# Fermi National Accelerator Laboratory 

## MINOS Detector Steel Magnetic Measurements

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February 1999

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# MINOS DETECTOR STEEL MAGNETIC MEASUREMENTS 

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#### Abstract

Magnetic measurements were made on one steel plate of the MINOS far detector. The conventionally used technique of measuring sense coil voltage induced by step changes in excitation current voltage was successful in providing stable, repeatable measurements. Measurements were made at several locations on the steel and the results are presented.


## MINOS DETECTOR STEEL MAGNETIC MEASUREMENTS

Some characteristics of the magnetic properties of iron or steel can be determined by measuring the fields induced in a sense coil by step current changes in a magnetizing coil. In the MINOS detector, a variety of sense coils with a judiciously chosen number of turns were installed on the steel detector plate located in NMS (see Figure 2). The sense coils were readout by the integrator shown schematically in Figure 1 below.


Figure 1
For this circuit,

$$
\begin{equation*}
E_{0}=\frac{-1}{R C} \int_{0}^{T} E_{1} d t \tag{a}
\end{equation*}
$$

where $\mathrm{E}_{1}$ is the induced voltage at the winding caused by the change in magnetizing current. From the Faraday induction law:

$$
E_{1}=-N \frac{d \Phi}{d t}
$$

where,

$$
\begin{aligned}
& \Delta \Phi=\text { change of flux } \\
& \mathrm{N}=\text { number of turns }
\end{aligned}
$$

Also, the magnetic flux is related to the magnetic flux density by:

$$
\begin{align*}
\Phi & =\int_{S} B \bullet d S \\
B & =\frac{\Delta \Phi}{S} \tag{b}
\end{align*}
$$

For (a) we can write:

$$
\int_{0}^{T} E_{1} d t=-N \int_{0}^{T} d \Phi / d t=-N(\Delta \Phi)
$$

and from (b) the output of the voltage integrator is related to the change in the flux density as follows:

$$
\begin{aligned}
E_{0} & =\frac{N}{R C} \Delta \Phi \\
E_{0} & =\frac{N S}{R C} B \\
B & =\frac{R C}{N S} E_{0}
\end{aligned}
$$

Since the product of R and C determines the final accuracy of the integrator, it was measured by applying a precisely known (amplitude and width) pulse to the amplifier input and measuring the output. Based upon this calibration, a value of $\mathrm{RC}=4.945 \times 10^{-2}$ was determined and will be used throughout the subsequent discussion.

A series of induction measurements were made on a single plane of the MINOS detector steel to evaluate the efficacy of the measurement scheme and to compare the measurements to those determined by FEA. Initially, five coils were installed on the plane; one single turn coil from center to outside and four additional five turn coils placed equally on radius. As a result of the measurements using these coils, it was decided to add several additional coils in an attempt to better understand the field non-uniformity and the apparent effect of the plane support steel on the field shape. Figure 2 shows the final coil arrangement and identification setup. Figure 3 shows the schematic of the setup for the WHT coil.

As was stated above, several simplifying assumptions were made to evaluate the data; among them uniform flux through the steel volume and no air gaps was assumed. Clearly this is not the case for the MINOS detector plate. Furthermore, the non-uniformity of the flux in the MINOS plate makes it difficult to make a precise estimate of H. Consequently, the plots that follow are of B versus I in amperes. In spite of these limitations, the discussion that follows was used to determine the relative measures of the magnetic flux density versus the exciting current and plot the BI curves shown on pages 5 through 9 .


Figure 2


Figure 3

For the integrator used and the area of the MINOS detector plate sense coils:

$$
\begin{aligned}
& R=10^{4} \text { ohms } \\
& C=4.945 \times 10^{-6} \mathrm{farads} \\
& \text { WHT, GRN, GRY and } 45 \text { DEG WEST RED coils } \\
& \quad N=1 \\
& \quad S=0.074 \mathrm{~m}^{2} \\
& \text { BRN, ORG, VIO and BLK coils } \\
& \quad N=5 \\
& S=0.018 \mathrm{~m}^{2} \\
& \text { IBEAM WEST ORG and IBEAM EAST RED } \\
& N=10 \\
& S=0.011 \mathrm{~m}^{2}
\end{aligned}
$$

