilon}_{22} = \frac{\dot{\varepsilon}_{eq}^h}{3} \qquad (5)$$

regardless of the value of x_{10}, since $\delta_{22} = 1$, and $M_{22} = 0$. The ratio of axial strain rate to lateral strain rate is then given by

$$\frac{\dot{\varepsilon}_{11}}{\dot{\varepsilon}_{22}} = 1 - x_{10} \qquad (6)$$

which can be used to determine the healing anisotropy and the number of mechanisms (kinetic terms) present in the healing process. Eq. (6) suggests that a plot of axial strain, ε_{11}, versus lateral strain, ε_{22}, should yield a linear relation with a slope of $1 - x_{10}$ when healing is dominated by a single mechanism. The value of the slope thus provides an indication of the anisotropy of the healing process. Isotropic damage healing occurs when $x_{10} = 0$, or a slope of unity in the plot of axial strain versus lateral strain; otherwise damage healing is anisotropic. Furthermore, the number of healing mechanisms can be inferred based on the shape of the ε_{11} versus ε_{22} plot. A straight line in the strain plot would indicate healing by a single mechanism, while a bilinear curve would indicate the presence of two healing mechanisms with different values of x_{10}.

3.1 Multiple Healing Mechanisms in WIPP Salt

In a recent paper, Brodsky and Munson (1994) reported results of a series of damage healing experiments for WIPP salt under hydrostatic compression. In these experiments, damage was introduced by pre-straining cylindrical specimens of WIPP salt in axial compression at 25°C under a strain rate of 1×10^{-6} sec^{-1} and 0.5 MPa confining pressure. The predamaged specimens were then compressed under a hydrostatic pressure of 15 MPa at 20°, 46°, and 70°C. The amount of volumetric strain and the ultrasonic wave velocity and attenuation recovered during damage healing were recorded. These experimental results have been used as the basis for comparison against the healing model and for evaluating healing anisotropy in WIPP salt.

Previous work indicated that the healing of volumetric strain in WIPP salt appeared to obey a first-order kinetic equation (Brodsky and Munson, 1994). On this basis, the volumetric strain due to healing, ε_v^h, is obtained as

$$\varepsilon_v^h = \varepsilon_{vo} \exp\left[-\frac{t}{3\tau_1} \frac{(I_1 - x_{10}\sigma_1)H(I_1 - x_{10}\sigma_1)}{\mu} \right] \quad (7)$$

where ε_{vo} is the initial volumetric strain at the beginning of the healing process, μ is the shear modulus, and τ_1 is the characteristic time constant. Eq. (7) may be rearranged to obtain

$$\ln\left\{ 1 - \frac{\varepsilon_{vo} - \varepsilon_v^h}{\varepsilon_{vo}} \right\} = -\frac{t}{3\tau_1} \frac{(I_1 - x_{10}\sigma_1)}{\mu} \quad (8)$$

which indicates that a semi-log plot of the normalized volumetric strain term in the left-hand-side of Eq. (8) versus time, t, of healing should yield a linear plot if a single mechanism with one characteristic time constant dominates the healing process. Plotting the experimental data in this manner indicated that bilinear plots of two different slopes were obtained for healing at 20°, 46°, and 70°C. The result for 70°C is presented in Figure 1, which suggests that two healing mechanisms (Mechanisms 1 and 2) with different characteristic times (τ_1 and τ_2) appear to be present in WIPP salt.
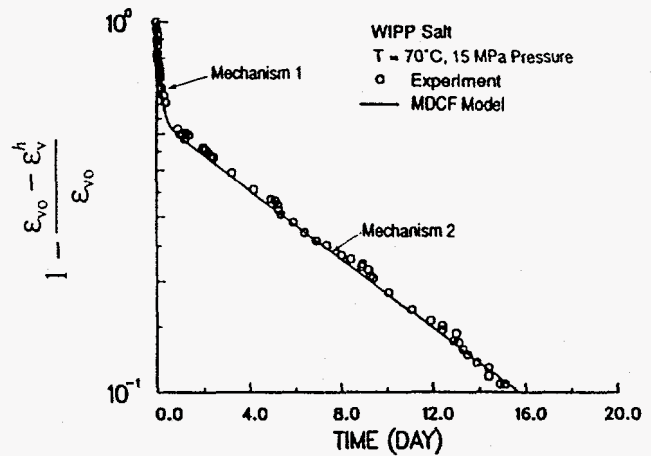

FIGURE 1. Semi-log plot of normalized volumetric strain versus time for damage healing of WIPP salt at 70°C under a hydrostatic pressure of 15 MPa, and comparison against model calculation. The bilinear experimental data indicates the presence of two healing mechanisms with different characteristic time constants.

