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UNDULATOR TUNABILITY AND RING-ENERGY

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1. Introduction

An Undulator has two properties which make it an extremely attractive source of electromagnetic radiation.[1] The first is that the radiation is concentrated in a number of narrow energy bands known as harmonics of the device. The second characteristic is that under favorable operating conditions, the energy of these harmonics can be shifted or "tuned" over an energy interval which can be as large as two or three times the value of the lowest energy harmonic.

Both the photon energy of an undulator as well as its tunability are determined by the period, λ, of the device, the magnetic gap, G (which is larger than the minimum aperture required for injection and operation of the storage ring), and the storage ring energy, E_R. Given the photon energy, E_P, the above parameters ultimately define the limits of operation or tunability of the undulator.

For the specific case of the Advanced Photon Source, the original user requirement for an undulator with a 20 keV fundamental photon energy can be met at a ring energy of 6 GeV and the ring aperture proposed [2]. However, as more additional capabilities of undulator sources were investigated [3,4], it became evident that at 6 GeV, the tunability of undulators with 1st harmonic energies above 10 keV is rather limited. As a result, as many as five undulators would be needed to span the photon energy interval of 5 to 20 keV at a single straight section. The expense and complication of having so many undulators on a given

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keV. A second undulator should be tunable from 14 to 20 keV. Again, although two devices per straight section are considered reasonable, the 3rd harmonic intensities of the first undulator might be large enough in certain cases to eliminate the second device whenever possible.

3.1 General Considerations

The energy in keV of the i^{th} harmonic of an undulator for an observation point along the midplane axis of the device is given by:

$$E_{Pi} = \frac{0.949\ E_R{}^2 i}{\lambda(1 + K^2/2)} \qquad (1)$$

Where E_{Pi} is photon energy in keV, E_R is the ring energy in GeV, λ is the undulator period in cm and i is the harmonic number. We will consider only the first harmonic $i = 1$ and $E_{Pi} = E_P$. The deflection parameter K is given in terms of the undulator period (in cm) and the peak magnetic field B_o (in Tesla) by

$$K = 0.934\ \lambda B_o \qquad (2)$$

For hybrid magnets based on permanent magnet blocks and vanadium permandur pole tips, B_o is given by [6]

$$B_o = 0.95\ a\ \exp\ (-G/\lambda(b-cG/\lambda)) \qquad (3)$$

where G is the magnet gap of the undulator in cm.

In Eq. 4, the factor 0.95 represents the "filling factor" which takes into account the packing factor of high-permeability blocks in the undulator assembly. The constants a, b and c depend on the magnetic material and are given in Table 1 for two permanent magnet candidate materials, $SmCo_5$/permendur (REC-hybrid) and Nd-Fe-B.

TABLE 1

Constants used in Eq. 3 for Hybrid Magnetic based on REC or Nd-Fe-B

	REC/Hybrid	Nd-Fe-B
a (T)	3.33	3.44
b	5.47	5.08
c	1.8	1.54

Equation 3 for the peak field is valid in the interval $0.07 \leq G/\lambda \leq 0.7$. Although the upper limit for G/λ does not define the maximum operational gap of the device, we have taken this as the maximum gap in our calculations for the reason that the K values encountered for $G>0.7\lambda$ are small and hence the intensities of the photon beam are too small. Secondly, higher than expected field errors have been observed for gaps larger than 0.7λ. We have taken the ratio $R = G/\lambda = 0.7$ as a conservative and "safe" upper limit.

Equations 1, 2, and 3 form a set of coupled equation which determine the photon energy of a given harmonic as a function of gap, device period and ring energy. At a given ring energy E_R, the undulator period and gap, determine the on-axis 1st harmonic energy, E_P. Decreasing the gap increases B_o and hence the value of K resulting in a lower photon energy.

