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ESTIMATES OF FREQUENCY-DEPENDENT COMPRESSIBILITY FROM A QUASISTATIC DOUBLE-POROSITY MODEL

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1 Introduction

Gassmann's [1951] relationship between the drained and undrained bulk modulus of a porous medium is often used to relate the dry bulk modulus to the saturated bulk modulus for elastic waves, because the compressibility of air is considered so high that the dry rock behaves in a drained fashion and the frequency of elastic waves is considered so high that the saturated rock behaves in an undrained fashion. The bulk modulus calculated from ultrasonic velocities, however, often does not match the Gassmann prediction. Mavko and Jizba [1991] examined how local flow effects and unequilibrated pore pressures can lead to greater stiffnesses. Their conceptual model consists of a distribution of porosities obtained from the strain-versus-confining-pressure behavior. Stiff pores that close at higher confining pressures are considered to remain undrained (unrelaxed) while soft pores drain even for high-frequency stress changes. If the pore shape distribution is bimodal, then the rock approximately satisfies the assumptions of a double-porosity, poroelastic material. Berryman and Wang [1995] established linear constitutive equations and identified four different time scales of flow behavior: (1) totally drained (K), (2) soft pores are drained but stiff pores are undrained ($K[u^{(1)}]$), (3) soft and stiff pores are locally equilibrated, but undrained beyond the grain scale (K_u), and (4) both soft and stiff pores are undrained (K_{EB}). The relative magnitudes of the four associated bulk moduli (in parentheses above) will be examined for all four moduli and illustrated for several sandstones.

2 Constitutive Equations

Berryman and Wang [1995] expressed the constitutive equations for a double-porosity, poroelastic medium in matrix form:

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix} \tag{1}$$

where e is volumetric strain, p_c is confining pressure (equal to the negative of the mean stress with the convention that extensional stresses are positive), $p_f^{(1)}$ is the pore pressure of the stiff component, $p_f^{(2)}$ is the pore pressure of the soft component, $\zeta^{(1)}$ is the increment of fluid content in the stiff component, and $\zeta^{(2)}$ is the increment of fluid content in the soft component. The coefficient matrix is symmetric. The coefficients a_{12} and a_{13} are *poroelastic expansion coefficients*, analogous to thermal expansion. The submatrix elements a_{22}, a_{33}, and a_{23} are *storage coefficients* of the matrix. The coefficient a_{11} is simply the compressibility of the drained (elastic) material.

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The case of drained soft porosity and undrained stiff porosity is defined by $\delta p_f^{(2)} = 0$, while $\delta\zeta^{(1)} = 0$. The pore pressure buildup in the stiff porosity fraction is

$$B[u^{(1)}] \equiv \left.\frac{\partial \delta p_f^{(1)}}{\partial \delta p_c}\right|_{\delta\zeta^{(1)}=\delta p_f^{(2)}=0} = -\frac{a_{21}}{a_{22}}. \tag{2}$$

The effective unrelaxed bulk modulus is found to be

$$\frac{1}{K[u^{(1)}]} \equiv -\left.\frac{\partial \delta e}{\partial \delta p_c}\right|_{\delta\zeta^{(1)}=\delta p_f^{(2)}=0} = a_{11} - \frac{a_{21}^2}{a_{22}} = a_{11} + a_{21}B[u^{(1)}]. \tag{3}$$

Another compressibility similar to Eqn. 3 is one defined by no local flow at all. The pore pressure buildups in the stiff and soft pore fractions are, respectively,

$$B_{EB}^{(1)} \equiv \left.\frac{\partial \delta p_f^{(1)}}{\partial \delta p_c}\right|_{\delta\zeta^{(1)}=\delta\zeta^{(2)}=0} = \frac{a_{23}a_{13} - a_{12}a_{33}}{a_{22}a_{33} - a_{23}^2}, \tag{4}$$

$$B_{EB}^{(2)} \equiv \left.\frac{\partial \delta p_f^{(2)}}{\partial \delta p_c}\right|_{\delta\zeta^{(1)}=\delta\zeta^{(2)}=0} = \frac{a_{23}a_{12} - a_{13}a_{22}}{a_{22}a_{33} - a_{23}^2}. \tag{5}$$

The compressibility associated with the unequilibrated pore pressure buildups is

$$\frac{1}{K_u^{EB}} \equiv -\left.\frac{\partial \delta e}{\partial \delta p_c}\right|_{\delta\zeta^{(1)}=\delta\zeta^{(2)}=0} = a_{11} + a_{12}B_{EB}^{(1)} + a_{13}B_{EB}^{(2)}. \tag{6}$$

A final compressibility is the one associated with globally undrained conditions, but equilibration by local flow between the stiff and soft pores. The pore pressure buildup is

$$B \equiv \left.\frac{\partial \delta p_f}{\partial \delta p_c}\right|_{\delta\zeta^{(1)}+\delta\zeta^{(2)}=0} = -\frac{a_{21} + a_{31}}{a_{22} + 2a_{23} + a_{33}}, \tag{7}$$

where $\delta p_f \equiv \delta p_f^{(1)} = \delta p_f^{(2)}$. The long-time, globally undrained compressibility is

$$\frac{1}{K_u} \equiv -\left.\frac{\partial \delta e}{\partial \delta p_c}\right|_{\delta\zeta^{(1)}+\delta\zeta^{(2)}=0} = a_{11} + (a_{12} + a_{13})B. \tag{8}$$

