Reduction of Turbulence by Zonal Flows

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Three-dimensional gyrokinetic simulations of ion temperature gradient driven turbulence in magnetically confined toroidal plasmas support the view that turbulence-driven fluctuating $\mathbf{E} \times \mathbf{B}$ zonal flows can significantly reduce turbulent transport. Random shearing of turbulent eddies by zonal flows is analytically studied. It is shown that the fast time varying components of $\mathbf{E} \times \mathbf{B}$ flows, while they typically contribute significantly to the instantaneous $\mathbf{E} \times \mathbf{B}$ shearing rate, are less effective in suppressing turbulence. This is because the shear flow pattern changes before the eddies get distorted enough. We analytically derive the effective $\mathbf{E} \times \mathbf{B}$ shearing rate capturing this important physics, thereby extending the theory of $\mathbf{E} \times \mathbf{B}$ shear suppression of turbulence in toroidal geometry [Phys. Plasmas **2**, 1648 (1995)].

I. INTRODUCTION

There is accumulating evidence that $\mathbf{E} \times \mathbf{B}$ shear suppression of turbulence is the most likely mechanism to be responsible for confinement enhancement [1]. Theory of $\mathbf{E} \times \mathbf{B}$ shear suppression has been developed in cylindrical geometry first [2]. It is valid when the time variation of E_r is much slower than the eddy turn-over time. Extension to a shaped tokamak geometry [3] has been useful for comparisons to experimental data [1] which indeed show such relatively slow time variation of the macroscopic E_r.

Recent experimental data from tokamaks [4] show evidence of small radial scale $\mathbf{E} \times \mathbf{B}$ flows that cannot be explained by the existing neoclassical (Coulomb collisional) theory.

DISCLAIMER

These observations point to the possibility of a spontaneous generation of $\mathbf{E} \times \mathbf{B}$ zonal flows which in turn regulate the turbulence. In this paper, we report the recent simulation results on the zonal flow dynamics studied via a fully three-dimensional global gyrokinetic toroidal code (GTC) [5]. The code uses a general geometry Poisson solver [6] and Hamiltonian guiding center equations of motion in magnetic coordinates to treat realistic numerical MHD equilibria. By use of low-noise numerical algorithms and a massively parallel computer, we have been able to reproduce key features of turbulent transport observed at the core of tokamak plasmas [5].

In this paper, we report the recent simulation results on the zonal flow dynamics and the analytical estimation of the random shearing due to zonal flows. Zonal flows observed in simulations contain significant components with radial scales and frequencies comparable to those of the turbulence and play a crucial role in regulating nonlinear saturation and transport levels. These are in qualitative agreements with the previous toroidal gyrofluid [7,8] and gyrokinetice [9] simulations of ion-temperature-gradient (ITG) turbulence in a flux-tube geometry. Our analytical estimation of the radial correlation length reduction due to zonal flows is in qualitative agreement with simulation results.

II. ZONAL FLOW DYNAMICS IN GYROKINETIC SIMULATIONS

Turbulence-generated zonal flows in toroidal plasmas are driven by the flux-surface-averaged radially local charge separation, and mainly in the poloidal direction for high aspect ratio devices. Rosenbluth and Hinton [10] pointed out that the undamped component of poloidal flows must be predicted accurately to determine the transport level in nonlinear turbulence simulations and provide an illuminating test for predicting the residual flow level in response to an initial flow perturbation introduced by a flux-surface-averaged charge separation. We reproduced this test in gyrokinetic particle simulations by solving the toroidal gyrokinetic equation [11,12] with an initial source that is constant on a flux surface and introduced a perturbation of the poloidal flow. This flow was relaxed through the transit time magnetic pumping effect followed by a slower damped oscillation with a characteristic frequency corresponding to that of the geodesic acoustic mode (GAM) [13]. The residual level of this flow measured from the simulation agrees well with the theoretical prediction (0.12 for simulation parameters) [10] as shown in Fig. 1.

