Large Aperture Fresnel Telescopes

R. A. Hyde

July 16, 1998

This is an informal report intended primarily for internal or limited external distribution. The opinions and conclusions stated are those of the author and may or may not be those of the Laboratory.

Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

DISCLAIMER

This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or the University of California, and shall not be used for advertising or product endorsement purposes.

This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from the Office of Scientific and Technical Information P.O. Box 62, Oak Ridge, TN 37831 Prices available from (423) 576-8401

Available to the public from the National Technical Information Service U.S. Department of Commerce 5285 Port Royal Rd., Springfield, VA 22161

Large Aperture Fresnel Telescopes

Roderick A Hyde Lawrence Livermore National Laboratory

Successfully imaging extra-solar planets requires the use of very large aperture (VLA) space telescopes. Fielding such huge (50-100 meter) telescopes is an immense challenge; not only do they require maintaining optical precision over large distances, but the apertures must be delivered to and fielded in space, i.e., they must be lightweight, packagable for launch, and deployable upon arrival.

The mass of astronomical mirrors is dictated not by the need to reflect light (which can be done with a very thin metal film), but instead by accuracy requirements. High precision imaging requires wavefront control of $\sim \lambda/10$, which means that mirror surfaces must be accurate to $\sim \lambda/20$; for visible imaging this demands accuracies of about 250 Angstroms. The traditional approach to attaining such precision (using thick and rigid substrates) is unaffordably massive for VLA telescopes; modern designs emphasize a combination of more efficient rigidization and active adaptive optics. Most of the material in thick, monolithic, substrates is not necessary and can be done away with; with this approach the Hubble primary was thinned to a 180 kg/m² mass, and current prototypes extend this method down to ~ 10 kg/m². While still much heavier than the physical limit set by the reflective-layer ($\sim 10^4 \text{ kg/m}^2$), this technique should eventually reach the \sim 1 kg/m² values at which VLAs start to become mass-affordable. Because packaging and deployment of such locally-stiff mirrors will be quite challenging, there is also interest in the opposite approach; namely using a flexible, non-rigid material, and achieving its surface accuracy via active control. Here, the reflective coating is placed on a thin polymer film, which is both very light ($\sim 10^{-2} \text{ kg/m}^2$) and flexible enough so that it can be compactly folded up into a launchable volume. While this approach would solve the problem of launching and deploying a VLA, it must rely upon active control to meet the severe precision requirements. Not having the assistance of local stiffness greatly increases the challenge involved (namely the time and spatial frequency requirements) in achieving active control. Hence, while approaches to extend reflective telescopes up into the VLA regime exist and are being pursued, success is hardly guaranteed.

At Livermore we've spent the last two years examining an alternative approach towards VLA telescopes, one based upon transmissive Fresnel lenses rather than on mirrors. Thin optical films are the key to making VLAs launchable; not only are they lightweight, but they are also flexible and so are inherently packagable and deployable. The problem with membrane mirrors, of course, is that it's hard to control their shape to the $\lambda/20$ type of precision needed to attain $\lambda/10$ optical tolerances. However, this difficulty arises because the film is reflecting light, thereby doubling the effect of surface errors; for a thin transmissive surface, pathlength errors arriving at the surface are cancelled (not reinforced) as the light departs. So a transmissive primary (i.e., a lens) should be virtually insensitive to shape errors. In practice, this pathlength cancellation is not perfect since (because the film must focus light) the incoming and outgoing light are not traveling in quite the same directions. If the light is bent through angle θ , then the pathlength error in a mirror is amplified (above the physical shape error) by a factor of $(1+\cos\theta)$, while that in a thin lens is reduced by $(1-\cos\theta)$. Lenses therefore have a $(1+\cos\theta)/(1-\cos\theta)$ tolerance advantage over mirrors. By making the lens weak, i.e., keeping θ small, then this tolerance gain becomes huge, scaling as $4/\theta^2$, i.e., as $16F_{\#}^2$. A slow thin lens having $F_{\#}$ of 100 can tolerate 160,000 times greater shape errors than a similar mirror; this raises visible-light tolerances from a few hundred angstroms up to nearly a centimeter, a tremendous practical advantage when trying to field a thin optical-quality membrane in space.

