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Ambient-Temperature Passive Magnetic Bearings: Theory and Design Equations

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Abstract

As described previously (R. F. Post, D. D. Ryutov, J. R. Smith, and L. S. Tung, Proc. of MAG '97 Industrial Conference on Magnetic Bearings, p. 167), research has been underway at the Lawrence Livermore National Laboratory to build a theoretical and experimental base for the design of ambient-temperature passive magnetic bearings for a variety of possible applications. In the approach taken the limitations imposed by Earnshaw's theorem with respect to the stability of passive magnetic bearing systems employing axially symmetric permanent-magnet elements are overcome by employing special combinations of elements, as follows: Levitating and restoring forces are provided by combinations of permanent-magnet-excited elements chosen to provide positive stiffnesses (negative force derivatives) for selected displacements (i.e., those involving translations or angular displacement of the axis of rotation). As dictated by Earnshaw's theorem, any bearing system thus constructed will be statically unstable for at least one of the remaining possible displacements. Stabilization against this displacement is accomplished by using periodic arrays ("Halbach arrays") of permanent magnets to induce currents in close-packed inductively loaded circuits, thereby producing negative force derivatives stabilizing the system while in rotation. Disengaging mechanical elements stabilize the system when at rest and when below a low critical speed. The paper discusses theory and equations needed for the design of such systems.

I) Introduction

There are many examples of rotating machinery, e.g., modular flywheel energy storage systems (electromechanical batteries) where it would be highly advantageous to be able to employ "passive" magnetic bearing systems. Compared to "active" magnetic bearings (those employing position sensors, electronic amplifiers, and control magnets) passive bearing systems could be less complex, less subject to failure, and, possibly, far lower in cost. Passive magnetic bearings must, however, be able to overcome the well-known consequences of Earnshaw's theorem [1] This theorem asserts the impossibility of statically levitating systems employing only permanent magnets or electromagnets with fixed currents. One approach, pursued by Argonne National Laboratory [2] and by other groups, is to employ superconducting elements in the bearing system. Owing to their diamagnetic and other characteristics, superconductors evade Earnshaw's theorem, and thus offer a way to resolve this problem. This solution, however, necessarily involves the use of cryogenic systems, with their attendant power requirements and complexity. Passive bearing systems employing ambient-temperature permanent-magnet elements would seem preferable to either active or superconductor-based bearing systems for a wide variety of possible applications.

Research has been underway for some time at the Lawrence Livermore National Laboratory to build a theoretical and experimental base for designing ambient-temperature passive magnetic bearings for various applications [3]. In brief summary of the working principles undergirding this particular approach to passive magnetic bearing systems, they are the following:

(1) It is sufficient in the applications intended if stability is only achieved in the rotating state. That is to say, a centrifugally disengaging mechanical system can be used to insure stable levitation at rest (when Earnshaw's theorem applies). This relaxation of requirements opens up the possibility of using dynamic effects to achieve stability, a possibility not included in the assumptions made in deriving Earnshaw's theorem.

(2) Stable levitation results if the vector sum of the force derivatives of the several elements of the bearing system, for axial, radial, and tilt-type displacements from equilibrium, is net negative (i.e., restoring). In this way it is possible to achieve Earnshaw-stable levitation using systems composed of multiple elements, no one of which is by itself stable against all of these displacements. Insuring stability then becomes a quantitative matter, where the destabilizing tendency, for a given displacement of one magnetic element is paired off against the (greater) stabilizing tendency of another element for displacements in that same direction, and so forth.

This article will be concerned with presenting the results of theoretical analyses, in the form of design equations, that we have developed to facilitate the design of Earnshaw-stable ambient-temperature passive bearing systems. Only brief comments will be made on the next level of instability-related problems encountered in rotating systems, rotor-dynamic instabilities. Our analyses of this latter problem (in the context of passive magnetic bearing systems), and the stability criteria that have resulted from these analyses will be the subject of future papers.

