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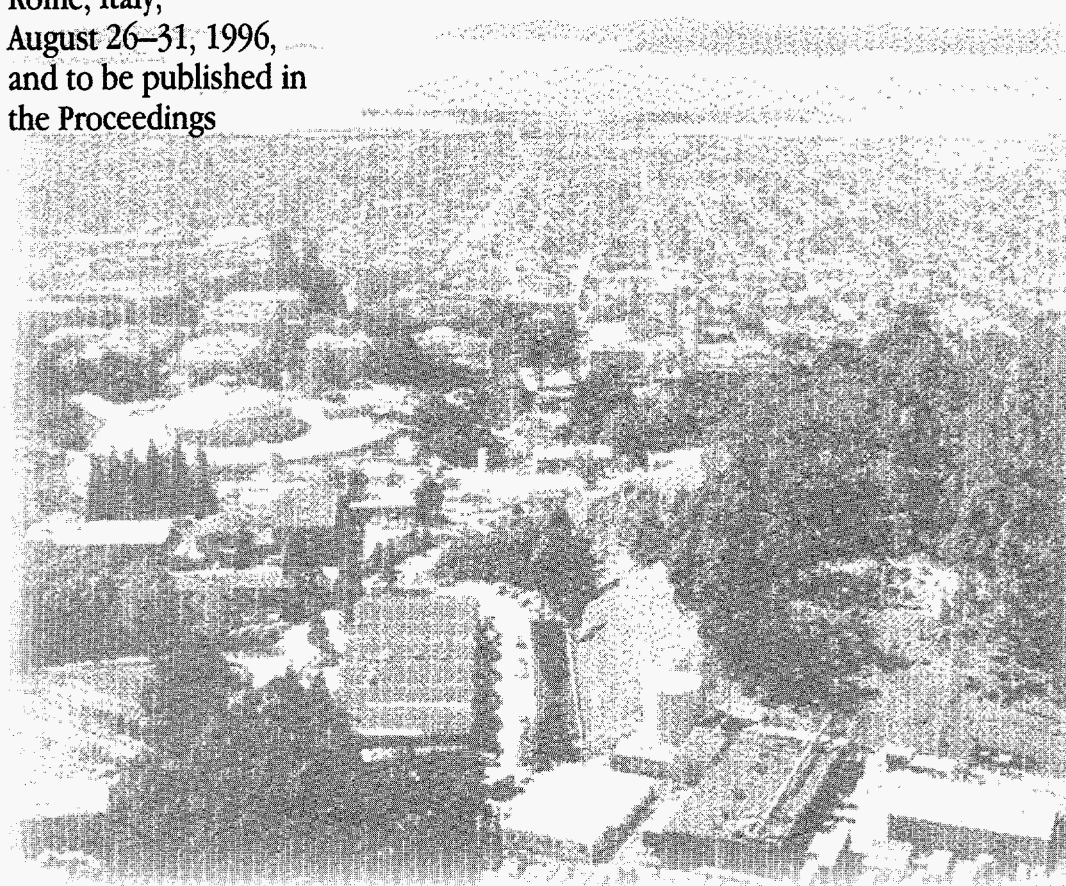

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Can a Free Electron Laser Operate with a Broad Momentum Spread?*

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1. Introduction

It is well-known that the gain in a usual free electron laser (FEL) vanishes when the energy spread of the electron beam is too broad. This is because electrons with positive detuning parameter ν are bunched around a certain phase contributing to a positive gain, while those with negative ν are bunched around another phase contributing to a negative gain. The net gain therefore vanishes for a beam of electrons distributed over a broad range of positive and negative values of ν. To avoid this problem, it was proposed recently [1] to use two undulators and to insert a device between them which separates the electrons of negative ν and to displace them in phase so that they contribute to an overall positive gain after passing through the second undulator. The device will be referred to as the "redistributor" in this paper. The scheme, illustrated in Figure 1, was an effort to extend the idea of the inversionless atomic laser to the FEL [2], and, if true, would have an important consequence for the future FEL development.