In order to use a lens as the primary of a VLA telescope, it must be a thin lens; not only is this essential for launchability reasons, but also to reap the optical tolerance benefits. Conventional refractive lenses are, unfortunately, not nearly thin enough to be useful. Diffractive optics, however, do provide a means to field truly thin transmissive lenses, since they chop the applied phase correction into 2π -thick radial bands.

There are two main complications to using slow Fresnel lenses for a VLA telescope. The first concern is that the telescope must be very long, and the second is that Fresnel lenses are strongly chromatic, and so are generally considered incapable of broad-band, diffraction-limited, imaging.

This form of transmissive VLA telescope must be long; the product of its large aperture and high $F_{\#}$ (responsible for its tolerance advantages) dictates primary focal lengths of ~ 10 km or more. Such lengths are impracticable for a single spacecraft, so the telescope is split into two separate vehicles, a Magnifying Glass and an Eyepiece. The Magnifying Glass vehicle is responsible for supporting the large aperture Fresnel lens, holding it in the proper shape and for swiveling it in order to point towards desired targets. Light is gently focused by the Fresnel lens, directed towards its distant focal point. There it is collected and imaged by a free-flying Eyepiece vehicle. This spacecraft is a much more conventional space telescope, rigidly built, and with a modest, few meter wide, aperture.

The Eyepiece is similar to other space telescopes in most regards, but does face two special requirements due to its cooperation with the Magnifying Glass. It is responsible for the stationkeeping necessary to keep the overall telescope in focus and properly pointed, and also performs the chromatic correction necessary to turn this Fresnel telescope from a monochromatic to a broad-band instrument.

The telescope's pointing and alignment is handled by two control loops; an outer, crude-level, of control is provided by the physical placement of the Eyepiece vehicle, and an inner, fine-level, of control is then handled by small adjustments in its internal optics. Because of this split-up, the Eyepiece's stationkeeping task is not particularly challenging; the telescope is in solar space, where gravity gradients are weak, and (due to the multi-kilometer separation) its position need only be controlled to near-meter accuracy. The Eyepiece's maneuvers (for stationkeeping and retargeting) do not have to be done quickly, so efficient, low-thrust, electric propulsion can be used.

The greatest optical challenge to using a Fresnel telescope is chromatic correction. The focal length of a single Fresnel lens varies inversely with wavelength, so it only accurately focuses light which lies within a small spectral bandwidth. To achieve $\lambda/10$ imaging, the $|\Delta\lambda/\lambda|$ bandwidth is limited to 1/(10N), where N is the number of Fresnel zones in the lens, i.e., the lens's optical depth measured in wavelengths. The Fresnel lenses needed for VLA visible telescopes have $\sim 100,000$ zones, and hence bandwidths of ~ 1 ppm. This bandwidth is far too small for an efficient telescope, so widening it is essential.

In principle, the chromatic errors introduced by one Fresnel lens can be corrected by passing the light through another, inverse, one. Hence, a Fresnel Corrector in the Eyepiece vehicle can be used to cancel the chromatic dispersion introduced by the Magnifying Glass. The keys to doing so successfully were presented by Faklis & Morris in 1989 (Optical Engineering, V28, p592). The primary requirement is that the second, correcting, Fresnel lens must be at an image site of the first, distorting, lens. Physically, the reason for this is clear; light leaves each point of the Magnifying Glass Fresnel lens in an angular spray, each color being sent into a different direction. As the light from this site travels towards the Eyepiece it spreads apart, diverging both spectrally and physically; both effects must be corrected. The physical reassembly is achieved first, by making the light pass through a reimaging telescope as it enters the Eyepiece. This internal telescope focuses

the surface of the Magnifying Glass onto that of the Fresnel Corrector, thereby physically recombining rays which left each site on the first Fresnel lens to a matching site on the second one. Now each site on the Fresnel Corrector sees an incoming angular/color spray corresponding to that from the departure site on the Magnifying Glass; by employing an inverse (defocusing) Fresnel profile, it can remove this angular/color spray. As a result, each ray bundle is now both physically and spectrally recombined; the set of bundles can then be brought to a common achromatic focus. This final focusing requires another optical element, since the light leaving the Fresnel Corrector is diverging (from a virtual image site); as Faklis & Morris showed, a hybrid element (combining the defocusing Fresnel profile with a focusing achromatic one) will bring the light to a common, real focus.