Turbulence-driven zonal flows are now self-consistently included in the nonlinear simulations of toroidal ITG instabilities. The flows are generated by the Reynolds stress [14] and can be considered as a nonlinear instability associated with inverse cascade of the turbulent spectra [15]. Our global simulations produce fluctuating $\mathbf{E} \times \mathbf{B}$ flows containing significant components with radial scales and frequencies comparable to those of the turbulence. These simulations used representative parameters of DIII-D H-mode core plasmas, which are described in Ref. [5]. We used an electron response with $\delta n_e/n_0 = e(\Phi - < \Phi >)/T_e$, where $< \cdots >$ represents a flux surface average. To illustrate the effects of these flows on transport, we also carried out simulations of the same set of parameters with $\mathbf{E} \times \mathbf{B}$ flows suppressed by forcing $< \Phi >= 0$. Comparison of the time history of χ_i from the simulation with turbulence-driven $\mathbf{E} \times \mathbf{B}$ flows included to that from the simulation with the flows suppressed shows that a significant reduction (up to an order of magnitude) of steady state ion thermal diffusivity occurs when $\mathbf{E} \times \mathbf{B}$ flows are retained.

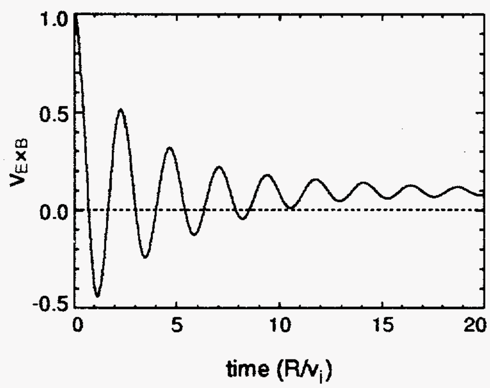

FIG. 1. Time evolution (solid line) of $\mathbf{E} \times \mathbf{B}$ flow which exhibits the linear damping, the GAM oscillation, and a finite residual level in very good agreement with the analytical prediction by Rosenbluth and Hinton. Dashed line is the zero level.

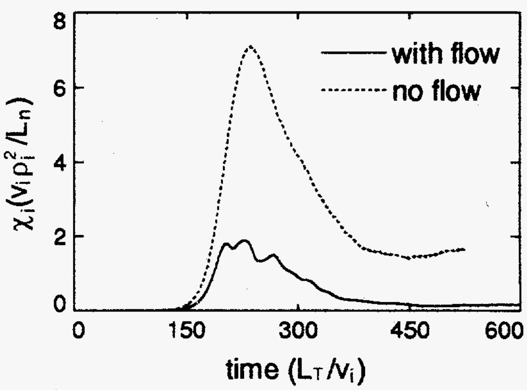

FIG. 2. Time history of ion thermal diffusivities with and without $\mathbf{E} \times \mathbf{B}$ flows in global gyrokinetic simulations.

III. REDUCTION OF TURBULENCE BY ZONAL FLOWS

The reduction of turbulence is estimated via a two point nonlinear analysis in the presence of time varying $\mathbf{E} \times \mathbf{B}$ flow shear. We consider a model problem in which the potential Φ associated with the zonal flows is a flux function, but has the following time dependence.

$$\Phi(\psi, t) = \Phi_0(\psi) exp(-i\omega_f(t - t_0)),$$

where t_0 specifies the initial phase. The corresponding radial shear of the time dependent angular frequency is given by

$$\Omega_E(\psi, t) \equiv -\frac{\partial^2}{\partial\psi^2}\Phi_0(\psi) exp(-i\omega_f(t - t_0)) \equiv \Omega_\psi exp(-i\omega_f(t - t_0)).$$

The two-point correlation evolution equation is then derived following the procedure described in [16]. Results show that the radial correlation length $\Delta r \equiv \Delta\psi/RB_\theta$, is reduced by the flow shear relative to its value $\Delta r_0 \equiv \Delta\psi_0/RB_\theta$, determined by ambient turbulence alone:

$$\left(\frac{\Delta\psi_0}{\Delta\psi}\right)^2 = 1 + \frac{\omega_{Eff}^2}{\Delta\omega_T^2}. \tag{1}$$

Therefore, we expect that order of magnitude fluctuation reduction occurs when the following effective shearing rate ω_{Eff} becomes comparable to the decorrelation rate of the ambient turbulence, $\Delta\omega_T$.

$$\omega_{Eff} \equiv \omega_E^{(0)} \frac{((1 + 3F)^2 + 4F^3)^{1/4}}{(1 + F)\sqrt{(1 + 4F)}} \tag{2}$$

Here $\omega_E^{(0)} \equiv \Omega_\psi \frac{\Delta\psi_0}{\Delta\phi}$, $R\Delta\phi$ is the correlation length in toroidal direction, and $F \equiv \omega_f^2/\Delta\omega_T^2$. When E_r varies slowly enough such that $F << 1$, we have $\omega_{Eff} = \omega_E^{(0)}$, and recover the previous result in general toroidal geometry [3]. On the other hand, when E_r varies fastly in time such that $F >> 1$, we have $\omega_{Eff} << \omega_E^{(0)}$. In this case, it is difficult to achieve turbulence suppression.

