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Title: ON THE RELIABILITY OF THE NERVOUS (Nv) NETS

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On the Reliability of the Nervous (Nv) Nets

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Abstract. This paper investigates the reliability of a particular class of neural networks, the Nervous Nets (**Nv**). This is the class of nonsymmetric ring oscillator networks of inverters coupled through variable delays. They have been successfully applied to controlling walking robots, while many other applications will shortly be mentioned. We will then explain the 'robustness' of **Nv** nets in the sense of their highly reliable functioning — which has been observed through many experiments. For doing that we will show that although the **Nv** net has an exponential number of periodic points, only a small (still exponential) part are stable, while all the others are saddle points. The ratio between the number of stable and periodic points quickly vanishes to zero as the number of nodes is increased, as opposed to classical finite state machines — where this ratio is relatively constant. These show that the **Nv** net will always converge quickly to a stable oscillatory state — a fact not true in general for finite state machines.

The model we shall discuss wants to duplicate the activity of the human brain [1]. This is made of living neurons composed of a cell body and many outgrowths. One of these is the *axon* — which may branch into several collaterals. The axon is the 'output' of the neuron. The other outgrowths are the *dendrites*. The end of the axons from other neurons are connecting to the dendrites through 'spines'. Active pumps in the nerve cell walls push sodium ions outside, while keeping fewer potassium ions inside. Therefore, their tendency is to keep the cell body at a small negative electric potential (–60mV). The electrical balance varies at the exit point of the axon. If the electrical potential of the cell becomes too positive (+10÷15mV), the potential suddenly jumps to about +60mV. After a short delay of 2÷3ms the potential returns to the normal negative value (–60mV). This change of potentials is sequential and is called an *action potential*. The action potential travels down the axon and its branches (with a speed in the range 1÷10m/s). This variation of potential represents the signal sent by one neuron to its neighbours. The generation of the signal is achieved by summing the signals coming from the dendrites. The strength of the action potentials travelling along an axon are identical, nevertheless, the effects to the neighbouring cells are different. This is due to the rescaling effect which takes place at the *synapse*.

Formally, a *network* is a graph having several input nodes, and some (at least one) output nodes [27]. If a synaptic *weight* is associated with each edge, and each node i computes the weighted sum of its inputs to which a nonlinear activation function is then applied (*i.e.*, *artificial neuron*, or simply *neuron*):

$$f_i(\boldsymbol{x}_i) = f_i(x_{i,1}, \ldots, x_{i,k}) = \sigma_i\left(\textstyle\sum_{j=1}^{k} w_j x_{i,j} + \theta_i\right), \tag{1}$$

the network is a *neural network* (NN), with the synaptic *weights* $w_j \in \mathbb{R}$, $\theta_i \in \mathbb{R}$ known as the *threshold*, $k \in \mathbb{N}$ being the *fan-in*, and σ_i a non-linear activation function. Three well known and widely used non-linear activation functions are: (*i*) the sigmoid function $\sigma(x) = 1/(1 + e^{-ax})$, where a is the gain, or amplification factor; (*ii*) the linear function $\sigma(x) = x$; (*iii*) the threshold function $\sigma(x) = sgn(x)$.

Such a description of the living nerve cells is a correct representation of the system [8]. A more detailed model includes different *delays* for the transmission and/or computation of the signals, *e.g.* a discrete-time and synchronous model would be:

$$f_i(\boldsymbol{x}_i, t+1) = \sigma_i\left\{\textstyle\sum_{j=1}^{k} w_j x_{i,j}(t) + \theta_i\right\}. \tag{2}$$

More complex time dependences could be continuous, and could include the modification of *weights* $w_i(t)$, and *thresholds* $\theta_i(t)$ — an aspect which relates to learning, *i.e.* modifying the *weights* and the *thresholds* such as to adapt to external stimuli.

If the underlying graph is *acyclic*, the network does not have feedback connections, and can be layered being known as a *multilayer feedforward neural network*, and are commonly characterised by two cost functions: *depth* (*i.e.*, number of layers) and *size* (*i.e.*, number of neurons). If the underlying graph is *cyclic*, the network has feedback connections and is characterised by *size*.

A particular class of feedback networks if formed by *rings*. The bi-directional (or symmetric) homogeneous ring has the following *weight* matrix:

$$\boldsymbol{W} = \begin{bmatrix} 0 & w_{2,1} & 0 & \ldots & 0 & w_{n,1} \\ w_{1,2} & 0 & w_{3,2} & \ldots & 0 & 0 \\ 0 & w_{2,3} & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots & \vdots \\ w_{1,n} & 0 & 0 & \ldots & w_{n-1,n} & 0 \end{bmatrix} \tag{3}$$

which shows that neuron i has only two inputs (*i.e.*, *fan-in* $k = 2$), as being connected only to its two adjacent neurons $i-1$ and $i+1 \pmod{n}$, with *weights* $w_{i,i-1}$ and $w_{i,i+1}$, respectively. Here n is the number of neurons (*i.e.*, the *size*). If the ring is non-symmetric, the *weight* matrix is:

$$\boldsymbol{W} = \begin{bmatrix} 0 & 0 & 0 \ldots & 0 & w_{n,1} \\ w_{1,2} & 0 & 0 \ldots & 0 & 0 \\ 0 & w_{2,3} & 0 \ldots & 0 & 0 \\ \vdots & \vdots & \vdots \ldots & \vdots & \vdots \\ 0 & 0 & 0 \ldots & w_{n-1,n} & 0 \end{bmatrix} \tag{4}$$

showing that neuron i has only one input (*i.e.*, *fan-in* $k = 1$) from neuron $i+1 \pmod{n}$ with a *weight* $w_{i,i+1}$.