This basic approach to chromatic correction can be refined for use in VLA Fresnel telescopes. The major requirement for our application is that we want the VLA telescope to have diffraction-limited performance. This requires that the imaging (mapping) between the two Fresnel lenses be precise, which in turn requires that the transfer optics be achromatic and aberration-free. Faklis & Morris demonstrated their Fresnel telescope by using a single refractive lens to perform the reimaging; this introduces too many physical and chromatic aberrations for use in VLA telescopes. The reimager is of course just a telescope; as with conventional telescopes, chromatic aberrations can be prevented by using reflective elements, and physical aberrations can be limited by using multiple, properly shaped, reflectors. We have successfully designed aberration-free reimagers which use just two reflectors, placed in a Cassegrain configuration; the Fresnel Corrector is placed at their image surface. The surfaces of the two reflectors are shaped to remove spherical and coma aberrations. The image of the Magnifying Glass formed by this two-element transfer telescope is (in general) still subject to aberrations from astigmatism and distortion. However, because the "objects" being imaged are sites on a diffractive lens, each emits a one-dimensional angular spray; only tangential rays are created, so sagittal astigmatism does not have to be corrected. This enables a simple design in which astigmatism can be eliminated by placing the Fresnel Corrector along the transfer telescope's tangential image surface. The remaining aberration, distortion, simply causes the imaging to be slightly nonlinear, rather than strictly proportional; it is easily dealt with by properly tailoring the Fresnel profile on the Corrector. Once these aberrations are suppressed, Fresnel telescopes have, as with other diffraction-limited telescopes, the same pathlength for all light rays, regardless of color or position.

The Eyepiece vehicle thus contains 3 basic optical elements. The first two are the Primary and the Secondary of the transfer telescope. They are designed to transfer an aberration-free image of the Magnifying Glass onto the surface of the third element, which is a hybrid of a defocusing Fresnel lens and a focusing mirror. This Fresnel Corrector, being contained within the Eyepiece vehicle, is much smaller than the large Magnifying Glass Fresnel lens; the magnification between them, η , is generally ~ 0.01. The Corrector's small size means that its Fresnel profile must, in order to apply the proper angular/color correction, have a much smaller $F_{\#}$ (by a factor of η) than that of the Magnifying Glass. The feature-sizes needed to construct a Fresnel lens scale with it's $F_{\#}$, so the Fresnel Corrector will require small, precisely located, surface features. Fortunately (precisely because of its small, sub-meter, size) the Corrector can be built from conventional optical materials and its Fresnel surface pattern can be generated by modern lithographic techniques; these factors will enable it to achieve the necessary precision.

After leaving the Fresnel Corrector, light has been chromatically corrected and physically focused towards the telescope's sensor array. In principle, this design should be chromatically corrected for all wavelengths, so the only limitation on spectral bandwidth should be set by the Magnifying Glass's wavelength dependent diffractive efficiency. In practice, the bandwidth is governed by the fact that the Eyepiece can only correct light which it sees. Although the Magnifying Glass is focused towards the Eyepiece, its focal length varies with wavelength, so short wavelength light is focused behind the Eyepiece and long wavelength light in front of it. Since the Eyepiece (being

built from conventional optical materials) is much narrower than the Magnifying Glass, much of this defocused light will be missed. This spectral-capture bandwidth is just the diameter ratio of the Eyepiece Primary to that of the Magnifying Glass. All light within this bandwidth is captured and properly corrected; when light is outside this range, then only central portions of the Magnifying Glass are seen, with reduced intensity and angular resolution. For instance, consider a VLA telescope using an 80 meter Magnifying Glass and a 4 meter Eyepiece Primary. This design would fully capture a $\pm 5\%$ spectral bandwidth; only 25% of light 10% off the design wavelength would be captured, and this (coming from only half the aperture) could be imaged to only half the angular resolution. Because of its limited intensity and poor resolution, out-of-band light is generally more trouble than it's worth, and can be rejected by spectral filters at the sensors.

