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EFFICIENT COMPUTATION OF PERIODIC AND NONPERIODIC GREEN'S FUNCTIONS IN LAYERED MEDIA USING THE MPIE

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Abstract: The mixed potential integral equation (MPIE) formulation is convenient for problems involving layered media because potential quantities involve low order singularities, in comparison to field quantities. For nonperiodic problems, the associated Green's potentials involve spectral integrals of the Sommerfeld type; in the periodic case, discrete sums over sampled values of the same spectra are required. When source and observation points are in the same or in adjacent layers, the convergence of both representations is enhanced by isolating the direct and quasi-static image contributions associated with the nearby layers. In the periodic case, the convergence of direct and image contributions may be rapidly accelerated by means of the Ewald method.

INTRODUCTION

The efficient evaluation of layered-media Green's functions is an important issue for the analysis and design of many structures. Nonperiodic structures include microstrip antennas and circuits, while periodic applications include frequency selective surfaces (FSS), polarizers, and leaky-wave antennas. The mixed potential integral equation (MPIE) method is an efficient technique for analyzing either periodic or nonperiodic structures of arbitrary shape in the space-domain, since the Green's functions for the scalar and vector potentials are less singular than for the electric and magnetic field components. In particular, the formulation of Michalski utilizing "traditional" forms of vector and scalar potentials [1] is convenient for the analysis of both planar and nonplanar currents in layered media.

NONPERIODIC LAYERED MEDIA

In the MPIE formulation for a *nonperiodic* source, the scalar and vector potential components are in the form of Sommerfeld integrals. For observation and source points in adjacent layers, the asymptotic behavior of the integrands can be extracted to improve the convergence. For some potential components, the Sommerfeld identity can be used directly to evaluate the extracted part in closed form. For other components the Sommerfeld identity is not applicable, but other mathematical identities can be used to evaluate the extracted parts. The extracted terms correspond to the direct source radiation in a homogeneous medium and to quasi-static image contributions arising from reflections from the nearest boundaries. After extraction of the homogeneous and image terms, the resulting integrands decay exponentially, unless the source and observations points both lie along an interface. In this case, the integrands exhibit an algebraic decay after the extraction. Extraction of a half-space Green's function can be used to further improve convergence, since the resulting integrands decay exponentially following the extraction. The half-space Green's function can be evaluated by using Lindell's exact image theory for the half-space [2].

PERIODIC LAYERED MEDIA

The extension of the MPIE formulation to *periodic* sources is straightforward, with the Sommerfeld integrals being replaced by double summations over Floquet modes (space harmonics). The homogeneous and quasi-static terms now correspond to the evaluation of the periodic Green's function in a homogeneous free space. One of the most efficient methods for the evaluation of the periodic free-space Green's function is the Ewald method [3], [4]. The Ewald method is a hybrid spatial/spectral method, which expresses the free-space periodic Green's functions as the sum of two series, one of which is a "modified spatial" series and the other one a "modified spectral" series. Both of these series involve the complementary error function, which results in both series converging exponentially fast with Gaussian-type decay.

The complete periodic Green's function for a typical potential quantity may be written in the form

$$G = \sum_{p,q}\left[\tilde{G}_{pq} - \sum_{i}\tilde{G}^{i}_{pq}\right] + \sum_{i}\left[\sum_{p,q}\tilde{G}'^{i}_{pq} + \sum_{r,s}(G'^{i}_{rs} - \delta_{r0}\delta_{s0}G^{i}_{rs})\right] + \sum_{i} G^{i}_{00} \ . \qquad (1)$$