Berryman and Wang [1995, 1998] used laboratory measurements of Coyner [1984] for various rocks to obtain the parameters necessary for the constitutive equations. The soft component parameters (including unjacketed solid compressibility and drained frame compressibility) were taken to be those measured at a confining pressure of 10 MPa, whereas the corresponding stiff component parameters were those measured at 25 MPa. The resulting four compressibilities, $1/K[u^{(1)}]$, $1/K_u^{EB}$, and $1/K_u$, together with $1/K$, are shown in Table 1 for three sandstones.

If it is assumed that the quasistatically *measured* value is the long-time, undrained limit for a double-porosity medium in which the pore pressures between the different subsets of pore shape have equilibrated, then $1/K_u$ in Table 1 would be the predicted value. Among the samples in Table 1, $1/K_u$ has been determined experimentally only for Berea sandstone [Hart and Wang, 1995]. The measured value $1/K_u = 0.063$ GPa^{-1} compares favorably with the value 0.076 GPa^{-1} in Table 1.

Time Scales	Compressibility	Weber	Berea	Navajo
long	$1/K$, GPa^{-1}	0.250	0.167	0.0769
intermediate	$1/K[u^{(1)}]$, GPa^{-1}	0.198	0.129	0.0638
short	$1/K_u$, GPa^{-1}	0.061	0.076	0.0541
very short	$1/K_u^{EB}$, GPa^{-1}	0.054	0.069	0.0518

Table 1: Compressibilities for different assumptions about pore pressure equilibration between soft and stiff pores, assuming that $K_f = 2.3$ GPa^{-1}. Values in the first row are drained frame compressibilities measured for 10 MPa confining pressure by Coyner [1984]. All other values are computed from formulas in the text.

3 Gassmann's Equation

A rigorous expression of the Gassmann relation for a single-porosity medium is [Brown and Korringa, 1975; Berge, 1998]

$$K_u - K = \frac{\alpha^2}{\frac{\alpha}{K_s} + \phi\left(\frac{1}{K_f} - \frac{1}{K_\phi}\right)}, \tag{9}$$

where K_u is the undrained bulk modulus, K is the drained bulk modulus, K_s is the unjacketed bulk modulus, K_f is the fluid bulk modulus, K_ϕ is the unjacketed pore bulk modulus, ϕ is the porosity, and $\alpha \equiv 1 - K/K_s$. Berge [1998] has shown that the assumption $K_\phi = K_s$ often fails for experimental values of $K_u - K$, even for monomineralic rocks, where in theory they are equal. A possible reason for the breakdown is that the pore structure of monomineralic rocks is not homogeneous, and the double-porosity poroelastic model may be more appropriate than the single-porosity poroelastic model.

Berge [1998] showed that K_ϕ may depend strongly on effective stress, with $K_\phi \to K_f$ at low stress and $K_\phi \to K_s$ at high stress. Stress dependence is routinely observed in laboratory measurements of B and K [e.g., Green and Wang, 1986; Fredrich et al., 1995; Hart and Wang, 1995]. At high confining pressures, K_ϕ values may represent the behavior of stiff pores in a double-porosity medium, whereas the K_ϕ values at low stresses may represent the response of the soft pores. In a single-porosity medium, K_ϕ is related to B and K by [Green and Wang, 1986; Berge, 1998]

$$\frac{1}{K_\phi} = \frac{1}{K_f} - \left(\frac{\alpha}{\phi K}\right)\left(\frac{1}{B} - 1\right). \tag{10}$$

In the double-porosity model, we can obtain two estimates of K_ϕ by considering first the case when soft pores are drained but stiff pores are undrained, and next the long-time, globally undrained case. In the first case, we can estimate K_ϕ using $B[u^{(1)}]$ in Eqn. 10, and we use B in the second case.

4 Discussion

The three compressibilities, $1/K[u^{(1)}]$, $1/K_u^{EB}$, and $1/K_u$, together with $1/K$, show that the equilibration of pore pressure affects the volumetric strain that results from the application of confining pressure. Achieving an approximation to the different boundary conditions in the definition of each compressibility depends on the permeabilities of the different subsets of porosity. The frequency of an elastic wave relative to these permeabilities will determine which set of boundary conditions is most appropriate for a particular rock.

The unrelaxed compressibility of Mavko and Jizba, $1/K[u^{(1)}]$, represents a frequency for which flow from the soft porosity is globally drained. It is stiffer than the totally drained case ($1/K$). The undrained case in which local flow equilibrates pore pressure between the soft and stiff porosity is represented by $1/K_u$, and it is lower except for the case of no local flow at all, which is represented by $1/K_u^{EB}$. The totally drained compressibility is largest, and the compressibility associated with no local flow at all is the smallest, and is associated with the highest frequency of elastic wave propagation. Pressure dependence of poroelastic parameters measured in the laboratory may be described using the double-porosity poroelasticity model.

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