So far we have discussed a simple, single spectral-band, telescope. What happens if we want to view a much broader spectral region? One obvious approach would be to take advantage of the variable focal length of Fresnel lenses, and to simply move the Eyepiece vehicle to the appropriate separation distance for the desired wavelength. Unfortunately, it is hard to keep the internal transfer telescope in focus as the vehicle separation changes; zoom optics can be designed to maintain a paraxial focus, but do not suppress the aberrations sufficiently to permit diffraction-limited viewing. Using multi-spectral Fresnel optics (for both the Magnifying Glass and the Fresnel Corrector) solves this problem (and permits simultaneous multi-spectral viewing), since the focal length and hence the vehicular separation need not change. The simplest multi-spectral Fresnel lens is one which images harmonic frequencies to a common focal point: A quadratically-blazed phase profile lens which is designed for a given upper wavelength, λ_1 , also focuses all of its harmonics, $\lambda_k = \lambda_l/k$, to the same focal point. For example, a lens designed for 2.0 µm operation will also function at 1.0, 0.67, 0.5, 0.4, etc., microns. Each spectral band is simultaneously color-corrected (since the crucial reimaging is achromatic, dependent only upon the system's geometry), and so the telescope can view a set of harmonic bands, each with a reasonable (~ 0.1) bandwidth. The only cost associated with this approach is the need to make the Fresnel features deeper and wider, i.e., matched to the longest wavelength-to-be-viewed, not the shortest. The precision tolerances (on optical depth and on radial dimensions) are set by the shortest wavelength-to-be-viewed, i.e., are the same as the simple visible-wavelength systems that are primarily discussed here. Another technique, particularly suitable for deep-IR wavelengths, is to use spectrally-selective zone-plates. These use a thin present/absent surface coating which is opaque only near the desired wavelength; this coating acts as a zone-plate for a given spectral region, but has no effect at other wavelengths. This technique can be used in combination with a (lower spectral regime) multi-harmonic phaseprofile lens, which then can be made thinner than if it were responsible for the entire spectral coverage.

For spectroscopic measurements, one would like continuous spectral coverage, not a set of windows with gaps in between. Multi-harmonic Fresnel telescopes can be designed to provide such a continuous spectral coverage, at least within fairly broad spectral limits. This is achieved by noting that as the harmonic index increases, the wavelengths get closer and closer together. Eventually, at some λ_+ , their spectral windows start to overlap; the telescope provides continuous spectral coverage for all wavelengths below this. In practice, there are two complications: The first is due to material dispersion; for short wavelengths these profiles are many waves thick, and eventually (below some λ) the difference between $n(\lambda_+)$ and $n(\lambda)$ becomes too severe to permit efficient focusing. This restricts coverage to a finite (λ,λ_+) range; but in practice the range can extend from the visible well into the IR. A second concern is that truly continuous coverage is harder to provide, the larger the Fresnel aperture becomes. This happens because (for a given limit on the size of the conventional optics in the Eyepiece vehicle) the width of each spectral window shrinks as the Fresnel aperture increases. This delays the start of continuous coverage to higher and higher harmonic indices, which for a given desired λ_+ value forces the lens' design wavelength,

and hence its thickness, to increase. In practice, this can dealt with by designing for a tolerable thickness and hence allowing coverage gaps; there is still coverage within the gaps, simply at an intensity and resolution corresponding to a smaller aperture. For most spectroscopic purposes this will be acceptable since extreme angular resolution is not their greatest priority. So while a moderately-sized (say 40 m) telescope can be fully corrected, a larger one (say 80 m) will have variable spectral coverage. About half of its spectrum will be inside coverage windows and hence benefit from the full 80 m aperture, while in the "gaps", imaging will be reduced to that obtainable from a 40 m telescope.

It is clear, and has been confirmed by detailed optical simulation, that a Fresnel VLA telescope will work if it can be built. Since the same is also true of more conventional reflective telescopes, then we must ask: Can a large Fresnel telescope be built, and if so, what are its advantages?

The reasons for pursuing large Fresnel telescopes have already been discussed: They provide a lightweight, readily packaged and deployable, method of obtaining a large optical aperture. In this they are similar to membrane reflectors and superior to panel-truss designs. In addition to their inherent launchability, they virtually eliminate the extremely tight (few hundred angstroms in the visible), out-of-plane shape tolerances required for reflectors. These are substantial advantages, so: Can large Fresnel lenses be built and fielded?