Based upon the simplifying assumptions and for these values,

$$
\begin{aligned}
& \Delta B=\frac{4.945 \times 10^{-2}}{(N)(S)} E_{0} \\
& \Delta B=0.6682 E_{0} \text { tesla for the WHT, GRN, GRY and } 45 \text { DEG WEST RED coils } \\
& \Delta B=0.5369 E_{0} \text { tesla for BRN, ORG, VIO and BLK coils } \\
& \Delta B=0.4495 E_{0} \text { tesla for IBEAM WEST ORG and IBEAM EAST RED coils }
\end{aligned}
$$

Table 1 and 2 are the direct results from the measurements for full cycle runs. At the start of each run, the current was run up to a maximum of 150 amperes to fully saturate parts of the steel ( 100 amperes is the expected maximum operating current). The current was then reduced in steps to zero. The power supply was reversed and the current was run up in steps to a maximum 150 amperes and then run down to zero current. The power supply polarity was once again reversed and the current was increased in steps to 150 amperes (the starting current value and polarity). The step sizes are shown in the Tables and were chosen merely to improve the data plots. The data was then adjusted to minimize the effect of integrator drift and to ensure symmetry. The results are shown on pages 4 through 9 .

Several observations can readily be made; the energizing coil placement is not optimal and the steel support structure strongly affects the field lines in the detector steel. Several measurements were taken in an attempt to quantify the effect of the support steel and the IBEAM sense coils show the results of these measurements. It is clear that a significant field exists and to some extent short circuits the flux in the detector plate.