We can now formally define the class of neural networks known as the "Nervous Net" or **Nv** [14, 15]. **Nv** is a non-symmetric ring (4) of n threshold neurons (1) with negative unit *weights*, *i.e.*, $w_{1,2} = w_{2,3} = \ldots = w_{n-1,n} = w_{n,1} = -1$, and *delay* on the transmission lines (2). Hence, for **Nv** eq. (2) becomes $f(x, t+\Delta) = sgn[-x(t) + \theta]$, where Δ is the *delay* associated to one neuron. In particular, because this *delay* can be modified, it can be seen as an additional parameter for the learning process.

There is clearly one important advantage of such NNs, namely the fact that the neurons are very easy realisable in hardware [2, 3]: the delay is represented by an RC high pass filter (R = resistance, C = capacitance), while an inverter (with Schmitt trigger) implements the threshold function [24].

Before going further, we shall shortly mentioned some of the applications where **Nv** can and have been used and tested [12, 13].

- Mine detection is an application where the **Nv** network has been used to *control the walking gaits and the direction of motion* for solar powered walking robots [4, 22, 23]. The robots react with the environment and navigate around or over obstacles while trying to detect the location of a mine.
- *Facility monitoring* and *clean room operations / tasks* are other applications for this technology.
- *For environmental clean-up and monitoring*, a group of autonomous diving robots have been designed using this technology.
- The **Nv** network is being studied as *a primary or backup controller* for satellite systems and other autonomous flying machines. Research and prototyping for nanosatellite systems is on-going [7, 9, 10, 11, 17, 26, 28, 29].
- Currently, *the study of these systems working together in a collective effort*, self-assembling to accomplish a common task is another area of research [18, 19].
- In the future, the *medical field* could use **Nv** networks in the design of manipulators or prosthetics.

The dynamics of discrete-time and synchronous continuum-state systems has been motivated by the possibility of constructing VLSI circuits for implementing such neural networks having prescribed fixed-points or periodic orbits [6, 16, 25]. For such networks, the dynamics is given by an iteration map $F: X^n \to X^n$. Using a slightly modified sigmoid function:

$$\psi(x) = 2\{\sigma(x) - 1/2\} = (1 - e^{-ax})/(1 + e^{-ax}) \tag{5}$$

Blum and Wang [5] have analysed nonsymmetric ring networks of n neurons. The state space is $X^n = [0, 1]^n$, while the state $x_n^{(i)}$ of neuron i at time $t = p$ (for $p = 0, 1, 2, \ldots$) is updated according to:

$$\begin{aligned}(x_{p+1}^{(1)}, x_{p+1}^{(2)}, \ldots, x_{p+1}^{(n)}) &= (f(x_{p+1}^{(1)}), f(x_{p+1}^{(2)}), \ldots, f(x_{p+1}^{(n)})) \\ &= F(x_{p+1}^{(1)}, x_{p+1}^{(2)}, \ldots, x_{p+1}^{(n)})\end{aligned} \tag{6}$$

Proposition 1 *Any closed periodic orbit of eq. (6) has minimum period* $q \le n$ *and* $q \mid n$. *Moreover, if* $x = (x^{(1)}, x^{(2)}, \ldots, x^{(n)})$ *is a periodic point, then each* $x^{(i)}$, $1 \le i \le n$, *must be a fixed point of the function f.*

If the nonlinear function f is ψ, the following has also been proved.

Proposition 2 *For the parameter* $a \le 2$, *the network described by eq. (4) has an attractor* $(0, \ldots, 0)$ *which is the only periodic point. For* $a > 2$, *the network has precisely* 3^n *periodic points, among which one is unstable (i.e.,* $(0, \ldots, 0)$*),* 2^n *are stable (all points that do not contain 0), and the others are saddles.*

For **Nv**, $a \gg 2$ as the neurons are threshold gates (inverter with Schmitt trigger). These explain the robustness of **Nv** nets in the sense of their highly reliable functioning (which has been observed through many experiments): although any **Nv** has an exponential number of periodic states (3^n), only a small (still exponential) part are stable (2^n), while all the others are saddle states ($3^n - 2^n - 1$). The ratio between the number of stable and periodic points is $(2/3)^n$, and vanishes to zero as the number of nodes is increased (these results will further be generalised to continuous states by quantization with a given step). On the other hand, for classical finite state machines this ratio is a constant as more than half of the states are stable states (we have supposed a classical design which uses binary encoding for the states). These shows that **Nv** nets will always converge (quickly) to one of the stable oscillatory states, while the statement does not hold (in general) for finite state machines.

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