Clearly building and fielding huge Fresnel lenses will be a challenging task, and until actually demonstrated, will be a major source of concern. Nonetheless, both jobs, the building and the fielding, do look feasible.

In building a large polymer-based Fresnel lens, we can take advantage of two things; the required feature sizes are large, and the lens can be seamed together from small sections. The large Magnifying Glass Fresnel lens has a high F_# (this, remember, is what's responsible for its large tolerances), so also has large radial feature sizes. For instance, an F_# 400 design has (for 0.5 µm light), a smallest feature-size of 400 µm; in relation to current manufacturing capabilities this is huge, no challenge at all. The challenge is presented by two other tolerances; the need to control the phase thickness of the polymer film, and to place the Fresnel zones in their correct radial positions. In order to achieve $\lambda 10$ imaging, the phase profile (pattern plus substrate) of the film must be accurate (for typical n = 1.5 polymers) to $\lambda/5$; for visible light this is about 0.1 μ m. Note that this is purely a manufacturing challenge, fielding issues primarily change the shape of the film, not its thickness. Typical commercially purchased thin films, to 1 mil thick, are not globally accurate to this level, although can be over meter scale regions. The required precision can, however, be reached in several different ways: The preferred approach is simply to improve the processing, giving thickness uniformity a much higher priority than it is typically assigned. Another, independent procedure, is to incorporate total phase-depth control into the Fresnel patterning step. A third, and also independent step, is to achieve local accuracy (over ~ 1 meter scales) by the first two methods, but to correct larger-scale nonuniformities in-the-field, using an adaptive optics system in the Evepiece.

Livermore is currently investigating two scalable techniques for applying the Fresnel pattern to the surface of a polymer film. The first process involves creating the pattern on the surface of a rigid and stable master, and then replicating it onto the polymer film. This approach is made feasible by the fact that the full lens can be assembled by seaming together smaller sections. Optical-surface-quality material and techniques (lithography and diamond turning) exist with which we can create the required feature-size patterns in meter-scale sizes. These can be used as masters for the replication-in-polymer of sections of the overall lens, which are then seamed together. A second process involves directly writing the pattern, via laser ablation, onto the surface of a polymer film. This procedure has been well studied using either UV or short pulse lasers. The challenge in

scaling it up to writing large area Fresnel lenses is neither the photonic cost nor the feature sizes, but instead is delivering the many laser spots needed. Beam delivery systems, in which many parallel spots are written simultaneously, should provide an effective patterning tool. The overall challenge in building large area Fresnel lens is likely to be as much one of pattern registration and film handling, than it is creating the proper phase profiles in the polymer.

Once a large Fresnel lens is built, it must be successfully fielded in space; it must be transported there, deployed into the proper configuration, and then held to the required optical precision. The desired shape is simply a flat disk, which is inherently simpler to form and maintain than are precisely curved reflecting mirrors, particularly given the loose (near-centimeter) flatness levels adequate for a slow Fresnel lens. There is, however, one substantial challenge involved in fielding the Magnifying Glass lens; namely that the Fresnel zones must be in the correct radial locations. Each band is responsible for applying a specific one- λ phase change to the incoming light; if the band is misplaced, then the phase correction is wrong. This concern leads to fairly tight radial tolerances, 10% of the zone sizes for $\lambda/10$ viewing. So at the rim of a visible-wavelength, F# 400 lens, we need about 40 µm in-plane precision; this is 1 ppm for an 80 meter aperture system. This radial precision requirement is, of course, not unique to Fresnel lenses; it applies to any lens or mirror. The radial coordinate governing an optical element's built-in phase profile must be the same as the geometric radius; radial distortion leads to phase errors. For reflectors, the challenge of meeting this radial tolerance is generally dwarfed by the difficulties in attaining the proper (few hundred angstrom) out-of-plane precision. Large, slow, Fresnel lenses effectively eliminate this out-of-plane demand (via F_#² scaling), but only relax (via F_#), not remove, the in-plane radial requirements.