II) Criteria for Earnshaw-Stability of Levitated Rotors

We first define criteria that, if met, will insure the Earnshaw-stability of a rotor supported by a passive bearing system. Figure 1 is a schematic drawing of such a system, in this case shown with the axis of rotation being vertical. The magnetic bearing components, A and B, shown above and below the rotor, may be composed of sub-elements, as described later. The combined characteristics of bearing components A and B are represented by stiffnesses K_A and K_B for lateral displacements (i.e., force derivatives with magnitudes $-K_A$ and $-K_B$). For axial displacements bearing components A and B will be characterized by stiffnesses $\beta_A K_A$ and $\beta_B K_B$. In the equations of motion of the rotor/bearing system we define transverse displacements of the center of mass by the variables x and y, and axial displacement by the variable z. Tilts of the axis can be characterized by two angles, θ_x, θ_y, representing tilts in the xz and yz planes, respectively. For small perturbations, lateral displacements of the axis in the bearings A and B can be represented as:

$$x_A = x + \theta_x d \qquad x_B = x - \theta_x d \qquad y_A = y + \theta_x d \qquad y_B = y - \theta_x d \qquad (1)$$

Perturbations of the potential energy with respect to the equilibrium state can be written as:

$$U = \frac{K_A x_A^2}{2} + \frac{K_A y_A^2}{2} + \frac{K_B x_B^2}{2} + \frac{K_B y_B^2}{2} + \frac{(\beta_A K_A + \beta_B K_B) z^2}{2} \qquad (2)$$

Using the Hamiltonian approach [4], and noting that the kinetic energy of the perturbations in the system, where we ignore gyroscopic effects, is a positive-definite quadratic form, one comes to the conclusion that the system is stable if and only if the potential energy (2) is also a positive-definite quadratic form, i.e., if

$$K_A > 0,\ K_B > 0,\ \text{and}\ (\beta_A K_A + \beta_B K_B) > 0 \qquad (3)$$

Our task in defining an Earnshaw-stable passive bearing system will be to find combinations of bearing elements satisfying (3), recognizing that these conditions are more stringent than required in all situations.

In summary to this point, achieving Earnshaw-stability in a passive magnetic bearing system is an exercise in defining a bearing system composed of various elements, no subset of which is required to be stable against all displacements from equilibrium. One is not here dealing with individual elements that are intrinsically stable, but rather with a bearing system achieving stable levitation by the satisfaction of quantitative requirements on its various elements.

III) Axially Symmetric Permanent-Magnet Elements

To provide levitation and/or centering forces in our passive bearing system it utilizes axially symmetric bearing elements employing permanent magnet material. The simplest form of such elements are permanent magnets in the form of thin discs or annuli, magnetized in the axial direction and polarized so as either to attract or to repel. When facing each other, such elements in the attracting polarization provide radial centering, but are, per Earnshaw, unstable against axial displacements, and vice-versa for the repelling polarization.

Before presenting approximate analytical formulae for the forces and stiffnesses of the above-described elements, we note a property of all such axially symmetric elements. That is, the absolute value of the radial and axial stiffnesses of such elements for small displacements are in ratio of 1:2, while their signs are opposite. Thus, in calculating the axial stiffness of such an element one can be assured that the radial stiffness will be of opposite sign and of half the magnitude. This result is a consequence of the curl-free nature of the vacuum magnetic fields, and of the fact that in calculating the radial stiffness one must take a cosine-average of the forces, whereas for axial displacements no such averaging is required.

For the case of two equal-diameter magnetized discs with radius = b (m.) and thickness = h (m.), facing each other at a separation distance = 2a (m.), where a << h and a << b, the magnitude of the axial force exerted by one disc on the other one is given approximately by the expression:

$$F_z = \frac{2B_r^2 bh}{\mu_0}\left\{(1+a/h)\ln[1+a/h]-(1+2a/h)\ln[\tfrac{1}{2}(1+2a/h)]+(a/h)\ln(a/h)\right\}\ \text{Newtons} \qquad (4)$$

Here B_r (Tesla) is the remanent field of the permanent magnetic material (e.g., 1.4 Tesla for high-flux NdFeB magnet material), and $\mu_0 = 4\pi \times 10^{-7}$ hy/m.

In the limit of zero separation between the magnets, the limiting axial force becomes:

$$F_z(\max) = 2\ln(2)\left[\frac{B_r^2 bh}{\mu_0}\right]\ \text{Newtons} \qquad (5)$$

The axial force derivative (negative of the axial stiffness) is (for attracting magnets):

$$\frac{dF_z}{dz} = \frac{B_r^2 b}{\mu_0}\left\{2\ln\left[\tfrac{1}{2}(1+2a/h)\right] - \ln\left[(a/h)(1+a/h)\right]\right\} \quad \text{Newtons/meter} \qquad (6)$$

For repelling magnets the magnitude of the force and of its axial derivative remain the same, but the signs of both are changed. Also, as noted, the magnitude of the force derivative for displacements transverse to the axis is half of the value given by equation (6), and of opposite sign.

One can deduce directly from the above results obtained for repelling and attracting axially symmetric magnets that the best one can do with any combination of such elements is to achieve a meta-stable state, i.e., one in which the force derivatives of the various elements add to zero, a situation of no practical value. Using only such elements it is impossible to satisfy criterion (3).