WHT C OL


BR N C OL


ORG C aL


VIO C aL



Page 7


W EST 45 DEG RED


IBEAM EAS T RED


IBEAM W EST ORG


| Integrator: $\mathrm{R}=10 \mathrm{~K}, \mathrm{C}=4.945 \mathrm{uF}$ |  |  |  |  | Current: 1V = 20A |  |  |  | Date: | 12/1/98 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature @ start= |  | 67 deg F |  |  |  |  |  |  | Name: |  |
| Sensor | WHT |  |  |  | Sensor | BRN |  |  | Sensor | ORG |
| \# of turns | 1 |  |  |  | \# of turns | 5 |  |  | \# of turns | 5 |
| Area= | 0.0737 | sq meters |  |  | Area= | 0.018425 | sq meters |  | Area= | 0.01843 |
| Polarity | 1 | 1 | Integrator | Polarity | 1 | 1 | Integrator | Polarity | 1 | 1 |
| (Reset) | amps | volts | volts |  | amps | volts | volts |  | amps | volts |
| $F$ (reset) | 150.0 | 7.50 | 0.00 | $F$ (reset) | 150.0 | 7.50 | 0 | $F$ (reset) | 150.0 | 7.50 |
| 67 degF | 130.0 | 6.50 | -0.01 |  | 130.0 | 6.50 | -0.031 |  | 130.0 | 6.50 |
|  | 108.4 | 5.42 | -0.03 |  | 108.4 | 5.42 | -0.067 |  | 108.4 | 5.42 |
|  | 89.8 | 4.49 | -0.04 |  | 90.0 | 4.50 | -0.109 |  | 90.0 | 4.50 |
|  | 70.0 | 3.50 | 0.06 |  | 70.0 | 3.50 | -0.1604 |  | 70.0 | 3.50 |
|  | 50.0 | 2.50 | -0.09 |  | 50.0 | 2.50 | -0.227 |  | 50.0 | 2.50 |
|  | 40.0 | 2.00 | -0.11 |  | 40.0 | 2.00 | -0.27 |  | 40.0 | 2.00 |
|  | 29.8 | 1.49 | -0.14 |  | 30.0 | 1.50 | -0.336 |  | 30.0 | 1.50 |
|  | 20.0 | 1.00 | -0.19 |  | 20.0 | 1.00 | -0.44 |  | 20.0 | 1.00 |
|  | 10.0 | 0.50 | -0.29 |  | 10.0 | 0.50 | -0.663 |  | 10.0 | 0.50 |
|  | 0.0 | 0.00 | -0.49 |  | 0.0 | 0.00 | -1.22 |  | 0.0 | 0.00 |
| R | -10.0 | -0.50 | -0.48 | R | -9.0 | -0.45 | -3.53 | R | -9.0 | -0.45 |
|  | -20.0 | -1.00 | -1.14 | 95 degF | -20.0 | -1.00 | -4.41 |  | -20.0 | -1.00 |
|  | -30.0 | -1.50 | 2.17 |  | -30.0 | -1.50 | -4.7 | 106 degF | -30.0 | -1.50 |
|  | -40.0 | -2.00 | 2.38 |  | -40.0 | -2.00 | -4.85 |  | -40.0 | -2.00 |
|  | -50.0 | -2.50 | 2.47 |  | -50.0 | -2.50 | -4.92 |  | -50.0 | -2.50 |
|  | -70.0 | -3.50 | -2.57 |  | -70.0 | -3.50 | -5.01 |  | -70.0 | -3.50 |
|  | -90.0 | -4.50 | -2.63 |  | -91.4 | -4.57 | -5.08 |  | -91.4 | -4.57 |
|  | -110.0 | -5.50 | -2.67 |  | -110.0 | -5.50 | -5.13 |  | -110.0 | -5.50 |
|  | -130.0 | -6.50 | -2.71 |  | -130.0 | -6.50 | -5.17 |  | -130.0 | -6.50 |
| 77 degF | -150.0 | -7.50 | -2.74 |  | -150.4 | -7.52 | -5.2 |  | -150.4 | -7.52 |
|  | -130.0 | -6.50 | -2.72 |  | -130.0 | -6.50 | -5.17 |  | -130.0 | -6.50 |
|  | -110.0 | -5.50 | -2.70 |  | -110.0 | -5.50 | -5.13 |  | -110.0 | -5.50 |
|  | -90.0 | -4.50 | -2.68 |  | -90.0 | -4.50 | -5.08 |  | -90.0 | -4.50 |
|  | -69.8 | -3.49 | -2.65 |  | -69.8 | -3.49 | -5.027 |  | -70.0 | -3.50 |
|  | -49.8 | -2.49 | -2.62 |  | -49.8 | -2.49 | -4.96 |  | -50.0 | -2.50 |
|  | -39.4 | -1.97 | -2.59 |  | -40.0 | -2.00 | -4.91 |  | -40.0 | -2.00 |
|  | -29.8 | -1.49 | -2.57 |  | -30.0 | -1.50 | -4.85 |  | -30.0 | -1.50 |
|  | -20.0 | -1.00 | -2.51 |  | -20.0 | -1.00 | -4.75 |  | -20.0 | -1.00 |
|  | -9.8 | -0.49 | -2.41 | 98 degF | -9.8 | -0.49 | -4.54 |  | -10.0 | -0.50 |
|  | 0.0 | 0.00 | -2.19 |  | 0.0 | 0.00 | -3.95 |  | 0.0 | 0.00 |
| F | 10.0 | 0.50 | -1.61 | F | 10.0 | 0.50 | -1.91 | F | 10.0 | 0.50 |
| 84 degF | 20.0 | 1.00 | -0.96 |  | 20.0 | 1.00 | -0.77 | 110 degF | 20.0 | 1.00 |
|  | 30.0 | 1.50 | -0.53 |  | 30.0 | 1.50 | -0.45 |  | 30.0 | 1.50 |
|  | 40.0 | 2.00 | -0.30 |  | 40.0 | 2.00 | -0.29 |  | 40.0 | 2.00 |
|  | 50.0 | 2.50 | -0.19 |  | 50.0 | 2.50 | -0.204 |  | 50.0 | 2.50 |
|  | 70.0 | 3.50 | -0.08 |  | 70.0 | 3.50 | -0.102 |  | 70.0 | 3.50 |
|  | 90.6 | 4.53 | -0.01 |  | 90.0 | 4.50 | -0.033 |  | 90.0 | 4.50 |
|  | 110.0 | 5.50 | 0.04 |  | 110.0 | 5.50 | 0.02 |  | 110.0 | 5.50 |
|  | 130.0 | 6.50 | 0.09 |  | 130.0 | 6.50 | 0.068 |  | 130.0 | 6.50 |
| 86 degF | 150.0 | 7.50 | 0.14 | 100 degF | 150.0 | 7.50 | 0.11 |  | 150.0 | 7.50 |

