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Coupled Spheromak-Helicity Injector in the Sustained Spheromak Physics Experiment, SSPX

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Coupled spheromak-helicity injector in the Sustained Spheromak Physics Experiment, SSPX*

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Abstract. The Sustained Spheromak Physics Experiment, now under construction at LLNL, will be sustained by helicity injected from a coaxial plasma source. The ideal MHD equilibrium of the coupled spheromak and source plasmas is calculated in this report, with the plasma in the injector treated in the force free approximation. The two regions are linked by a current-carrying edge plasma region, with field lines which end on the injector electrodes and a width determined by the ratio of the poloidal flux applied to the injector by external coils and that generated in the spheromak. The safety factor, q, on the magnetic axis and its profile within the spheromak separatrix are determined primarily by the ratio of the external and internal values of $\lambda = j_{||}/B$ and by the internal profile of λ. The q-profile is expected to have significant effect on the operation of the experiment, particularly on the stability of the equilibrium and the associated magnetic dynamo.

Spheromak magnetic fields and currents were sustained in CTX [1] and other experiments by helicity injected into a flux conserver from coaxial plasma sources. As was shown theoretically by Taylor [2] and experimentally by Barnes, et al. [3] (among others), the injected helicity is determined by the product of external magnetic flux and voltage applied to the discharge in the source. Although the helicity can be considered as applied to the spheromak across the injector gap in the flux conserver wall, the boundary conditions at the gap are not well defined, so helicity more properly is taken to be injected at the injector electrodes. The resulting plasmas in the injector and flux conserver are strongly coupled, especially in the SSPX geometry and the equilibrium should be modeled as a single problem.

The model [4] used to describe the CTX coaxial gun (helicity injector) was based on a MHD plasma flow. In the model, the flow velocity becomes Alfvénic at the maximum in the magnetic field which develops where the field is constricted at the exit to the gun, resulting in a magnetic nozzle. The plasma is accelerated by **j**×**B** forces in the gun plasma, which play a major role in the description of the flow. This model is a good description of the start-up phase of the spheromak when the plasma is injected into an empty flux conserver, and also applies to the MPD arc when the plasma flows into a vacuum or low density plasma [5]. However, once the spheromak is formed, there will be substantial plasma on the open fieldlines which surround the spheromak separatrix. The resulting back pressure along the field lines will change the flow condition at the field maximum, thus greatly reducing the flow from the injector into the flux conserver. This reduced flow is consistent with the difficulty within the CTX model of fitting the Hall parameter and recycling to the experimental spheromak data [4].

In SSPX we are primarily interested in the sustainment phase of the spheromak, where the injector and edge plasmas (between the spheromak and flux conserver) are better approximated as force-free. The current in these regions is parameterized by

$$\mu_0 \mathbf{j} = \lambda_{edge} \mathbf{B} \qquad (1)$$

with λ_{edge} constant, independent of the magnetic flux. The injector is also characterized by the applied bias magnetic flux, Φ_{gun}, which for the SSPX pulse length (a few milliseconds) is frozen into the conducting wall. The total discharge current is thus $I_{gun} = \lambda_{edge}\Phi_{gun}$.

The spheromak inside the separatrix is determined by its λ-profile, the plasma pressure, and the spheromak toroidal current, I_{tor}. The force-free part of the current is found from

$$\lambda = \lambda_0\left(1+\sum_1^3 a_n\psi^n + a_4\psi^{nasp}\right) \tag{2}$$

where the normalized flux lies between zero (magnetic axis) and unity on the separatrix. The coefficients are to be specified by physics outside the ideal MHD model. The exterior and interior values of λ are matched on the separatrix. The plasma pressure is a flux function, although we set $p = 0$ in this report to focus on the magnetic field characteristics. The value of the spheromak current is determined by the physics of the magnetic dynamo, which is poorly understood; we thus treat the current amplification factor, I_{tor}/I_{gun}, as a parameter in our calculations. The MHD calculation determines λ_0 as an eigenvalue in the boundary-value solution [6] of the Grad-Shafranov equation using the TEQ package [7] of the LLNL Corsica integrated physics code [8].

An example of the resulting magnetic configuration in SSPX [9], is shown in Fig. 1. The shape of the separatrix and flux surfaces are insensitive to the parameterization of λ, so this is representative of the expected experimental operation.