Equations (4) and (6) are but examples of the types of equations that will be needed for the design of the systems we will describe. There are, of course, other axially symmetric magnet configurations that could be employed, such as concentrically nested annular magnets, and circular-pole magnets energized by permanent-magnet material. The relative stiffnesses and forces of these alternate configurations could be determined analytically, by the use of computer codes, or by measurements, and the data obtained plugged into the overall design to arrive at an Earnshaw-stable situation, employing the "stabilizer" elements to be described in the sections to follow.

IV) Halbach-Array Stabilizers

As per the previous discussion, to achieve an Earnshaw-stable system levitated by ambient-temperature permanent magnets it is necessary to add another ingredient. That is, one must introduce at least one element that will have a ratio of transverse to longitudinal force derivatives that deviates from the -2:1 or 2:-1 stiffness ratios of the axially symmetric elements, in such a way and of a sufficient magnitude that the bearing system taken as a whole can satisfy the requirements of criterion (3) or its generalizations (for example, in order to include gyroscopic effects).

The stabilizer elements to be described employ periodic arrays of permanent magnets, configured in "Halbach arrays", named after the physicist who pioneered their analysis and use, Klaus Halbach. These configurations, employing only permanent-magnet bars in their construction, represent optimally efficient ways to assemble such bars, creating a strong periodically varying magnetic field at one face of the array, while nearly canceling the field on the back face of the array. Devised by Halbach for use in particle accelerators and free-electron lasers, they also turn out to be ideally suited for the stabilizers described here.

Halbach-array stabilizers take two geometrically different forms; "transverse" and "axial." Figure (2) is a schematic representation of one form of a transverse stabilizer. A rotating multi-pole Halbach array is shown surrounding a close-packed array of inductively loaded electrical circuits, (only the outer conductor of each such circuit is shown). The rotating Halbach array produces a time-varying flux in each circuit. Above a low critical speed (determined by the circuit resistance and inductance), the induced current approaches a phase shift of nearly 90 degrees relative to the inducing flux. This current, interacting with the magnetic field of the Halbach array, then exerts a net transverse restoring force on the rotating magnet array, thereby providing the needed stabilizing force derivative for magnet systems that would be otherwise unstable radially. At the same time, the axial force derivative of such an element is very low, arising only from weak end effects, so that the stabilizer will not itself contribute any appreciable destabilizing effect in the axial direction.

Since they employ non-axially symmetric fields, and since they involve dynamic (induction) effects, Halbach array stabilizers are not subject to the constraints of Earnshaw's theorem. Thus either alone, or in combination with axially symmetric permanent-magnet elements, they enable the design of Earnshaw-stable systems (for operation above a critical speed)[5].

The magnetic field produced by a N-pole Halbach array [6] as a function of radial position, $\rho < a$, and azimuthal angle, ϕ, is given by the following equations:

$$B_\rho = B_0 \left[\frac{\rho}{a}\right]^{N-1} \cos(N\phi) \quad (7)$$

$$B_\phi = -B_0 \left[\frac{\rho}{a}\right]^{N-1} \sin(N\phi) \quad (8)$$

$$B_0 = B_r \left\{\frac{N}{N-1} \left[1 - \left(\frac{a}{b}\right)^{N-1}\right] C_N\right\} \quad (9)$$

$$C_N = \cos^N(\pi/M)\left[\frac{\sin(N\pi/M)}{(N\pi/M)}\right], \quad N > 1 \quad (10)$$

In these expressions the quantity a (m.) is the inner radius of the Halbach array, and b (m.) is its outer radius. In the expression for C_N, the quantity M is the total number of magnets in the array. In Figure 2, and for the type of array shown, there are 4 magnets per pole (i.e., 4 magnets per wavelength in the azimuthal direction), so that M = 4N., i.e. N = 6 in the figure.

We consider two types of circuits for the windings of the stabilizer. The simplest type of circuit is a rectangular "window frame." The outer leg of this rectangular circuit, located at radius ρ_2, corresponds to one of the conductors shown in the figure, while the inner leg (not shown in the figure) is located at radius $\rho_1 < \rho_2$. The inductance of each circuit (self-inductance plus the effect of mutual inductance with adjacent circuits) is taken to be equal to L_0 (henrys), and its resistance is R (ohms). The conductor itself is "litzendraht" (litz) wire. That is, it is composed of a multi-stranded bundle of fine strands of insulated copper wire. As later discussed, the use of litz wire greatly reduces the power losses associated with internal eddy currents in the wires.