3.2 Healing Anisotropy in WIPP Salt

Whether or not two healing mechanisms are present in WIPP salt can be verified by a plot of axial strain versus lateral strain. Such a plot of ε_{11} versus ε_{22}, which also provides information about possible healing anisotropy, is presented in Figure 2 for damage healing of WIPP salt at 70°C. Figure 2 shows a bilinear curve with a negative slope at low lateral strains, but a slope of zero at higher lateral strains. The discontinuity at low axial strains was caused by a problem in the extensometer, which led to an apparent shift of the lateral strains. Ignoring that portion of the data, the x_{10} value for Mechanism 1 was determined to be 1.2, while $x_{10} = 1$ for Mechanism 2. The finding confirmed that two healing mechanisms, which exhibited different characteristic times and healing anisotropies, were present during healing of WIPP salt.
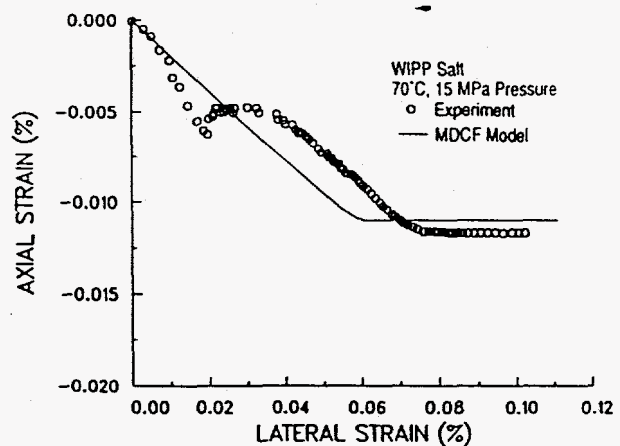

FIGURE 2. Experimental data of axial strain versus lateral strain and comparison against model calculation. The experimental data indicates the presence of two healing mechanisms with different x_{10} values. An instrument problem caused an offset in the experimental strain data at 0.02%.

A value of $x_{10} > 1$ for Mechanism 1 means damage healing in the lateral direction is accompanied by geometrical readjustment of the microcracks in the axial direction. A possible mechanism that fits this characteristic is closing of open crack surfaces of axially aligned microcracks. Such a mechanism reduces the volumetric strain but may not change the damage variable. A value of $x_{10} = 1$ for Mechanism 2 indicates that the volumetric strain is reduced with zero strain rate in the maximum principal stress direction, which is similar to, but in a reverse manner, the way damage-induced flow is generated (Chan et al. 1992, 1994a, 1994b). Thus, Mechanism 2 appears to be a damage healing process that involves reduction of both the volumetric strain and the damage variable.

3.3 Kinetic Equations for Damage Healing

The experimental observation suggests that two healing mechanisms might be present in WIPP salt. Each of the two healing mechanisms may be described by a first-order kinetic equation. The first mechanism, Mechanism 1, has a much smaller time constant, τ_1, and is assumed not to change the damage variable while the second mechanism, Mechanism 2, has a larger time constant, τ_2, and reduces the damage variable. The overall healing strain rate is the sum of these two healing mechanisms; the kinetic equation for damage healing is given by

$$\dot{\varepsilon}_{eq}^h = \dot{R}_1 + \dot{R}_2 \tag{9}$$

where

$$\dot{R}_1 = \frac{k_1 \sigma_{eq}^{h_1} H\left(\sigma_{eq}^{h_1}\right)}{3\tau_1 G} \exp\left[-\frac{\sigma_{eq}^{h_1} H\left(\sigma_{eq}^{h_1}\right) t_h}{3\tau_1 G}\right] \tag{10}$$

with $\sigma_{eq}^{h_1} = I_1 - x_{10}\sigma_1$ with $x_{10} = 1.2$ for Mechanism 1, and

$$\dot{R}_2 = \frac{k_2 \omega \sigma_{eq}^{h_2} H\left(\sigma_{eq}^{h_2}\right)}{3\tau_2 G} \tag{11}$$

with $\sigma_{eq}^{h_2} = I_1 - x_{10}\sigma_1$ and $x_{10} = 1.0$ for Mechanism 2. The term, $\dot{R}_1$, represents closing of crack surfaces without actual healing (Mechanism 1) and the $\dot{R}_2$ term represents removal of damage by healing (Mechanism 2). The parameter, t_h, represents the time of healing; k_1 and k_2 are material constants; H( ) is the Heaviside function with the argument in parentheses.