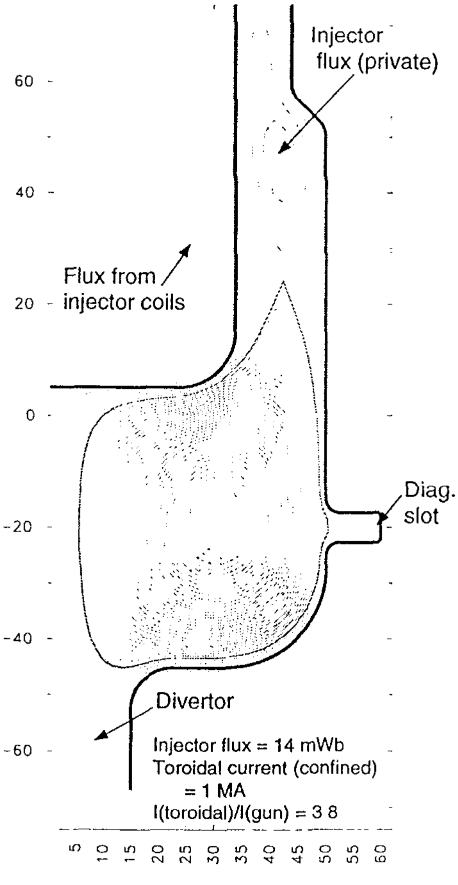

Fig. 1. Magnetic geometry for SSPX; the heavy lines are the flux conserver. The injector discharge is formed in the coaxial region at the top, with the central section assumed to be the cathode. Thus, current flows down in the metallic cathode wall, with a current emission per unit area proportional to the normal magnetic field at the surface as required by Eq. (1). The spheromak interior shown is described by $a_1 = a_2 = a_3 = 0$, $a_4 = 1$, and $nasp = 10$. This results in $\lambda_0 = 8.6\ \text{m}^{-1}$ (on the magnetic axis) with $\lambda_{edge} = 2\lambda_0$. The toroidal discharge current inside the separatrix has been set at 1 MA. The bias flux in the injector has been set to 34 mWb, with a distribution on the injector electrodes (and flux conserver) calculated from the coils used in the experiment.

Two example λ-profiles are shown in Fig. 2. The profile of the safety factor, q, for each of these examples is also shown. The value of q on the magnetic axis is $2/(R_{axis}\lambda_0)(2\kappa/(1+\kappa^2))$, with the ellipticity, $\kappa \approx 1$. Because of the eigenvalue nature of the λ-profile, a large value on the open field lines yields a low value on the magnetic axis, and thus a higher value of q_0. If the ratio of exterior to interior λ is too large, the safety value on axis will reach or exceed unity. The value of q will also diverge logarithmically on the separatrix, and thus pass through unity at a normalized flux close to one. Such an equilibrium will presumedly be unstable to a disruptive kink. A cyclic behavior in which q on the magnetic axis approaches unity, collapses to about 0.5, and then increases again has been observed using an interior magnetic probe array in the cold plasma in the FACT spheromak [10], and it may have been observed in SPHEX [11].

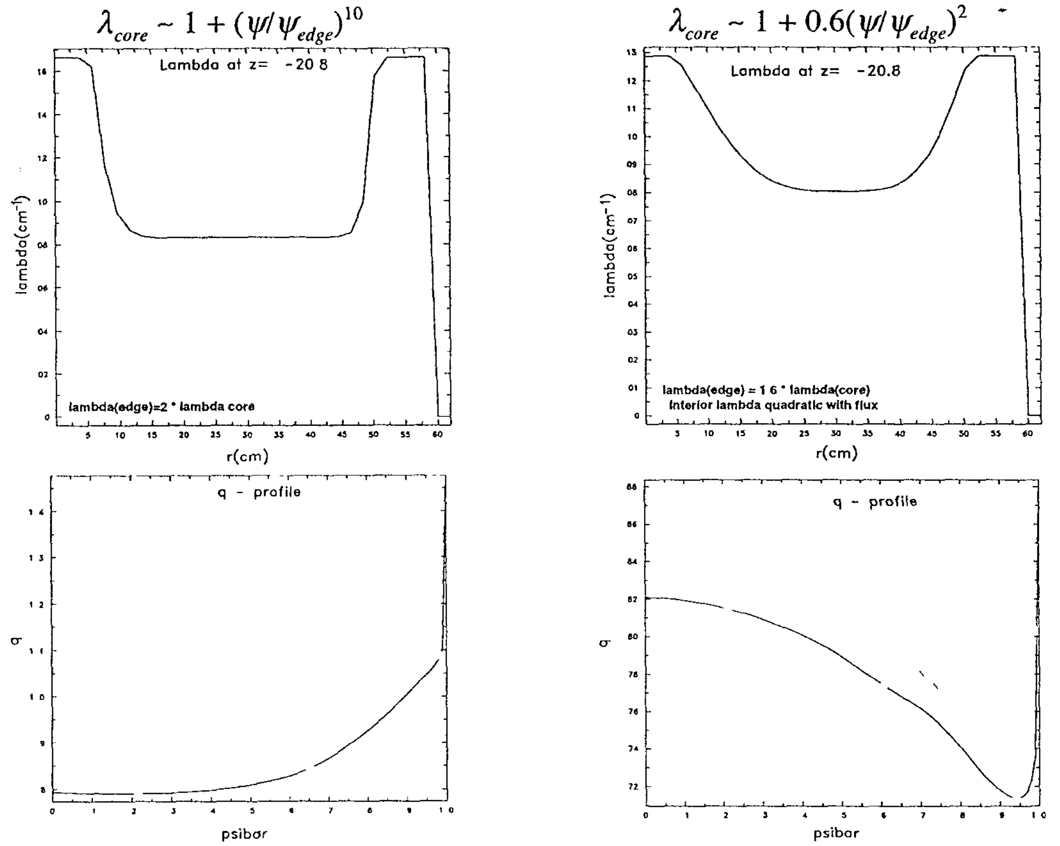
$\lambda_{core} \sim 1 + (\psi/\psi_{edge})^{10}$ $\lambda_{core} \sim 1 + 0.6(\psi/\psi_{edge})^2$