The current induced in the circuits by the rotating Halbach array can be calculated from the flux produced by the array fields (equations 7 - 10). At low speeds the current leads the flux by 90 degrees, resulting in drag forces but little repulsion. As the speed increases the phase lags until it approaches that of the flux, at which point the repelling force is maximal, and the drag torque is greatly reduced (varying inversely with the speed). The "transition speed," defined as the rotation speed where the repelling force has reached half its limiting value, is given by the relationship:

$$\omega_T = \frac{1}{N}\left[\frac{R}{L_0}\right] \quad \text{radians/sec.} \quad (11)$$

For typical stabilizers, this transition speed can be as low as a few hundred RPM. If it is desirable to lower the transition speed, inductive loading can be added to each of the circuits (we have used small powder-core toroids for this purpose).

From the analysis the expression for the stiffness, K_x, of this form of stabilizer for displacements transverse to the axis of rotation is:

$$K_x = \frac{(2N-1)\lambda M}{4\rho_2}\left[\frac{B_0^2 a h^2}{N L_0}\right]\left\{1 - \left(\frac{\rho_1}{\rho_2}\right)^N\right\}\left[\frac{\rho_2}{a}\right]^{2N-1} \quad \text{Newtons/m.} \quad (12)$$

The quantity λM corresponds to the total number of circuits, and the quantity h (m.) is the axial length of the Halbach array bars.

The analysis may also be extended to evaluate the ohmic power losses in the circuits relative to the stiffness. The expression derived is:

$$\frac{K_x}{P_0} = \frac{N(N-1)L_0}{2R\rho_2^2}\left\{\frac{1}{1 - \left(\frac{\rho_1}{\rho_2}\right)^N}\right\} \quad \text{Newtons m}^{-1}\text{ watt}^{-1} \quad (13)$$

When it is desirable to minimize the power losses associated with the stabilizer, reference [3] describes a version in which ohmic losses approach "zero" in the centered position. In this version, the window-frame circuits have their legs on opposite sides of the stator, and the Halbach array has an even order (N = 2, 4, 6, etc.). In this case there is flux cancellation in the centered condition and (except for residual currents arising from mechanical and magnetic tolerances associated with the Halbach array and the windings) the induced currents approach zero. For this case, the stiffness is given by the equation:

$$K_x = \frac{4\pi}{\mu_0}\left\{\frac{B_0^2 h^2 N}{P}\right\}\left[\frac{c}{a}\right]^{2N-1} \quad \text{Newtons/meter} \quad (14)$$

Here the quantity c (m.) is the radius of the cylindrical stator, and the quantity, P (m.), is the perimeter of each circuit. This term arises from inserting into the expression the results of an evaluation of the effect of the adjacent circuits on the inductance of each circuit, assuming no added inductive loading is used. As a result, and in the case that a high-order Halbach array is used ($N >> 2$), the stiffness values that can be attained can be quite high, in excess of 10^7 Newtons/m.

The stabilizers just described address the problem of stabilization of a bearing system that is unstable for transverse displacements, but stable axially. An example would be a rotor levitated vertically between two sets of repelling magnets. For those cases where transverse stability is present (for example when attracting magnet pairs or their equivalent are used) and the system is unstable axially, a stabilizer employing planar Halbach arrays can be used. Figure 3 is a schematic representation of such a stabilizer. As shown, a planar circuit array is positioned midway between planar Halbach arrays, coupled to each other at their inner radii so that they rotate together, and oriented azimuthally so that their axial fluxes cancel at the midway between them. Thus, when positioned midway between the Halbach arrays no currents are induced in the circuits, but currents and restoring forces arise from any axial displacement from this position. The design equation derived for this case is the following:

$$K_z = \frac{B_{0p}^2 N_c}{L_0}\left[bG(\alpha_b) - aG(\alpha_a)\right]^2 \quad \text{Newtons/meter} \qquad (15)$$

In this expression a (m.) is the inner radius of the planar Halbach array, b (m.) is its outer radius, the parameters $\alpha_a = Nh/a$ and $\alpha_b = Nh/b$, N_c is the number of circuit wires, and the function $G(\alpha)$ is defined by the relationship:

$$G(\alpha) = \left[1 + \alpha \ln(\alpha) + \alpha(1 - C)\right], \; C = \text{Euler's const.} = 0.577.... \qquad (16)$$

The quantity B_{0p} represents an effective mean value of the peak value of the magnetic field at the midplane between the two Halbach arrays at the inner radius, a. This quantity was calculated from results derived by Halbach for a linear array such as would be used as a "wiggler" in a free-electron laser. The result is given by (M = total number of magnet bars in each array):

$$B_{0p} = 2B_r\left\{1 - \exp\left[-\frac{N\,t}{a}\right]\right\}\left[\frac{\sin(\pi N/M)}{(\pi N/M)}\right]\exp(-Nh/a) \quad \text{Tesla} \qquad (17)$$

In a later section an example of the design of a complete Earnshaw-stable bearing system employing design equations given here will be outlined. We will now discuss a loss-related issue associated with residual eddy currents in the litz wire of the stabilizer circuits.