Table 1


Table 2

| Integrator: $\mathrm{R}=10 \mathrm{~K}, \mathrm{C}=4.945 \mathrm{uF}$ |  |  |  |  |  |  | Current: 1V = 20A |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature @ start= |  | 67 deg F |  |  |  |  |  |  |  |  |  |
| Sensor | TOP GRY |  |  |  |  | Sensor | BOTTOM GRN |  |  |  |  |
| \# of turns | 1 |  |  |  |  | \# of turns | 1 |  |  |  |  |
| Area= | 0.0737 | sq meters |  |  |  | Area= | 0.0737 | sq meters |  |  |  |
| Polarity | 1 | 1 | Tesla | Int adj | Integrator | Polarity | 1 | 1 | Tesla | Int adj | Integrator |
| (Reset) | volts | amps | 0.6682 |  | volts |  | volts | amps | 0.6682 |  | volts |
| F (reset) | 7.5 | 150 | 0.71 | 1.060 | 0.000 | $F$ (reset) | 7.5 | 150 | 1.280 | 1.915 | 0.000 |
|  | 6.5 | 130 | 0.70 | 1.055 | -0.004 |  | 6.5 | 130 | 1.262 | 1.888 | -0.025 |
|  | 5.5 | 110 | 0.70 | 1.048 | -0.009 |  | 5.5 | 110 | 1.240 | 1.856 | -0.055 |
|  | 4.5 | 90 | 0.70 | 1.041 | -0.015 |  | 4.5 | 90 | 1.215 | 1.819 | -0.090 |
|  | 3.5 | 70 | 0.69 | 1.031 | -0.023 |  | 3.5 | 70 | 1.184 | 1.772 | -0.135 |
|  | 2.5 | 50 | 0.68 | 1.020 | -0.033 |  | 2.5 | 50 | 1.142 | 1.710 | -0.195 |
|  | 2.0 | 40 | 0.68 | 1.012 | -0.040 |  | 2.0 | 40 | 1.111 | 1.663 | -0.240 |
|  | 1.5 | 30 | 0.67 | 0.998 | -0.052 |  | 1.5 | 30 | 1.070 | 1.602 | -0.299 |
|  | 1.0 | 20 | 0.65 | 0.976 | -0.073 |  | 1.0 | 20 | 0.995 | 1.489 | -0.410 |
|  | 0.5 | 10 | 0.62 | 0.928 | -0.120 |  | 0.5 | 10 | 0.866 | 1.297 | -0.600 |
|  | 0.0 | 0 | 0.55 | 0.816 | -0.230 |  | 0.0 | 0 | 0.665 | 0.994 | -0.900 |
|  | -0.5 | -10 | 0.18 | 0.265 | -0.780 |  | -0.5 | -10 | 0.115 | 0.172 | -1.720 |
|  | -1.0 | -20 | -0.18 | -0.267 | -1.31 |  | -1.0 | -20 | -0.407 | -0.610 | -2.500 |
|  | -1.5 | -30 | -0.42 | -0.628 | -1.670 |  | -1.5 | -30 | -0.756 | -1.132 | -3.020 |
|  | -2.0 | -40 | -0.53 | -0.799 | -1.840 |  | -2.0 | -40 | -0.945 | -1.414 | -3.300 |
|  | -2.5 | -50 | -0.59 | -0.881 | -1.920 |  | -2.5 | -50 | -1.040 | -1.556 | -3.440 |
|  | -3.5 | -70 | -0.64 | -0.961 | -1.999 |  | -3.5 | -70 | -1.128 | -1.688 | -3.570 |