The purpose of this paper is to show that the scheme does not work as proposed. The reason is simple: consider an initial electron distribution which is uniform extending to a large positive and negative detuning where the FEL interaction vanishes. The distribution, because of Liouville's theorem, will remain the same after the first undulator. Clearly, the redistributor would have no effect on this distribution, and the gain of the total system vanishes. What is then wrong with the reasoning in the first paragraph in the above? It is because we did not take into account the contributions of *all electrons*. The electrons with positive detuning after the first half of the undulator consist of two groups; one started out with positive detuning at the beginning of the undulator and another started out with negative detuning. In the argument above, the contribution from the second group of electrons is not taken into account. The first group contributes to a positive net gain. However, this is canceled by the negative net gain due to the second group, irrespective of how the phase of electrons in the region of negative detuning is displaced relative to those in the positive detuning.

In section 2, we present a mathematical proof of the statement. In section 3, we interpret the result with a simple physical picture. Section 4 contains some comments about a momentum selector.

2. Derivation

Let us introduce the dimensionless field amplitude a(z) as a function of the dimensionless distance z along the undulator, and the electrons' phase space distribution function $F(\theta,\nu,z)=F_0(\nu)+F_1(\theta,\nu,z)$, where F_0 and F_1 are, respectively, the initial and the perturbed electron distribution functions, and θ is the electron phase variable. The coupled Vlasov-Maxwell equations describing the FEL interaction to the first order are:

$$\frac{\partial f(\nu,z)}{\partial z}+i\nu f(\nu,z)-a(z)\frac{dF_0(\nu)}{d\nu}=0, \tag{1}$$

$$\frac{da(z)}{dz}=\int f(\nu,z)d\nu, \tag{2}$$

where

$$f(\nu,z)=\frac{1}{2\pi}\int_0^{2\pi}F_1(\theta,\nu,z)e^{-i\theta}d\theta. \tag{3}$$

Solving eq. (1) and eq. (2) in perturbation theory, one obtains f and a at an arbitrary point z in terms of initial values $a(z_1)$ and $f(\nu,z_1)$

$$f(\nu,z)=e^{-i\nu(z-z_1)}f(\nu,z_1)+a(z_1)\frac{dF_0(\nu)}{d\nu}\int_{z_1}^{z}dz'e^{-i\nu(z-z_1)}, \tag{4}$$

$$a(z)=a(z_1)\left\{1+\int d\nu\frac{dF_0(\nu)}{d\nu}\int_{z_1}^{z}dz'\int_{z_1}^{z'}dz''e^{-i\nu(z'-z'')}\right\}+\int_{z_1}^{z}dz'\int d\nu e^{-i\nu(z'-z_1)}f(\nu,z_1). \tag{5}$$

For an unbunched electron beam, $f(\nu,0)$ vanishes at the undulator entrance, z=0. After an undulator of length L, we obtain from eqs. (4) and (5),

$$f(\nu,L)=a(0)\frac{dF_0(\nu)}{d\nu}\int_0^{L}dz'e^{-i\nu(L-z')}, \tag{6}$$

$$a(L) = a(0)\left\{1 + \int_0^0 dz' \int_0^{z'} dz'' \int d\nu e^{-i\nu(z'-z'')} \frac{dF_0(\nu)}{d\nu}\right\}. \tag{7}$$

Equation (7) leads to

$$|a(L)|^2 = |a(0)|^2 \left\{1 + \int d\nu \frac{dF_0(\nu)}{d\nu} U(\nu, L)\right\}. \tag{8}$$

Here,

$$U(\nu, L) = \left|\int_0^L dz e^{-i\nu z}\right|^2 = \left(\frac{\sin(\nu L/2)}{\nu/2}\right)^2 \tag{9}$$

is the spectrum of the spontaneous radiation. The total gain is

$$G_L = \frac{|a(L)|^2 - |a(0)|^2}{|a(0)|^2} = -\int d\nu F_0(\nu) \frac{d}{d\nu} U(\nu, L). \tag{10}$$