In summary, the largest photon energy occurs at the largest gap for a given undulator period and has both the smallest K and intensity. The energy may be shifted down from this maximum by decreasing the gap. The tunability from the maximum to minimum photon energy is limited by the maximum gap determined by

$R=0.7$ and the minimum gap determined by the ring aperture. In addition, the tunability interval in the photon energies depend on the storage ring energy.

3.2 Analysis

From Eqs. 1, 2 and 3, it follows that any three of the parameters λ, E_R, E_P, and gap uniquely determined the forth. For example, a given undulator period, ring energy and disired photon energy specify the required gap. In Figs. 1, the gap values necessary for a set of undulator periods are shown plotted against the ring energy for the photon energies indicated. The upper bounds on the gap for 14 keV (Fig. 1a) and 7 keV (Fig. 1b) are determined by the condition $R=0.7$. If a minimum or closed-gap line is drawn in each figure, then only those devices which fall above this line can provide the indicated E_P at any given ring energy. For example, an undulator with a period of 3.7 cm will require gaps of 2.6 and 1.5 cm to provide 14 and 7 keV respectively at 8 GeV.

This graphical analyses which defined allowable device periods within the gap constraints can be extended to determine the tunability of an undulator in a more general way. That is, at any given E_R, the maximum desired photon energy, Eu and minimum deflection parameter, K_U, at the open gap (G_U) position determined the required device period <u>at each ring energy</u>. This value of λ is the maximum one at E_R capable of producing E_U and is also the one with the <u>largest</u> tunability range consistent with E_U and K_U because it has the smallest closed-gap to λ-ratio. Once this period is determined for E_U, then the gap at each E_R required to produce any E_P smaller than E_U can be determined.

For reasons mentioned above, the value of K_U was taken as that given by the open gap relation of $R=0.7$. Following this procedure, the gap values as a functions of ring energy for 14, 7 and 4.7 keV photon energies are shown in Fig. 2 for REC-hybrid magnet undulators. Each curve is for the constant photon energy indicated. The maximum photon energy is 14 keV. The period of the "optimum" device determined at 14 keV and $G/\lambda = 0.7$ is shown for each E_R on the top of the figure. The two horizontal lines at G = 1.5 and 1.0 cm are the initial and final phase operation minimum gap values. It is obvious that these minimum gap sets the lower limit on the tunability at a given E_R.

The intersection of any constant gap line and one of the constant photon energy curves (7 or 4.7 keV in this case) corresponds to the minimum ring-energy necessary to achieve the tunability between the maximum E_P and the selected lower photon energy. One sees immediately from Fig. 1 that at an initial phase minimum gap of 1.5 cm, the minimum ring-energy necessary to provide tunability between 14 and 7 keV is approximately 8 GeV for the REC-Hybrid undulator. AT a minimum gap of 1.4 cm, E_R-min is reduced to 7.8 GeV. At a final phase minimum gap of 1.0 cm, tunability between 14 and 4.7 keV will occur at 7.4 GeV minimum ring energy.

The tunability analysis can also be presented in a different fashion in which the photon energy for a constant gap value is plotted as a function of E_R. These results are shown in Fig. 3 for a REC-Hybrid device. The photon energy interval over which a device can be tuned for a given minimum gap value and a ring energy is directly obtainable from the figure for an undualtor which produced maximum of 14 keV photons. As an example, at 7.4 GeV the tunability ranges for various minimum gap values are present in Table 2.

TABLE 2

The range of photon energies achievable for a REC hybrid device with 7.4 GeV positrons as a function of the values of minimum magnet gap.

Minimum Gap (cm)	Range of Photon energy (keV)
1.5+	9.4 to 14.0
1.4	8.5 to 14.0
1.0++	4.7 to 14.0

+Initial phase
++Final phase

The results show that a final phase operation tunability of 14 to 4.7 keV would be achievable with 7.4 GeV. However, this ring energy is too low to permit 7 keV at 1.5 cm or even 1.4 cm gap during the initial phase of the operation of the ring. A ring energy of 7.5 GeV would be adequate if the photon energy range for initial operation is between 9 and 14 keV.