Spectral terms are denoted with a tilde, and the first bracketed term on the right-hand side of this equation is the spectral summation for the layered media Green's function (terms $\tilde{G}_{pq}$) with the homogeneous term and up to two quasi-static image terms removed. These latter terms have the spectral representation

$$\sum_{p,q} \tilde{G}_{pq}^{i} = \frac{\Gamma_i}{A} \sum_{p,q} \frac{1}{2jk_{zpq}} e^{-j\mathbf{k}_{tpq}\cdot(\mathbf{r}-\mathbf{r}_i')} e^{-jk_{zpq}|z-z_i'|} , \tag{2}$$

where A is the unit-cell area, and have an alternative spatial representation given by

$$\sum_{r,s} G_{rs}^{i} = \Gamma_i \sum_{r,s} e^{-j\mathbf{k}_{t00}\cdot\mathbf{r}_{irs}} \frac{e^{-jkR_{irs}}}{4\pi R_{irs}} . \tag{3}$$

In the second bracketed term on the right-hand side of (1), the homogeneous and quasi-static contributions are evaluated by the Ewald method. The prime denotes that the original spatial or spectral terms are replaced by the corresponding Ewald terms. The first summation in the second term on the right-hand side of (1) is the Ewald spectral series,

$$\sum_{p,q} \tilde{G}_{pq}^{\prime i} = \frac{\Gamma_i}{A} \sum_{p,q} \frac{e^{-j\mathbf{k}_{tpq}\cdot(\mathbf{r}-\mathbf{r}_i')}}{4jk_{zpq}} \left[e^{-jk_{zpq}|z-z_i'|} \operatorname{erfc}\left(\frac{jk_{zpq}}{2E} - |z-z_i'|E \right) + e^{jk_{zpq}|z-z_i'|} \operatorname{erfc}\left(\frac{jk_{zpq}}{2E} + |z-z_i'|E \right) \right], \tag{4}$$

and the second involves the Ewald spatial series,

$$\sum_{r,s} G_{rs}^{\prime i} = \Gamma_i \sum_{r,s} \frac{e^{-j\mathbf{k}_{t00}\cdot\mathbf{r}_{irs}}}{8\pi R_{irs}} \left[e^{jkR_{irs}} \operatorname{erfc}\left(R_{irs}E + \frac{jk}{2E} \right) + e^{-jkR_{irs}} \operatorname{erfc}\left(R_{irs}E - \frac{jk}{2E} \right) \right], \tag{5}$$

where erfc(x) is the complementary error function and E is the so-called Ewald splitting parameter. In the spatial series, the nonperiodic homogeneous medium Green's function due to the (0,0) reference source element has also been removed and then added back as the last term on the right-hand side. This subtraction removes the singularity from the Ewald spatial summation due to the (0,0) source *and its images*, resulting in a smoother contribution for numerical processing. The remaining homogeneous medium Green's function for the nonperiodic image source terms is handled by standard methods.

The selection of the Ewald splitting parameter E determines the rate of convergence of the Ewald spatial series relative to the spectral series. A simple formula exists for the optimum Ewald parameter, $E_0 = \sqrt{\pi / A}$, which balances the asymptotic rate of convergence of the two series [4]. A further improvement can be obtained by modifying the Ewald parameter slightly to achieve equal errors in the two series for a given summation limit. Once this is done, sufficient convergence is usually obtained in the Ewald summations by taking only three terms in each sum, corresponding to $p,q \in (-1,0,1)$ and $r,s \in (-1,0,1)$.

Results will be presented to demonstrate the rapid rate of convergence for the Ewald method, and to show how this implementation of the MPIE approach can efficiently analyze scattering and radiation from rather complex periodic structures in layered media.

RESULTS

Fig. 1 shows the convergence of the Ewald method for a typical planar periodic array of sources in free-space. For both the modified spatial and spectral series, the error is shown relative to the magnitude of the total periodic Green's function, expressed in percent. The summation limits are from $-N$ to $+N$ in the calculation of either series. Note that very accurate results are obtained with $N = 1$. In Fig. 1a, the asymptotically-optimum Ewald parameter E_0 is used, while in Fig. 1b the Ewald parameter has been modified to optimize the accuracy for $N = 1$, which results in $E = 1.13E_0$.