Fig. 2. Example λ- and q -profiles ("psibar" is the normalized flux, ψ/ψ_{edge}).

As seen in Fig. 3, the interior q-profile is only weakly dependent on the gun flux, most of which passes through the hole on the geometric axis. The actual operating point will be determined by the current-amplification factor, I_{tor}/I_{gun}, which is expected to be determined by the balance between the strength of the magnetic dynamo and resistive losses in the plasma interior. A cold, sustained spheromak will have a high interior ohmic resistivity, and thus operate at a low interior λ. If this value becomes low enough that $q = 1$ on the magnetic axis, there will thus be in danger of disruptive behavior. Operation at as low a level of gas and impurities as possible is thus essential to successful sustainment and energy confinement in the spheromak.

The strength of the magnetic dynamo in the spheromak core depends on the level of magnetic turbulence and thus most likely on resistive tearing modes in the plasma. The q-profiles shown in Fig. 2 have no interior $m = 1$ mode resonant surfaces (except in the separatrix region), and thus differ from the RFP which has several $m = 1$, current-driven modes which are unstable and interact strongly. It has been recognized in numerical

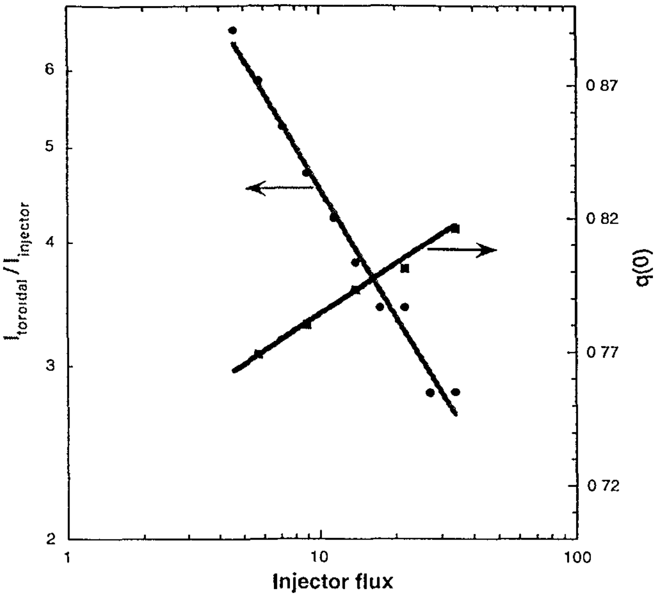

Fig. 3. Current amplification factor vs. gun flux for SSPX.

modeling of the RFP that the $m = 1$ modes differ from higher order modes in that they are essentially global in nature ("non-constant ψ") [12, 13]. Thus, if the sustained spheromak can operate with profiles that avoid the internal kink expected when $q = 1$ on the magnet axis, and with sufficient magnetic (or rotation) shear that the higher-order modes are localized, it should have fluctuation and confinement characteristics very different from those observed in the RFP.

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