V) Eddy-Current Losses in the Stabilizer Windings

In the stabilizer configurations described the legs of the litz-wire circuits closest to the Halbach arrays are exposed to a rotating vector magnetic field, whether or not there is cancellation of the flux linked by these windings. This rotating field will induce residual eddy currents in the strands of the litz wire, thus leading to losses. However, since these residual losses vary as the fourth power of the diameter of these strands, the use of litz wire composed of many strands of fine wire can reduce these losses to the fractional-Watt level. It is straightforward to calculate the eddy-current loss per meter in a litz wire strand. The expression derived is:

$$\frac{P_{ec}}{L} = \frac{\pi}{4}\left[\frac{B^2\omega^2 a^4}{\rho_c}\right] \quad \text{Watts/meter of conductor} \qquad (18)$$

Here ω (rad./sec.) = $N\omega_0$, where ω_0 is the angular frequency of the rotating system), ω is the frequency of the rotating field of magnitude B (Tesla), a (m.) is the radius of the conductor strand, and ρ_c (ohm-meter) is its resistivity.

VI) Comments on the Stabilization of Rotor-Dynamic Instabilities

While the emphasis in this paper is on the design of Earnshaw-stable passive magnetic bearing systems, some remarks are in order on the use of passive elements to stabilize rotor-dynamic modes. Two generically different techniques exist for their stabilization: (1) eddy-current dampers, and, (2) anisotropic radial stiffness. The former involves, for example, stationary conducting sheets exposed to axially symmetric fields from circular poles excited by permanent magnets. This technique has been employed, for example, by Fremerey [7]. The second technique, believed novel to our approach, is to introduce anisotropic stiffness in the radial stabilizers of the types described by equations (12) and (14). This result can be accomplished by modulating the spacing of the circuits as a function of azimuth, or by modulating their inductive loading as a function of azimuth, or by using a stator of elliptical cross-section. As described in reference [3], the degree of anisotropy required for this type of stabilization depends on the magnitude of the displacement-dependent drag terms, as follows:

$$\frac{K_x}{K_y} < \left[1 - \frac{2\sqrt{\alpha_x \alpha_y}}{K_y}\right] \text{ , stable, } K_x < K_y \qquad (19)$$

Here α_x and α_y (Newtons/meter), the displacement-dependent drag-force terms, have been assumed also to be anisotropic for generality. When these terms are small (as they are likely to be in many situations), the degree of anisotropy predicted to be required for stabilization would be minimal. Note that in addition to the stabilization introduced by anisotropy, displacement-dependent ohmic losses in the stabilizer windings can be expected to introduce some damping of radial oscillations.

VII) Example System Design

To illustrate the use of the design equations that have been presented we outline the design of a vertical-axis system supporting a mass of 10 kilograms. (F_z = 100 Newtons, approximately). Of the two alternatives, we choose to levitate the rotor between two sets of repelling disc-shaped magnets, so that the system is stable axially but unstable radially. Stabilization for transverse displacements is then to be achieved by using upper and lower Halbach-array transverse stabilizers.

The parameters of the upper and lower repelling bearing sets are chosen by requiring that they both have the same magnitude of (unstable) transverse stiffness, subject to the requirement that the difference in their axial forces should equal 100 Newtons (to provide levitation). We further assume that the ratio of the half-gap to the magnet thickness, i.e., the parameter (a/h), is the same for both magnet sets. It then remains to select the radius, b, and the relative thickness of the upper and lower magnet sets in order to satisfy the two requirements.