Our global gyrokinetic simulations [5] show that the instantaneous $\mathbf{E} \times \mathbf{B}$ shearing rate which varies roughly on the turbulence time scale is much larger than the maximum linear growth rate for a significant portion of the simulation domain as shown in Fig. 3. Maximum linear growth rate for this run is $\gamma = 0.1 v_{Ti}/L_n$. This is similar to the recent analysis of flux-tube gyrofluid simulations [7].

The high k_r components of flows oscillate faster in time. If the flow pattern changes before eddies get distorted enough, the shearing effect is reduced. This effect is quantified by the factor in Eq. (2). Therefore, the fast varying components, while they typically contribute the most to the instantaneous $\mathbf{E} \times \mathbf{B}$ shearing rate, are not effective in suppressing turbulence. Therefore, GAM may not play an important role in regulating turbulence and transport because of its high frequency. The effective $\mathbf{E} \times \mathbf{B}$ shearing rate which is much lower than the instantaneous shearing rate is possibly of the order of the maximum linear growth rate.

The fact that the breaking of turbulent eddies by $\mathbf{E} \times \mathbf{B}$ flows results predominantly in the reduction of the radial correlation length [2,3] is also reflected in the observed flow-induced broadening of the k_r spectrum of fluctuations as shown in Fig. 4. These trends are in qualitative agreement with our theoretical predictions [16]. We also observe that the flow-induced broadening of the k_r spectrum is accompanied by the reduction in fluctuation level, although we have not studied the relation between them in detail. Finally, the $\mathbf{E} \times \mathbf{B}$ flows also broaden the oscillation frequency of the individual mode via random shearing [5].

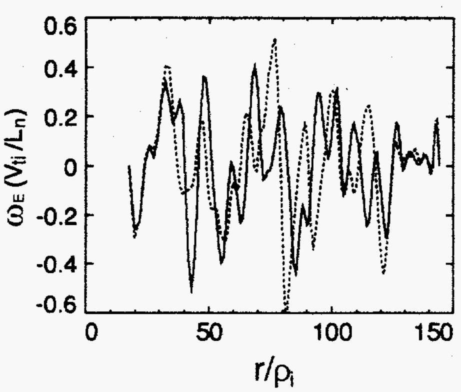

FIG. 3. Instantaneous shearing rates associated with small-scale turbulence generated flow from global gyrokinetic simulations. Two curves are separated by approximately one turbulent decorrelation time.

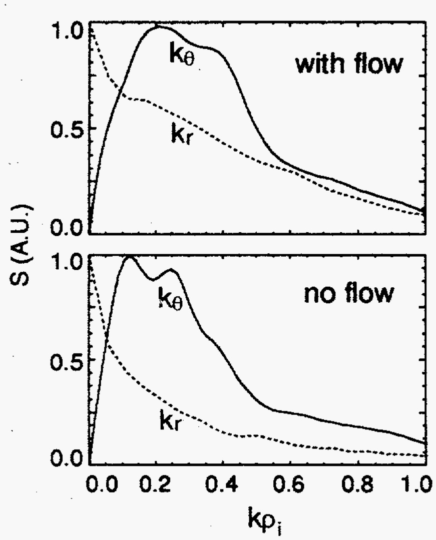

FIG. 4. k_r and k_θ spectra with (solid) and without (dotted) $\mathbf{E}\times\mathbf{B}$ flows in global gyrokinetic simulations.

ACKNOWLEDGMENTS

We would like to thank M.A. Beer, L. Chen, P.H. Diamond, G.W. Hammett, Y.B. Kim, and W.M. Nevins for useful conversations. This work was supported by the U.S. Department of Energy Contract No. DE–AC02–76–CHO-3073.

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