For a mono-energetic beam, $F_0(\nu) = \delta(\nu - \nu_0)$, and the gain is given by the Madey's theorem

$$G(\nu_0) = -\frac{d}{d\nu} U(\nu, L)\Big|_{\nu=\nu_0}. \tag{11}$$

These are all well-known results. We are ready to analyze the scheme proposed in Reference [1], shown schematically in Figure 1. After the first undulator, the electron distribution and the field are given by eqs. (6) and (7), respectively. Before entering the second undulator, however, the redistributor modifies the electron distribution: the electrons with $\nu>0$ are displaced in phase by $\Delta+$ and those with $\nu<0$ by $\Delta-$. This is equivalent to (for simplicity we assume that the redistributor has a vanishing length):

$$F_0(\nu) \rightarrow F_0(\nu), \tag{12}$$

$$f(\nu) \rightarrow S(\nu) f(\nu), \tag{13}$$

where

$$S(\nu) = e^{-i\Delta_+}\Theta(\nu) + e^{-i\Delta_-}\Theta(-\nu) . \tag{14}$$

Here Θ is the step function.

Inserting the modified electron distribution and the field amplitude as the initial conditions, the field amplitude at the end of the second undulator z=2L is obtained by applying eq. (5) one more time. The result is

$$a(2L) = a(0) \left\{ 1 + \int d\nu \frac{dF_0(\nu)}{d\nu} \left[\int_0^L dz' \int_0^{z'} dz'' + \int_L^{2L} dz' \int_L^{z'} dz'' \right] e^{-i\nu(z'-z'')} \right.$$
$$\left. \int d\nu S(\nu) \frac{dF_0(\nu)}{d\nu} \int_L^{2L} dz' \int_0^L dz'' e^{-i\nu(z'-z'')} \right\}. \tag{15}$$

The gain in the total system can be written as

$$G = \frac{|a(2L)|^2 - |a(0)|^2}{|a(0)|^2} = 2G_L + 2G_{int}, \tag{16}$$

where G_L is given by eq. (10) and

$$G_{int} = -\int d\nu \; F_0(\nu) \frac{d}{d\nu} \left\{ R_e\left(S(\nu) e^{-i\nu L}\right) U(\nu, L) \right\}. \tag{17}$$

The first term in the RHS of eq. (16) is the gain from two undulators operating independently. The second term may be considered as the interference gain. Both terms vanish in the limit of broad energy spread, i.e., as $F_0(\nu)$ becomes independent of ν. This is a mathematical proof of the result stated in the introduction.

3. Interpretation

To be specific, let us choose $\Delta_+=0$ and $\Delta_-=\pi$. The interference gain becomes

$$G_{int} = -\int d\nu F_0(\nu) \{ g(\nu) + g(-\nu) \} \tag{18}$$

where

$$g(\nu) = \Theta(\nu)\left(\frac{d}{d\nu}\cos\nu L\right)U(\nu,L) + \delta(\nu)U(0,L) \tag{19}$$

The first term in eq. (18) is due to the part of the electron distribution with $\nu>0$ at the end of the first undulator, while the second term is from the part with $\nu<0$ and displaced by a half wavelength in phase. Each of these contributions can be further divided into two groups corresponding to the two terms in eq. (19). For $\nu>0$, the first term arises from those electrons that started out with a positive detuning at the beginning of the first undulator. These electrons may be referred to as the "body" electrons. Similarly, the body electrons for $\nu<0$ are those with negative detuning before the first undulator. The total gain from the body electrons can easily be shown to be non-vanishing and positive. The scheme proposed in reference [1] would therefore work if only the body electrons are considered. However, there are also the "edge" electrons contributing the second term in eq. (19). For $\nu\geq0$ ($\nu\leq0$), these are the electrons that started out from a negative (positive) detuning at the beginning of the undulator, giving rise to the second term in eq. (18). The edge electrons do contribute a finite gain, which cancels the gain from the body electrons for a broad electron distribution. The cancellation occurs separately in each half of the momentum space, $\nu\geq0$ and $\nu\leq0$.

The edge electrons give rise to a δ-function gain for a mono-energetic electron beam. The phenomenon was first observed in numerical simulation by M. Xie [3].