We begin by evaluating the equations for the axial force, equation (4), and equation (6) for the axial stiffness (from which the transverse stiffness can be evaluated) in the case that the parameter (a/h) is fixed at the value 0.1 (thus satisfying the small-parameter assumption made in deriving the equations):

$$F_z\,(a/h = 0.1) = 0.97515\left[\frac{B_r^2 bh}{\mu_0}\right] \text{ Newtons} \qquad (20)$$

$$K_z\,(a/h = 0.1) = 1.1856\left[\frac{B_r^2 b}{\mu_0}\right] \text{ Newtons/meter} \qquad (21)$$

Recalling the 2:-1 ratio of stiffnesses for axially symmetric permanent-magnet elements that was discussed in Section III, we have for the transverse stiffness the relationship:

$$K_x\ (a/h = 0.1) = -\ 0.5928 \left[\frac{B_r^2 b}{\mu_0}\right] \quad \text{Newtons/meter} \qquad (22)$$

We will also use in the design equation (5), defining the maximum (repelling) force, occurring as the gap, a, approaches zero. Putting in numerical values for the coefficient we have:

$$F_z\ (\max) = 1.3863 \left[\frac{B_r^2 bh}{\mu_0}\right] \quad \text{Newtons} \qquad (23)$$

We further assume that the relative magnet thickness of the lower magnet element (the one that provides the levitating force), is h/b = 0.2, and that the remanent field B_r = 1.25 Tesla (standard grade NdFeB material). For the upper magnet we will leave the thickness, h, as a variable to be determined. With these assumptions equations (20) and (23) become:

$$F_z\ (a/h = 0.1,\ h/b = 0.2,\ B_r = 1.25\ T) = 2.4250 \times 10^5\, b^2 \quad \text{N (lower mag.)} \qquad (24)$$
$$F_z\ (\max) = 3.4474 \times 10^5\, b^2 \quad \text{N (lower mag.)} \qquad (25)$$
$$F_z\ (a/h = 0.1,\ B_r = 1.25\ T) = 1.2125 \times 10^6\, bh \quad \text{N (upper mag.)} \qquad (26)$$
$$K_x\ (a/h = 0.1,\ B_r = 1.25\ T) = -\ 7.3709 \times 10^5\, b \quad \text{Newtons/meter} \qquad (27)$$

Imposing the requirement that the transverse stiffness of both the top and bottom bearing elements should be equal implies that both magnets have the same radius (their thicknesses will not be equal, however). The common radius can be determined by establishing a value for F_z (max). To provide a robust value of the levitating axial force we take this maximum value to be 400N (4 times the weight that is to be levitated). With this assumption one finds for the magnet radii, b = .03406 m. The thickness of the upper magnet element can be determined by imposing the condition that the net levitating force should be 100 Newtons, yielding the equation:

$$2.4250 \times 10^5\, b^2 - 1.2125 \times 10^6\, b^2 (h/b) = 100 \text{ Newtons} \qquad (28)$$

Inserting the previously determined value of b we find (h/b) = 0.1289, for the upper magnet, to be compared to (h/b) = 0.2 for the lower magnet.

To complete the design we need only determine parameters for radial stabilizers whose positive transverse stiffness sufficiently exceeds the negative stiffness of the levitator magnets to yield a desired net positive stiffness value. Inserting the value of b into equation (27) we find for the negative stiffness of each magnet set the value

$$K_x\ (a/h = 0.1,\ B_r = 1.25\ T) = -2.511 \times 10^4 \quad \text{Newtons/meter} \qquad (29)$$

If we employ stabilizers of the type represented by equations 7,8,9,10, and 14 with the parameters B_r = 1.25 T, a/b = 0.8, h = 0.05 m. , P = 3h, N = 6, and c/a = 0.95, we obtain a positive stiffness value $K_x = 4.2 \times 10^5$ Newtons/meter, or about 16 times the negative stiffness of the levitating elements. It should be apparent that the stiffness of the Halbach array stabilizers can readily be made to overcome the negative stiffnesses of the levitating bearings.

VIII) Summary and Conclusions

We have outlined the theory and presented some design equations that can be used to perform the design of ambient-temperature passive magnetic systems that satisfy criteria for Earnshaw-stability. We have further sketched some approaches to the stabilization of rotor-dynamic instabilities in such systems. We have concluded by using the design equations to arrive at an example set of parameters for a vertical-axis system whose mass is 10 kilograms, finding reasonable values for all of the required parameters. The results presented are being incorporated in models that will explore practical issues that are sure to be encountered in converting the theoretical results into working systems.