It is also apparent from Fig. 3 that the minimum ring energy needed to achieve a given tunability interval is a steep function of the minimum magnet gap. For example, if the closed gap is 1.1 rather than 1.0 cm, the minimum E_R increases from 7.4 to 7.7 GeV. This emphases the necessity of precisely defining the closed gap tolerances.

The other aspect of this analysis has to deal with the photon flux or brilliance which is closely related to K values. At the maximum photon energy, the K-value is lowest and is determined by the maximum gap condition of $G/\lambda = 0.7$ for each ring energy. It can be shown that in this case

$$\frac{K}{\lambda} = 0.1551 \quad cm^{-1} \qquad \text{REC-Hybrid} \tag{4a}$$

$$\frac{K}{\lambda} = 0.1854 \quad cm^{-1} \qquad \text{Nd-Fe-B} \tag{4b}$$

The maximum photon energy to be derived from an undulator determines the λ value for a given ring energy. At lower photon energies (i.e., at smaller gaps), the K-value depends on the actual gap and ring energy. The resulting K-values for a 14 keV undulator operating at 7 and 4.7 keV closed gap mode are shown as a function of ring energy in Fig. 4. As can be seen, the K at maximum gap condition and 14 keV is less than 0.5 for a ring energy which is less than about 7.5 GeV. At the closed gap energies of 7 and 4.7 keV and with 7.5 GeV stored beam, the K-values are approximately 1 and 2, respectively. This means that the 3rd harmonic of the device operating between 7 and 4.7 keV has non-negligible intensity which would be useful for some experiments needing 14 to 20 keV radiation. One very important aspect of this result is that in this situation, a separate 20 keV undulator may not be necessary to cover the range from 14-20 keV. A comparitive study of the 3rd harmonic brilliance of the 14 keV device versus the 1st harmonic one of the 20 keV undulator will be presented in a later LS-note.

The last part of the criteria was the feasibility of obtaining 20 keV radiation in the 1st harmonic of a device at the initial and final phase of operation. The constant energy curve of gap versus ring energy plots for a 20 keV undulator is shown in Fig. 5 for devices capable of delivering maximum of 20 keV radiation. These curves show that at 1.5 cm gap, the minimum ring energy necessary to produce 20 keV photons in the 1st harmonic is approximately 7 GeV. At the final phase with a minimum gap of 1.0 cm, the lowest photon energy achievable at 7 GeV is approximately 16.5 keV as seen from Fig. 6. At higher ring energies, the tunability interval increases.

In Fig. 7, the K-values for several first harmonic photon energies are shown as a function of ring energy. As can be seen, at the maximum gap limit and 20 keV, the K-values are less than 0.5 for ring energies up to 8 GeV and hence the 3rd harmonic intensity from these devices will be of little use.

3.3 Ring Energy Independent Solutions

From Fig. 1a, it is clear that the maximum gap/period ratio of 0.7 determines the undulator period capable of producing 14 keV radiation at each ring energy. Also from Fig. 1b, the minimum gap condition specifies the undulator period at each ring energy necessary for a device to produce 7 keV radiation. In Fig. 8, those periods are plotted verses ring energy for 14 keV at R = 0.7 and 7 keV at minimum gaps of 1.5 and 1.3 cm. As can be seen intersects occur at $E_R \simeq 8$ GeV and $\lambda \simeq 3.7$ cm for a gap of 1.5 cm and 7.5 GeV and λ Z 3.2 cm at 1.3 cm minimum gap. These values correspond to the smallest ring energies and undulator periods capable of spanning the tunability range 14 to 7 keV within the specified gaps. At smaller ring energies, the lower photon energy cannot be reached at the respective minimum gap.