3.4 Evolution of Damage With Healing

Healing influences both the kinetic and the damage evolution equations. Damage development in the MDCF model is described in terms of an evolution equation given by (Chan et al. 1992, 1994)

$$\dot{\omega} = g\left(\omega, T, \sigma_{eq}^{\omega_i}, \xi_i\right) - h(\omega, T, \sigma_{eq}^h) \tag{12}$$

where $g\left(\omega, T, \sigma_{eq}^{\omega_i}, \xi_i\right)$ describes the growth of damage, and $h(\omega, T, \sigma_{eq}^h)$ describes the removal of damage. The damage growth function, g, is given by (Chan et al, 1994a, 1994b)

$$g = \frac{x_4}{t_o} \omega \left[\ln\left(\frac{1}{\omega}\right)\right]^{\frac{x_4+1}{x_4}} \left\{\left[\frac{\sigma_{eq}^{\omega_i} H\left(\sigma_{eq}^{\omega_i}\right)}{\xi_s}\right]^{x_{3s}}\right.$$
$$\left. + \left[\frac{\sigma_{eq}^{\omega_i} H\left(\sigma_{eq}^{\omega_i}\right)}{\xi_t}\right]^{x_{3t}}\right\} \tag{13}$$

where x_{3i}, x_4, ξ_i (with $i = s$ or t for shear or tensile damage, respectively) are material constants, and t_o is a reference time. Motivated by the experimental observations (Brodsky and Munson 1994), the healing function for WIPP salt by Mechanism 2 is taken to be a first-order kinetic equation given by

$$h = \frac{\omega \sigma_{eq}^{h_2} H\left(\sigma_{eq}^{h_2}\right)}{\tau_2 \mu} \tag{14}$$

which is then combined with Eqs. (12) and (13) to give the damage evolution equation for damage development with healing. In this approach, healing by Mechanism 1 is assumed not to contribute to the reduction of the damage variable, ω.

4. APPLICATION OF MODEL TO WIPP SALT

The healing term was incorporated into the MDCF model and used to compute the inelastic response during damage healing of WIPP salt. A complete description of the creep and damage terms of the model is given elsewhere (Chan et al. 1994b). The material constants for these calculations are presented in Tables 1 and 2. The input for the numerical simulations closely followed the actual experimental conditions.

TABLE 1. Material Constants for WIPP (Clean) Salt

Elastic Properties

μ = 12.4 GPa
E = 31.0 GPa
ν = 0.25

M - D Model	Damage Model
A_1 = 8.386 E22 sec^{-1}	x_2 = 9.0
Q_1 = 1.045 x 10^5 J/mol	x_3 = 5.5
n_1 = 5.5	x_4 = 3.0
B_1 = 6.086 E6 sec^{-1}	ξ_{3s} = 100 MPa
A_2 = 9.672 E12 sec^{-1}	x_6 = 0.75
Q_2 = 4.18 x 10^4 J/mol	x_7 = 1 MPa
n_2 = 5.0	x_8 = 0.1
B_2 = 3.034 E-2 sec^{-1}	c_0 = 5.0 E4
σ_0 = 20.57 MPa	c_2 = 850.0
q = 5.335 E3	c_3 = 10.0
R = 8.3143 J/mol·K	c_4 = 6.0
m = 3.0	c_5 = 25.0 MPa
K_0 = 6.275 E5	n_3 = 3
c = 0.009198 K^{-1}	ω_0 = 1.0 E-4
α = -8.263	
β = -5.448	
δ = 0.58	

TABLE 2. Material Constants in Healing Term of the MDCF Model for WIPP Salt

Parameter	20°C	46°C	70°C
x_{10} (Mech. 1)	1.28	1.2	1.2
x_{10} (Mech. 2)	1.0	1.0	1.0
τ_1 (sec)	5	10	10
τ_2 (sec)	922	890	644.4
k_1	1x10^{-3}	1.4x10^{-3}	1.6x10^{-3}
k_2	28.0	23.7	18

In both experiment and model calculations, WIPP salt was prestrained to 1.5% strain under a strain rate of 1 x10^{-6} sec^{-1} and 0.5 MPa confining pressure. After reaching 1.5% total strain, the axial stress was reduced to 0.5 MPa and subsequently the hydrostatic pressure was increased to 15 MPa. Under hydrostatic compression, the computed inelastic strain rates were solely due to healing of damage, since the creep and damage terms were zero.