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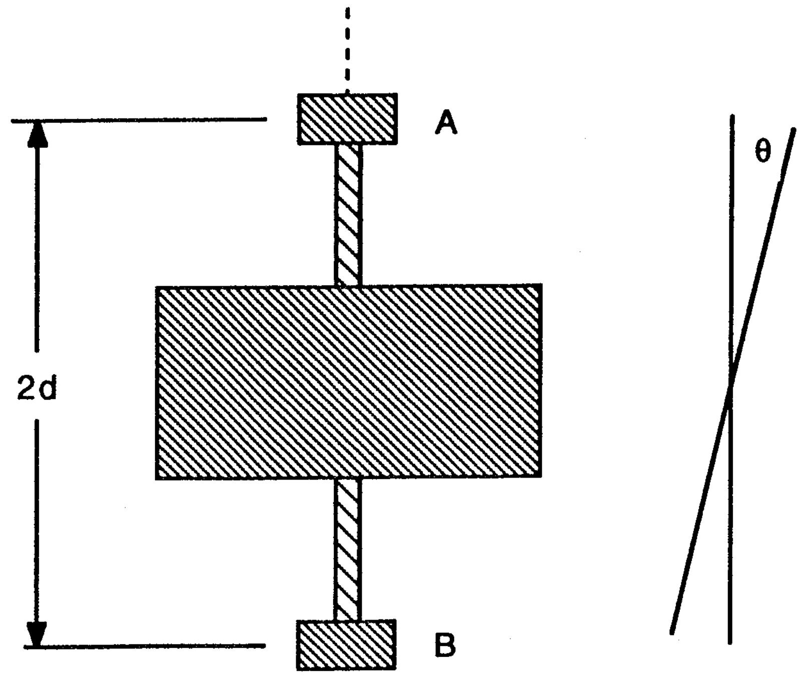

Fig. 1

Schematic drawing of rotor-bearing system

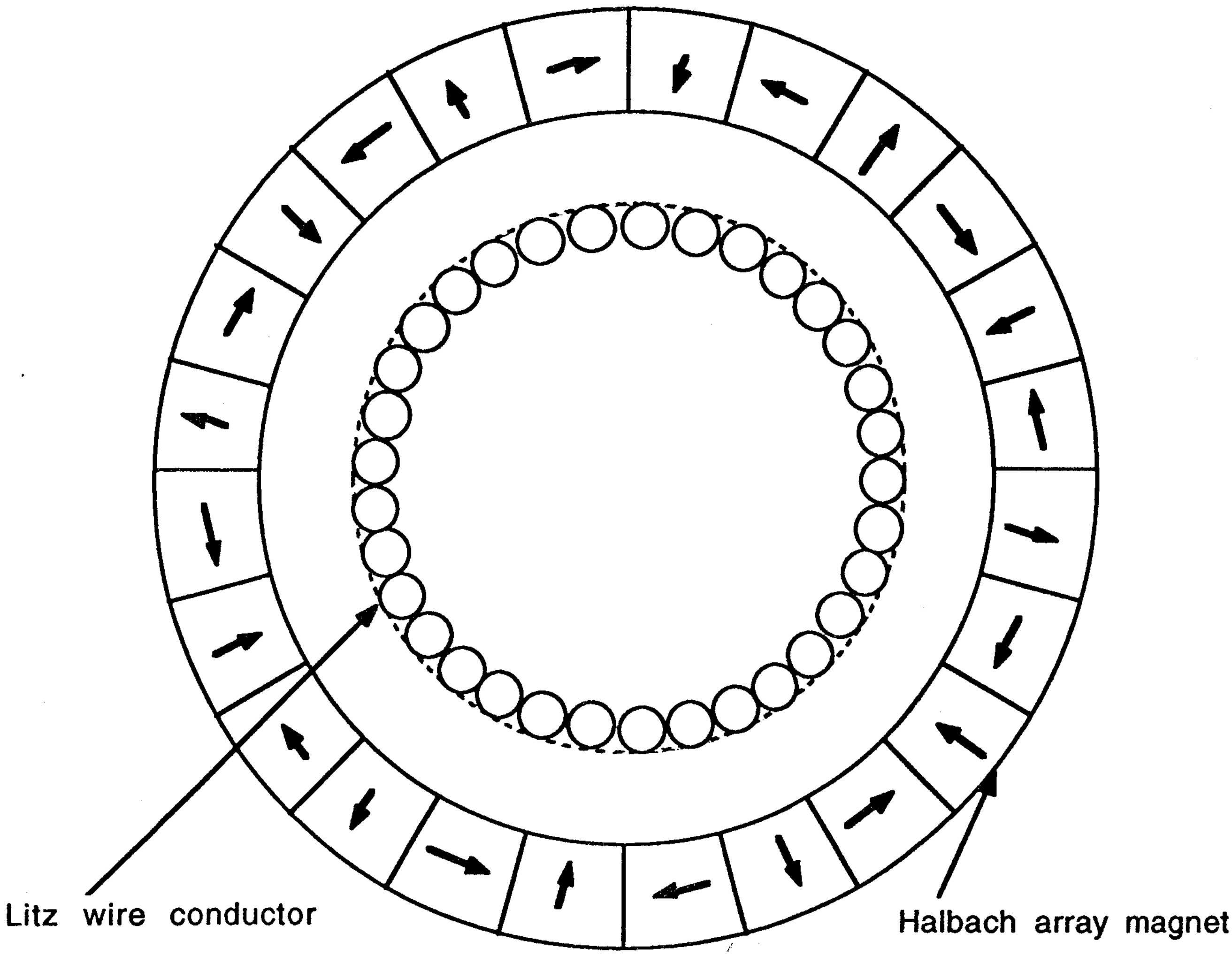

Fig. 2.