|  | -4.5 | -90 | -0.66 | -0.994 | -2.030 |  | -4.5 | -90 | -1.189 | -1.780 | -3.660 |
|  | -5.5 | -110 | -0.68 | -1.025 | -2.060 |  | -5.5 | -110 | -1.231 | -1.842 | -3.720 |
|  | -6.5 | -130 | -0.71 | -1.056 | -2.090 |  | -6.5 | -130 | -1.272 | -1.904 | -3.780 |
|  | -7.5 | -150 | -0.73 | -1.088 | -2.120 |  | -7.5 | -150 | -1.307 | -1.956 | -3.830 |
|  | -6.5 | -130 | -0.72 | -1.079 | -2.110 |  | -6.5 | -130 | -1.288 | -1.928 | -3.800 |
|  | -5.5 | -110 | -0.72 | -1.070 | -2.100 |  | -5.5 | -110 | -1.270 | -1.900 | -3.770 |
|  | -4.5 | -90 | -0.71 | -1.062 | -2.090 |  | -4.5 | -90 | -1.244 | -1.862 | -3.730 |
|  | -3.5 | -70 | -0.70 | -1.053 | -2.080 |  | -3.5 | -70 | -1.212 | -1.814 | -3.680 |
|  | -2.5 | -50 | -0.70 | -1.045 | -2.070 |  | -2.5 | -50 | -1.174 | -1.756 | -3.620 |
|  | -2.0 | -40 | -0.69 | -1.036 | -2.060 |  | -2.0 | -40 | -1.148 | -1.718 | -3.580 |
|  | -1.5 | -30 | -0.69 | -1.027 | -2.050 |  | -1.5 | -30 | -1.103 | -1.650 | -3.510 |
|  | -1.0 | -20 | -0.67 | -1.009 | -2.030 |  | -1.0 | -20 | -1.031 | -1.542 | -3.400 |
|  | -0.5 | -10 | -0.64 | -0.960 | -1.980 |  | -0.5 | -10 | -0.912 | -1.364 | -3.220 |
|  | 0.0 | 0 | -0.56 | -0.832 | -1.850 |  | 0.0 | 0 | -0.686 | -1.027 | -2.880 |
|  | 0.5 | 10 | -0.24 | -0.353 | -1.370 |  | 0.5 | 10 | -0.200 | -0.299 | -2.150 |
|  | 1.0 | 20 | 0.14 | 0.216 | -0.800 |  | 1.0 | 20 | 0.334 | 0.499 | -1.350 |
|  | 1.5 | 30 | 0.39 | 0.584 | -0.430 |  | 1.5 | 30 | 0.713 | 1.067 | -0.780 |
|  | 2.0 | 40 | 0.51 | 0.763 | -0.250 |  | 2.0 | 40 | 0.912 | 1.365 | -0.480 |
|  | 2.5 | 50 | 0.56 | 0.842 | -0.170 |  | 2.5 | 50 | 1.004 | 1.503 | -0.340 |
|  | 3.5 | 70 | 0.61 | 0.920 | -0.090 |  | 3.5 | 70 | 1.100 | 1.646 | -0.195 |
|  | 4.5 | 90 | 0.65 | 0.969 | -0.040 |  | 4.5 | 90 | 1.159 | 1.735 | -0.104 |
|  | 5.5 | 110 | 0.67 | 1.000 | -0.007 |  | 5.5 | 110 | 1.205 | 1.803 | -0.034 |
|  | 6.5 | 130 | 0.69 | 1.031 | 0.025 |  | 6.5 | 130 | 1.208 | 1.808 | -0.027 |
|  | 7.5 | 150 | 0.71 | 1.059 | 0.054 |  | 7.5 | 150 | 1.278 | 1.913 | 0.080 |