The axial, lateral, and volumetric strain recovered during damage healing of WIPP salt at 70°C were calculated. Calculated values of normalized volumetric strain as a function of time of healing are in good agreement with experimental results, as shown in Figure 1. The calculated axial strains and lateral strains are compared against the experimental data in Figure 2, which plots axial strain versus lateral strain. Comparison of the calculated and measured volumetric, lateral, and axial strains recovered during damage healing at 70°C is shown in Figure 3 as a function of time of healing. In all cases, the healing responses are well described by the model at both short and long durations of healing.
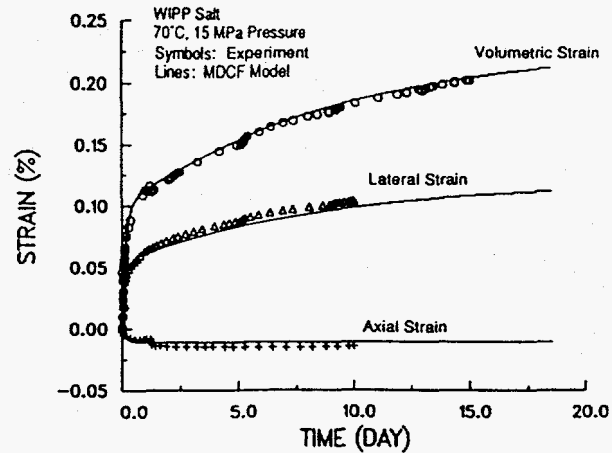

FIGURE 3. Measured strains recovered during damage healing of WIPP salt at 70°C under a hydrostatic pressure of 15 MPa and comparison against model calculation.

The presence of damage in WIPP salt was previously found to reduce the ultrasonic wave velocity and attenuation (Brodsky and Munson, 1994). A simplified analysis indicates that the wave velocity, $\overline{V}$, in a damaged material is related to the volumetric strain, ε_{kk}, and the damage variable, ω, according to

$$\frac{\overline{V}}{V} = \sqrt{(1 - \omega)(1 + \varepsilon_{kk})} \tag{15}$$

where V is the wave velocity in the pristine material. Using values of volumetric strain, ε_{kk}, and the damage variable, ω, calculated for damage healing of WIPP salt at 70°C, the normalized wave velocity was computed via Eq. (15). Comparison of the calculated and experimental results of wave velocities parallel to and perpendicular to the cylinder axis is presented in Figure 4, which shows fair agreement between model and experiment. However, the recovery of wave velocity at a short time of healing was less rapid in the calculation than in the experiment. The discrepancy appeared to be related to the assumption that Healing Mechanism 1, which had a short time constant, did not lead to reduction in the damage variable. Because of this, reduction of the damage variable with time was soley controlled by Healing Mechanism 2, which had a longer time constant. The longer time constant resulted in a larger value of ω and therefore a lower normalized wave velocity at short times. Despite the discrepancy, the model describes reasonably well the overall healing behavior of WIPP salt.
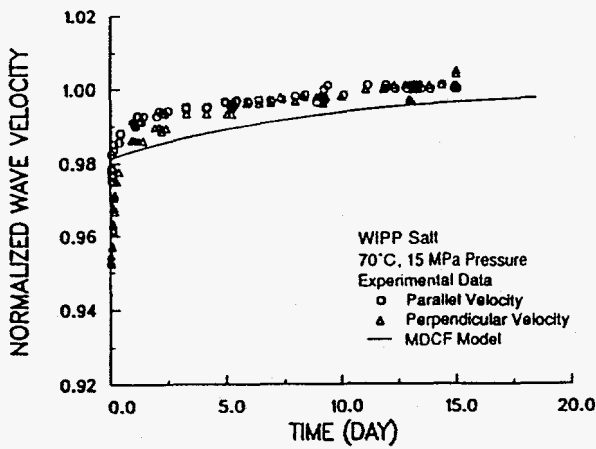
FIGURE 4. Comparison of calculated and measured values of normalized wave velocities observed during damage healing of WIPP salt at 70°C under a hydrostatic pressure of 15 MPa.

5. SUMMARY AND CONCLUSIONS

A damage healing term has been formulated and incorporated as an extension of the MDCF constitutive model for treating inelastic flow due to coupled creep, fracture, and healing in rock salt. The healing term has been developed on the basis that damage healing contributes, together with creep and damage mechanisms, directly to the macroscopic strain rate, and can be modeled in terms of a set of flow law, kinetic, and damage evolution equations formulated with pertinent power-conjugate equivalent stress and strain rate measures and an internal damage variable. Application of the model revealed that damage healing in WIPP salt occurred by two healing mechanisms with different characteristic time constants and anisotropies. Recovery of axial, lateral, and volumetric strains during healing of WIPP salt under hydrostatic compression was accurately represented by the model.

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