The reason that FEL gain vanishes from electron beam with broad detuning spread even with a redistributor is really very simple, as follows: consider an electron distribution uniform within a phase space square $-\pi \leq \theta \leq \pi$, $-\nu_1 \leq \nu \leq \nu_2$, as shown in Figure 2(a). If $\nu_1, \nu_2 >> 1/L$, then the edge $\nu = \nu_1, \nu = -\nu_1$ will not be affected by the FEL interaction. Also, the phase space density does not change because of Liouville's theorem. Thus the electron phase space distribution after the first undulator, shown in Figure 2(b), is identical to the initial phase space distribution before the undulator. Since the distribution is uniform, it remains uniform after passing through the redistributor. Therefore, the redistributor cannot affect the gain (which remains zero).

The dotted line in Figure 2(b) is that corresponding to the line $\nu=0$ before the first undulator. Thus, the electrons within the area between the line $\nu=0$ and the dotted line in Figure 2(b) are the edge electrons discussed in the previous paragraph, while the rest are the body electrons. It is clear that the gain by the body electrons will cancel that by the edge electrons.

4. Momentum Selector

The redistributor does not change the total number of electrons. If a fraction of the electrons are removed, then it is possible to obtain a non-vanishing gain. Thus consider a momentum selector which removes electrons with a negative detuning. A momentum selector before the undulator is clearly equivalent to preparing a better quality electron beam resulting in a higher gain. It is then a reasonable question to ask whether a momentum selector placed in the middle of the undulator would give a better result. The effect of the momentum selector can be studied in a similar fashion as in Section 2, and one finds that the gain for the case where the momentum selector is at the beginning is four times larger than the case where it is in the middle of the undulator.

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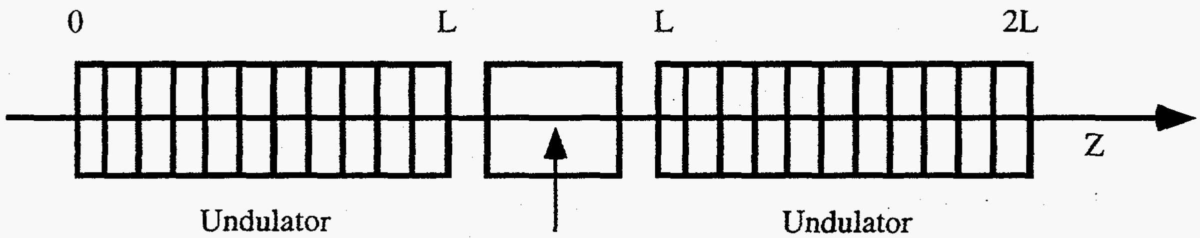
Figure Captions

Fig. 1. Illustration of the proposed scheme. The function of the redistributor is explained in the text.

Fig. 2. Electrons' phase space distribution before the undulator (a) and after the first undulator (b) for $\nu_1,\nu_2 >> 1/L$. The dotted line in (b) is that corresponding to the line $\nu=0$ in (a).



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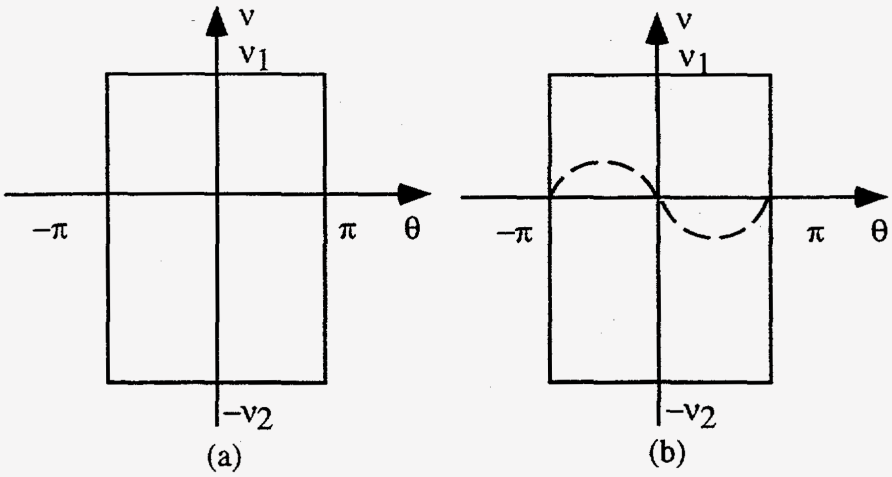

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