After being deployed in space, the Magnifying Glass lens will be kept flat by being held in tension. This flattens out elastic packaging wrinkles and raises the membrane's out-of-plane vibrational frequencies, making it less floppy. The dominant challenge in meeting the in-plane radial tolerances is not due to this stretching, (which is predictable and hence can be allowed for when writing the Fresnel pattern), nor is it from the presence of manufacturing seams; the largest distortions instead come from thermal strains. It is essential for the Magnifying Glass to be thermally "clean", that is, to avoid gradients in heating or in material properties that lead to nonuniform thermal stretching. Axial rotation is a particularly simple tensioning mechanism which is both self-deploying and thermally "clean"; it requires only the optical membrane, with no extrinsic support structure. Uniform temperature changes (due to the emplacement in space and/or by changes in the intercepted solar irradiation) are therefore not a problem, since they lead only to areally uniform stretching. For a Fresnel lens this just changes the focal length and hence is accommodated by a slight motion of the Eyepiece vehicle. With rotational tensioning, the largest source of radial distortion results from thermal stretching due to nonuniformities in the properties of the polymer film. These effects can most readily be kept small (without requiring extreme material uniformity) by employing low thermal-expansion-coefficient (low CTE) polymers for the lens. Other, more mechanically intensive, tensioning approaches can be used, but these will impose more tolerance challenges than are faced with the rotational approach.

There are, of course, other issues involved in building and fielding large thin-film Fresnel lenses. Here, let's discuss two of the most important: What's it built out of? And, what about noise from scattered sunlight?

A naked Magnifying Glass will be continually hit by sunlight, some portion of which will diffusely scatter and enter the distant Eyepiece. Unless specialized noise filters are used, this noise will be significantly (~ 10⁴) times greater than the signal from an extra-solar planet. Hence, for this application, it will be necessary to shade the Magnifying Glass from the Sun (as, of course, is planned to be done for the NGST). This sunshade must be somewhat larger than the Magnifying

Glass, but is also constructed from thin polymer film(s). It is much easier to fabricate and field, since it has a simple metal coating, with no complex optical pattern or tight dimensional tolerances.

The phase pattern for the Fresnel lens will be implemented as a surface relief (having physical cut depth of roughly twice the wavelength, i.e., about 1 µm for visible light) profile in a thin-film polymer. To be suitable for this application the polymer must be transparent at the operating wavelength, exhibit good phase uniformity, and must be stable in the space environment. In order to assist in meeting the in-plane dimensional tolerances, it is desirable that the polymer also has a low CTE. There are many transparent polymers, but most are attacked by one or more aspects of the space environment. Fortunately, our VLA telescope will be located in solar orbit (away from Earth's atomic oxygen and geomagnetic radiation belts) and will be sunshaded; this benign environment permits most traditional optical plastics to be considered. So polymers such as PMMA, polystyrene, Teflon-AF, polycarbonate, TPX, and others are all potential candidates (for visible imaging). The only common flaw to this group is that they all have fairly normal (i.e., > 10 ppm/°K) CTEs. Most thermal problems are alleviated by employing a sunshade, so such CTE values may be acceptable. However, the interaction of nonuniform material properties and a large temperature difference between fabrication and operational conditions (i.e., between Earth and space) may be troubling. In this event, it will be necessary to use a much lower CTE polymer; one attractive possibility is a class of low-CTE (< 1 ppm/°K), reasonably colorless, polyimides developed for the semiconductor industry.

In summary, Fresnel lenses are attractive for VLA telescopes because they are launchable (lightweight, packagable, and deployable) and because they virtually eliminate the traditional, very tight, surface shape requirements faced by reflecting telescopes. Their (potentially severe) optical drawback, a very narrow spectral bandwidth, can be eliminated by use of a second (much smaller) chromatically-correcting Fresnel element. This enables Fresnel VLA telescopes to provide either single band ($\Delta\lambda/\lambda\sim0.1$), multiple band, or continuous spectral coverage. Building and fielding such large Fresnel lenses will present a significant challenge, but one which appears, with effort, to be solvable.

| Technical Information Department • Lawrence Livermore National Laboratory University of California • Livermore, California 94551 |
|---|
| |
| |
| |
| |