Table 3

| Integrator: R=10K, C=4.945 uF |  |  |  |  |  |  | Current: 1V = 20A |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature @ start= |  | 67 deg F |  |  |  | Sensor |  |  |  |  |  | Sensor |  |  | - |  |  |
| Sensor | WEST 45 DEG RED |  |  |  |  |  | IBEAM EAST RED |  |  |  |  |  | IBEAM WEST ORG |  | from 12/8/98 data |  |  |
| \# of turns | 1 |  |  |  |  | \# of turns | 10 |  |  |  |  | \# of turns | 10 |  |  |  |  |
| Area= | 0.0737 | sq meters |  |  |  | Area= | 0.011 | sq meters |  |  |  | Area= | 0.011 | sq meters |  |  |  |
| Polarity | 1 | 1 | Tesla | Int adj | Integrator | Polarity | 1 | 1 | Tesla | Int adj | Integrator | Polarity | 1 | 1 | Tesla | Int adj | Integrator |
| (Reset) | volts | amps | 0.6682 |  | volts |  | volts | amps | 0.6682 |  | volts |  | volts | amps | 0.4495 |  | volts |
| F (reset) | 7.5 | 150 | 1.47 | 2.200 | 0.000 | $F$ (reset) | 7.5 | 150 | 0.448 | -0.670 | 0.000 | $F$ (reset) | 7.5 | 150 | 0.589 | 1.310 | 0.000 |
|  | 6.5 | 130 | 1.45 | 2.170 | -0.026 |  | 6.5 | 130 | 0.442 | -0.661 | 0.008 |  | 6.5 | 130 | 0.585 | 1.303 | -0.006 |
|  | 5.5 | 110 | 1.42 | 2.132 | -0.061 |  | 5.5 | 110 | 0.435 | -0.652 | 0.017 |  | 5.5 | 110 | 0.581 | 1.292 | -0.015 |
|  | 4.5 | 90 | 1.39 | 2.085 | -0.104 |  | 4.5 | 90 | 0.428 | -0.640 | 0.028 |  | 4.5 | 90 | 0.575 | 1.279 | -0.027 |
|  | 3.5 | 70 | 1.36 | 2.029 | -0.156 |  | 3.5 | 70 | 0.418 | -0.625 | 0.042 |  | 3.5 | 70 | 0.566 | 1.260 | -0.044 |
|  | 2.5 | 50 | 1.30 | 1.953 | -0.229 |  | 2.5 | 50 | 0.405 | -0.607 | 0.060 |  | 2.5 | 50 | 0.554 | 1.232 | -0.071 |
|  | 2.0 | 40 | 1.27 | 1.895 | -0.283 |  | 2.0 | 40 | 0.395 | -0.592 | 0.074 |  | 2.0 | 40 | 0.543 | 1.209 | -0.092 |
|  | 1.5 | 30 | 1.22 | 1.818 | -0.356 |  | 1.5 | 30 | 0.381 | -0.570 | 0.095 |  | 1.5 | 30 | 0.527 | 1.172 | -0.128 |
|  | 1.0 | 20 | 1.11 | 1.664 | -0.507 |  | 1.0 | 20 | 0.357 | -0.534 | 0.130 |  | 1.0 | 20 | 0.497 | 1.105 | -0.193 |
|  | 0.5 | 10 | 0.95 | 1.427 | -0.740 |  | 0.5 | 10 | 0.310 | -0.464 | 0.200 |  | 0.5 | 10 | 0.442 | 0.984 | -0.313 |
|  | 0.0 | 0 | 0.71 | 1.058 | -1.105 |  | 0.0 | 0 | 0.262 | -0.392 | 0.271 |  | 0.0 | 0 | 0.357 | 0.795 | -0.500 |
|  | -0.5 | -10 | 0.09 | 0.140 | -2.020 |  | -0.5 | -10 | 0.155 | -0.232 | 0.430 |  | -0.5 | -10 | 0.150 | 0.334 | -0.960 |
|  | -1.0 | -20 | -0.49 | -0.734 | -2.890 |  | -1.0 | -20 | 0.001 | -0.002 | 0.660 |  | -1.0 | -20 | -0.152 | -0.338 | -1.630 |
|  | -1.5 | -30 | -0.89 | -1.338 | -3.490 |  | -1.5 | -30 | -0.146 | 0.219 | 0.880 |  | -1.5 | -30 | -0.359 | -0.799 | -2.090 |
|  | -2.0 | -40 | -1.12 | -1.671 | -3.820 |  | -2.0 | -40 | -0.247 | 0.370 | 1.030 |  | -2.0 | -40 | -0.450 | -1.001 | -2.290 |
|  | -2.5 | -50 | -1.22 | -1.833 | -3.978 |  | -2.5 | -50 | -0.301 | 0.450 | 1.110 |  | -2.5 | -50 | -0.491 | -1.092 | -2.380 |
|  | -3.5 | -70 | -1.33 | -1.989 | -4.130 |  | -3.5 | -70 | -0.348 | 0.521 | 1.180 |  | -3.5 | -70 | -0.528 | -1.174 | $-2.460$ |
