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A BAYESIAN ANALYSIS OF THE SOLAR NEUTRINO PROBLEM

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We illustrate how the Bayesian approach can be used to provide a simple but powerful way to analyze data from solar neutrino experiments. The data are analyzed assuming that the neutrinos are unaltered during their passage from the Sun to the Earth. We derive quantitative and easily understood information pertaining to the solar neutrino problem.

1 Introduction

Solar neutrinos are of interest both in astro-physics as well as in particle physics. The discrepancy between measured and predicted solar neutrino fluxes[1] has prompted a great deal of activity in both fields over the last few years.

The solar models[1] that predict the solar neutrino fluxes use the *standard model* of elementary particles and the theory of stellar evolution. Table I gives a comparison between the neutrino fluxes measured by the four pioneering experiments[2–5] and the standard solar model predictions[1] for these experiments. In the past, these data have been analyzed using χ^2 methods[6]. In this paper we outline an alternate analysis in the Bayesian framework to extract more information in an elegant manner.

2 Bayesian Analysis and Results

Bayes' theorem gives us a prescription for calculating the posterior probability $P(\phi \mid s, I)$ for certain hypotheses about parameters ϕ, given measured quantities s and prior information I. According to the Bayes' theorem,

$$P(\phi \mid s, I) = \frac{L(s \mid \phi, I)P(\phi \mid I)}{\int L(s \mid \phi, I)P(\phi \mid I)}, \tag{1}$$

where L is the likelihood function assigned to s, $P(\phi \mid I)$ is the prior probability function for ϕ.

[a]Operated by the Universities Research Association, Inc., under contract with the U.S. Department of Energy.

Table 1: Measured and predicted solar neutrino fluxes in units of SNU ($=10^{-36}$ ν/atom/sec). Kamiokande data are relative to the Bahcall and Pinsonneault[1] predictions.

Experiment	Detection Technique and Threshold Energy E_{th}	Flux Rates	
		Measured	Predicted[1]
Homestake[2]	$\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$, E_{th}=0.814 MeV	2.56±0.21	9.3±1.41
GALLEX[4]	$\nu_e + {}^{71}Ga \rightarrow e^- + {}^{71}Ge$, E_{th}=0.233 MeV	70±7	137±8
SAGE[3]	"	72±14	"
Kamiokande[5]	$\nu_e + e^- \rightarrow e^- + \nu_e$, E_{th}=7.5 MeV	0.42±0.09	1.0±0.014

We are interested in extracting information about solar neutrino fluxes ϕ_B, ϕ_{Be} and ϕ_{PP}. We model here the measured solar neutrino rates as

$$s = R \cdot \phi, \text{ with } R = \begin{pmatrix} 7.36 & 1.24 & 0.0 & 0.7 \\ 16.1 & 37.3 & 69.7 & 13.1 \\ 1.0 & 0.0 & 0.0 & 0.0 \end{pmatrix}. \qquad (2)$$

The matrix elements of R are products of predicted energy-averaged neutrino fluxes from the core of the sun and the experimental detection efficiencies. The columns in matrix R correspond to B,Be, PP and *other* neutrino fluxes, respectively, and the rows correspond to Cl, Ga, and H_2O experiments. We assume that the data are uncorrelated so that the error matrix σ associated with the neutrino rates $s^T = (s_{Cl}, s_{Ga}, s_{H_2O}) = (2.56,\ 70,\ 0.42)$ is diagonal, with $\sigma^T = (0.21, 7.0, 0.09)$. We assume the likelihood function L to be of Gaussian form,

$$L(s \mid \phi, I) = \exp(-\frac{1}{2} z^T z), \quad \text{with} \quad z = \sigma^{-1} \cdot (s - R \cdot \phi). \qquad (3)$$

We take the prior probability function $P(\phi \mid I) = constant$. Our studies show that any reasonable function for $P(\phi \mid I)$ is acceptable. The posterior probability function $P(\phi_B, \phi_{Be}, \phi_{PP}, \phi_O)$, written as in Eq. (1), can be used to get the probability for one or more of the fluxes by marginlizing it with respect to all other fluxes.

Figures 1(a), (b) and (c) show, respectively, the normalized probability distributions for B, PP and Be neutrino fluxes and they indicate that all of them deviate from the standard solar model predictions. The PP flux is consistent with the standard model prediction within two standard deviations, whereas ^8B flux is down by more than a factor two. The probability for the ^7Be flux peaks at zero. At 95% confidence level, we find that ϕ_{Be} is less than 0.78.

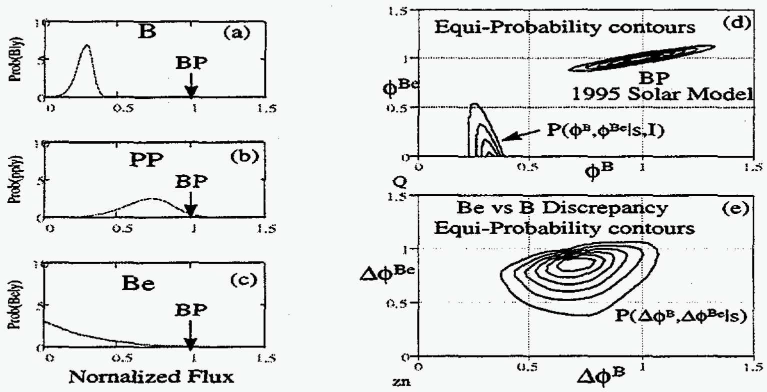

Figure 1: Normalized probability distributions of the solar neutrino fluxes from Bayesian analyses. In figures (a)-(d) the results of standard solar model[1] (BP) are also shown.

Figure 1(d) shows equi-probability contours for ^7Be versus ^8B fluxes. The results from the standard solar model[1] are also shown for comparison. The discrepancy plots between theory and experiment are shown in Fig. 1(e). In this analysis both theoretical as well as experimental uncertainties have been included.

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