The Ewald acceleration has been implemented in a general purpose, layered-media code for periodic and nonperiodic structures, FSS/EIGER. Results from one validation study, scattering from a periodic array of aperture-coupled patches, are presented in Figure 2, and are seen to agree quite well with the results of [5].

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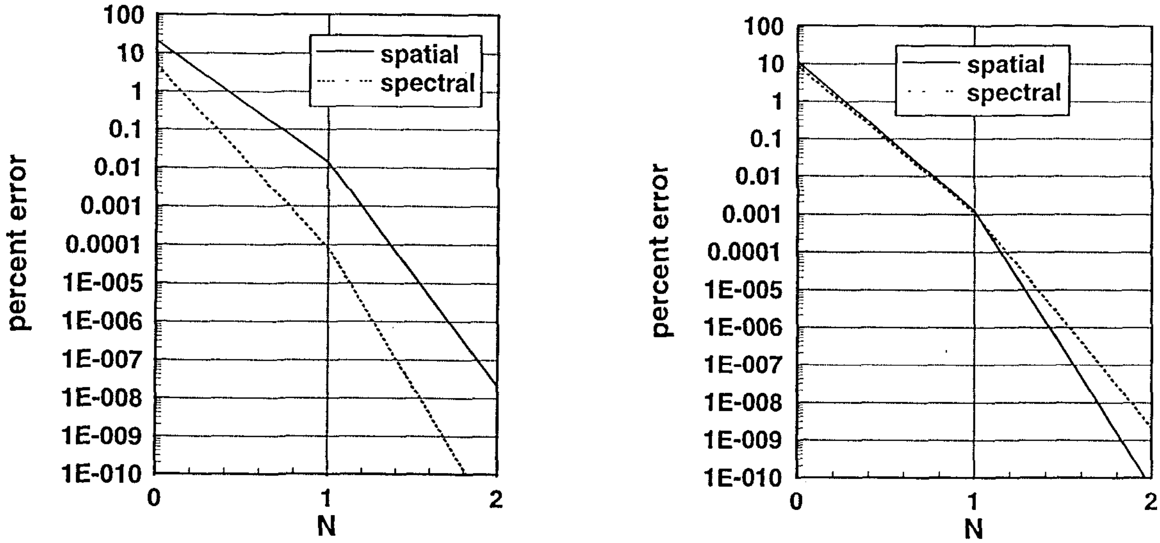
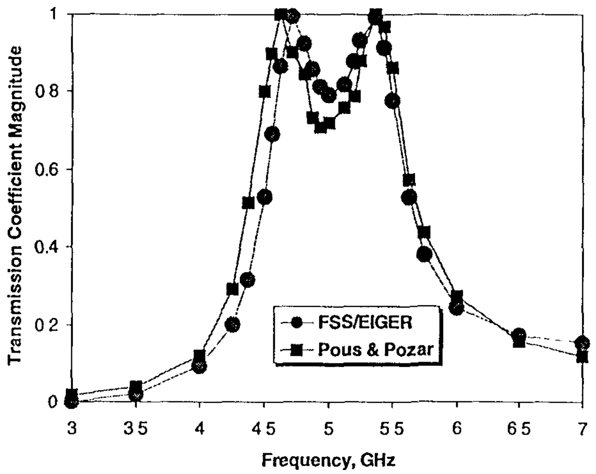


Fig. 1 Relative error (percent) in the Ewald spatial and spectral series versus the summation limits of the series. The results are for a square unit-cell lattice of dimensions $a = b = 0.5\lambda_0$, *with the (0, 0) source located at* $x' = y' = z' = 0$ *and the observation point located at* $x = y = 0.25\lambda_0$, $z = 0$. *The array is scanned to broadside, so that* $k_{x0} = k_{y0} = 0$. *(a) The Ewald parameter is chosen to be* $E = E_0$. *(b) The Ewald parameter is chosen to be* $E = 1.13E_0$.

Fig 2 A comparison of results for the transmission coefficient for a plane wave incident on a periodic array of aperture-coupled patches. The geometry is given in [5].



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