|  | -4.5 | -90 | -1.40 | -2.092 | -4.230 |  | -4.5 | -90 | -0.382 | 0.572 | 1.230 |  | -4.5 | -90 | -0.555 | -1.235 | -2.520 |
|  | -5.5 | -110 | -1.45 | -2.166 | -4.300 |  | -5.5 | -110 | -0.409 | 0.612 | 1.270 |  | -5.5 | -110 | -0.574 | -1.277 | -2.560 |
|  | -6.5 | -130 | -1.48 | -2.220 | -4.350 |  | -6.5 | -130 | -0.436 | 0.653 | 1.310 |  | -6.5 | -130 | -0.588 | -1.308 | -2.590 |
|  | -7.5 | -150 | -1.52 | -2.273 | -4.400 |  | -7.5 | -150 | -0.457 | 0.684 | 1.340 |  | -7.5 | -150 | -0.602 | -1.340 | $-2.620$ |
|  | -6.5 | -130 | -1.50 | -2.247 | -4.370 |  | -6.5 | -130 | -0.451 | 0.675 | 1.330 |  | -6.5 | -130 | -0.598 | -1.331 | -2.610 |
|  | -5.5 | -110 | -1.48 | -2.211 | -4.330 |  | -5.5 | -110 | -0.445 | 0.665 | 1.320 |  | -5.5 | -110 | -0.595 | -1.323 | -2.600 |
|  | -4.5 | -90 | -1.45 | -2.164 | -4.280 |  | -4.5 | -90 | -0.438 | 0.656 | 1.310 |  | -4.5 | -90 | -0.591 | -1.314 | $-2.590$ |
|  | -3.5 | -70 | -1.41 | -2.108 | -4.220 |  | -3.5 | -70 | -0.425 | 0.637 | 1.290 |  | -3.5 | -70 | -0.582 | -1.296 | -2.570 |
|  | -2.5 | -50 | -1.36 | -2.032 | -4.140 |  | -2.5 | -50 | -0.419 | 0.627 | 1.280 |  | -2.5 | -50 | -0.574 | -1.277 | -2.550 |
|  | -2.0 | -40 | -1.32 | -1.975 | -4.080 |  | -2.0 | -40 | -0.406 | 0.608 | 1.260 |  | -2.0 | -40 | -0.566 | -1.259 | $-2.530$ |
|  | -1.5 | -30 | -1.26 | -1.889 | -3.990 |  | -1.5 | -30 | -0.393 | 0.589 | 1.240 |  | -1.5 | -30 | -0.548 | -1.220 | -2.490 |
|  | -1.0 | -20 | -1.17 | -1.753 | -3.850 |  | -1.0 | -20 | -0.367 | 0.549 | 1.200 |  | -1.0 | -20 | -0.509 | -1.132 | -2.400 |
|  | -0.5 | -10 | -1.02 | -1.526 | -3.620 |  | -0.5 | -10 | -0.327 | 0.490 | 1.140 |  | -0.5 | -10 | -0.464 | -1.033 | -2.300 |
|  | 0.0 | 0 | -0.74 | -1.110 | -3.200 |  | 0.0 | 0 | -0.261 | 0.391 | 1.040 |  | 0.0 | 0 | -0.393 | -0.875 | -2.140 |
|  | 0.5 | 10 | -0.21 | -0.314 | -2.400 |  | 0.5 | 10 | -0.161 | 0.241 | 0.890 |  | 0.5 | 10 | -0.241 | -0.536 | -1.800 |
|  | 1.0 | 20 | 0.41 | 0.613 | -1.470 |  | 1.0 | 20 | 0.005 | -0.008 | 0.640 |  | 1.0 | 20 | 0.024 | 0.052 | -1.210 |
|  | 1.5 | 30 | 0.85 | 1.269 | -0.810 |  | 1.5 | 30 | 0.165 | -0.247 | 0.400 |  | 1.5 | 30 | 0.275 | 0.611 | -0.650 |
|  | 2.0 | 40 | 1.07 | 1.605 | -0.470 |  | 2.0 | 40 | 0.252 | -0.376 | 0.270 |  | 2.0 | 40 | 0.395 | 0.879 | -0.380 |
|  | 2.5 | 50 | 1.18 | 1.768 | -0.304 |  | 2.5 | 50 | 0.299 | -0.448 | 0.198 |  | 2.5 | 50 | 0.449 | 0.998 | -0.260 |
|  | 3.5 | 70 | 1.28 | 1.922 | -0.146 |  | 3.5 | 70 | 0.347 | -0.519 | 0.126 |  | 3.5 | 70 | 0.497 | 1.106 | -0.150 |
|  | 4.5 | 90 | 1.35 | 2.014 | -0.050 |  | 4.5 | 90 | 0.378 | -0.565 | 0.079 |  | 4.5 | 90 | 0.527 | 1.173 | -0.082 |
|  | 5.5 | 110 | 1.39 | 2.084 | 0.023 |  | 5.5 | 110 | 0.403 | -0.603 | 0.041 |  | 5.5 | 110 | 0.550 | 1.223 | -0.030 |
|  | 6.5 | 130 | 1.43 | 2.144 | 0.087 |  | 6.5 | 130 | 0.426 | -0.637 | 0.006 |  | 6.5 | 130 | 0.572 | 1.272 | 0.020 |
|  | 7.5 | 150 | 1.47 | 2.196 | 0.143 |  | 7.5 | 150 | 0.447 | -0.669 | -0.027 |  | 7.5 | 150 | 0.588 | 1.309 | 0.058 |

Table 4