Schematic drawing of transverse stabilizer

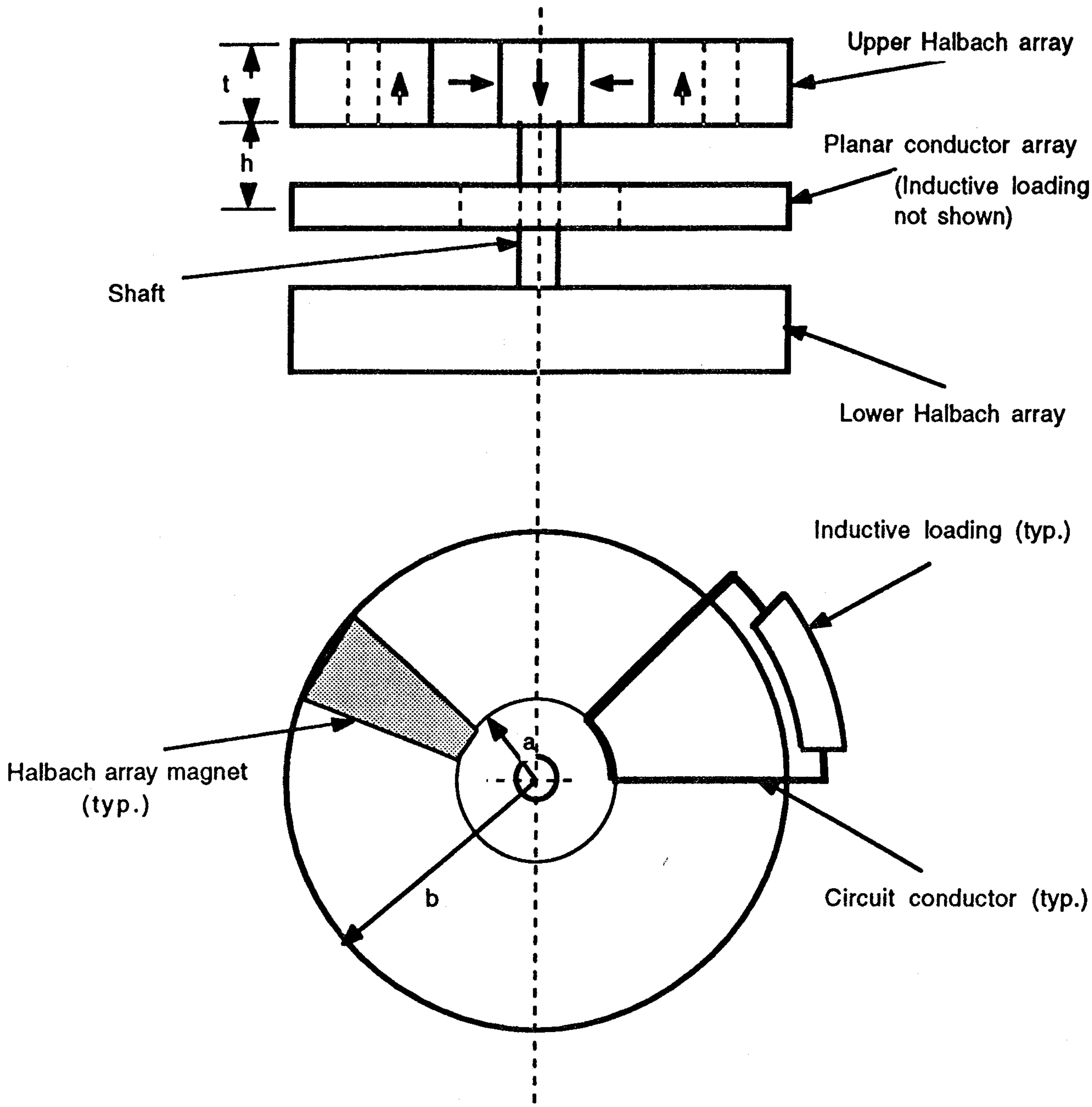

Fig. 3

Schematic drawing of axial stabilizer using planar Halbach arrays

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