From this we see that an equivalent way of analyzing the tunability vs. ring-energy question is to consider the solutions to Eq. 1 along the constraints imposed by the tunability. Let E_U be the maximum 1st harmonic photon energy desired at the maximum open gap G_U, and let E_L be the minimum photon energy at the closed gap position G_L of the undulator. Let $E_L = fE_U$ where $f<1$; then the on-axis energies of the undulators are:

$$E_U = \frac{0.949\ E_R^2}{\lambda\left(1 + \frac{K_U^2}{2}\right)} \qquad \text{at } G_U$$

$$E_L = f\ E_U = \frac{0.949\ E_R^2}{\lambda\left(1 + \frac{K_L^2}{2}\right)} \qquad \text{at } G_L$$

or

$$f\left(1 + \frac{K_L^2}{2}\right) = \left(1 + \frac{K_U^2}{2}\right). \qquad (5)$$

The value of K_U is either a minimum acceptable number or that determined by a maximum gap/λ value. In either case, if the minimum gap value is specified, then Eq. 5 has a unique solution for the value of the undulator period which is independent of the photon energy and the ring energy. This is the smallest period for which E_L will be achieved at G_L. It also yields the largest K-value at E_L. This minimum undulator period also corresponds to the minimum ring-energy at which the tunability range E_U - E_L will be achieved. At higher ring-energies, the same photon energy interval can be spanned with devices with periods larger than the minimum λ given by Eq. 5.

For example, consider the case of a REC-Hybrid undulator at R = 0.7. From Eq. 4a, K = 0.1551λ. Suppose the minimum gap is G_L = 1.5 cm, and the photon energy range (resulting from closed gap to open gap operation) is f = 0.5. Then the solution of Eq. 5 is λ = 3.73 cm. If 14 keV photons are required from this device at the open gap position, then this translates to a minimum ring energy of E_R = 8.01 GeV which is consistent with the results from the graphical analyses of the previous section. Following this procedure, in Table 3 has been constructed for several tunability ranges for gaps ranging from the value of R=0.7 for the maximum gap (G_U) to period ratio to the minimum gap specified (G_L).

TABLE 3

Values of minimum ring-energies E_R period λ and K-values for the various tunability intervals and minimum gap values, G_L for undulators. The largest gap considered, G_U, was determined by the maximum gap to period ratio of 0.7. The values for Nd-Fe-B devices are given in parenthesis below those for a REC-Hybrid one Δ.

E_U(keV)	E_L(keV)	G_L(cm)	G_U(cm)	λ(cm)	K_U	K_L	E_R(GeV)
14	14	1.5	-	2.143 (2.143)	0.33 (0.40)	-	5.78 (5.84)
14	7	1.5	2.61 (2.50)	3.730 (3.574)	0.57 (0.66)	1.63 (1.70)	8.01 (8.01)
14	7	1.4	2.49 (2.38)	3.559 (3.407)	0.55 (0.63)	1.62 1.67	7.78 (7.76)
14	7	1.3	2.37 (2.27)	3.386 (3.239)	0.53 (0.60)	1.60 1.65	7.54 (7.51)
14	8	1.5	2.48 (2.38)	3.541 (3.393)	0.55 (0.63)	1.42 (1.48)	7.75 (7.74)
14	9	1.5	2.35 (2.26)	3.363 (3.223)	0.52 (0.60)	1.24 (1.29)	7.51 (7.49)
14	4.7	1.0	2.31 (2.21)	3.297 (3.152)	0.51 (0.58)	2.19 (2.24)	7.42 7.38
20	20	1.5	- -	2.143 (2.143)	0.33 (0.40)	- -	6.90 (6.98)
20	14	1.0	1.70 (1.63)	2.434 (2.323)	0.38 (0.43)	1.03 (1.06)	7.41 (7.31)

As is evident from Table 3, the Nd-Fe-B material results in a larger K-values compared to REC hybrid, but does not significantly alter the minimum ring energy necessary to achieve the desired tunability range at the maximum gap condition if R=0.7 is maintained. Hence, a Nd-Fe-B device will always have a higher photon flux than the equivalent REC-Hybrid one, as expected from previous analyses.

The analysis follows the same procedure if a minimum K value were used at the upper photon energy rather than the maximum gap G/λ to period ratio. Obviously, K-values lower than those given by the maximum gap condition will result in smaller period devices and therefore smaller minimum ring energies than those given in Table 3. The opposite is true for larger K-values.

In the same way, the results depend on the choice of R (gap/period = 0.7). For example, in the case of the 14 keV undulator, increasing the R from 0.7 to 0.8 results in a decrease in E_R from 7.78 to 7.46 GeV. However, the minimum K-value also drops from 0.55 to 0.41, thus lowering the photon flux. A further increase of R to 0.9 results in an E_R of 7.31 GeV and a K_U of 0.32. In addition to lower K values, at large gaps, one may expect larger field errors which would deteriorate desired field profile. Again, the final minimum ring energy necessary will depend on the acceptable operating point of the device.

4. Conclusions

We have presented an algorithm for finding the minimum ring energy needed for a desired tunability photon energy range. The analysis shows that unique solutions are possible if the the minimum gap is specified and a maximum gap to period ratio (R) or minimum K-value is provided. All the criteria set forth by the National Task Group can be satisfied with a ring-energy of 7.5

GeV if (1) the initial phase operation criteria for obtaining 7 keV photons at 1.5 cm undulator gap is relaxed to a value of 9 keV or (2) the initial phase minimum gap is lowered from 1.5 cm to 1.3 cm.

At final phase operation a single undulator with a 14 keV 1st harmonic will have K-values ranging from about 1 to 2 in the 7 to 4.7 keV range. This means that the device will have non-negligible 3rd harmonic intensity for radiation from 14 to 21 keV. This intensity may be sufficient to satisfy the requirements for 20 keV radiation this eliminating the need for two devices on the same beamline. Further investigations of the Brilliance/Brightness for the devices will be presented in a separate LS note.

Figures

Fig. 1. Gap values (cm) as a function of ring energy (GeV) needed by an undulator of period (cm), λ to obtain a) 14 keV, b) 7 keV, and c) 4.7 keV photon energies. The upper bound on gap, G, is given by G/λ = 0.7.

Fig. 2. Constant photon energy (keV) plots of the gap values (cm) as a function of ring energy (GeV) needed by a 14 keV undulator to obtain the photon energies shown. The period (cm), λ, of the device is the maximum one permitted by the maximum gap condition $G/\lambda = 0.7$.

Fig. 3. Photon energies (keV) achievable at the minimum gaps shown as a function of ring energy (GeV) for the 14 keV undulator of Fig. 2. The horizontal lines represent the desired initial (7 keV) and final (4.7 keV) phase operation photon energies at the minimum gap.

Fig. 4. K-values as a function of ring energy (GeV) for different values of photon energies (of 14, 7, and 4.7 keV) for the 14 keV undulators. The corresponding gaps are given in Fig. 2.

Fig. 5. Constant photon energy (keV) plots of the gap values (cm) as a function of ring energy (GeV) needed by 20 keV undulators. The period (cm), λ, of the devices is that permitted by the maximum gap condition, $G/\lambda = 0.7$.

Fig. 6 Photon energies (keV) achievable at the minimum gaps shown as a function of ring energy (GeV) for the 20 keV undulator of Fig. 5.

Fig. 7. K-values as a function of ring energy (GeV) for the 20 keV undulator along the constant photon energies shown.

Fig. 8. A plot of the period (cm), λ, versus ring energy for a 14 keV device operating at the maximum gap $G/\lambda = 0.7$ and a 7 keV device operating at a minimum gap of 1.5 cm and 1.3 cm. The crossing points are the minimum period which will provide both photon energies at the specified gap. The ring energies are the minimum ones at which the tunability interval can occur.

References

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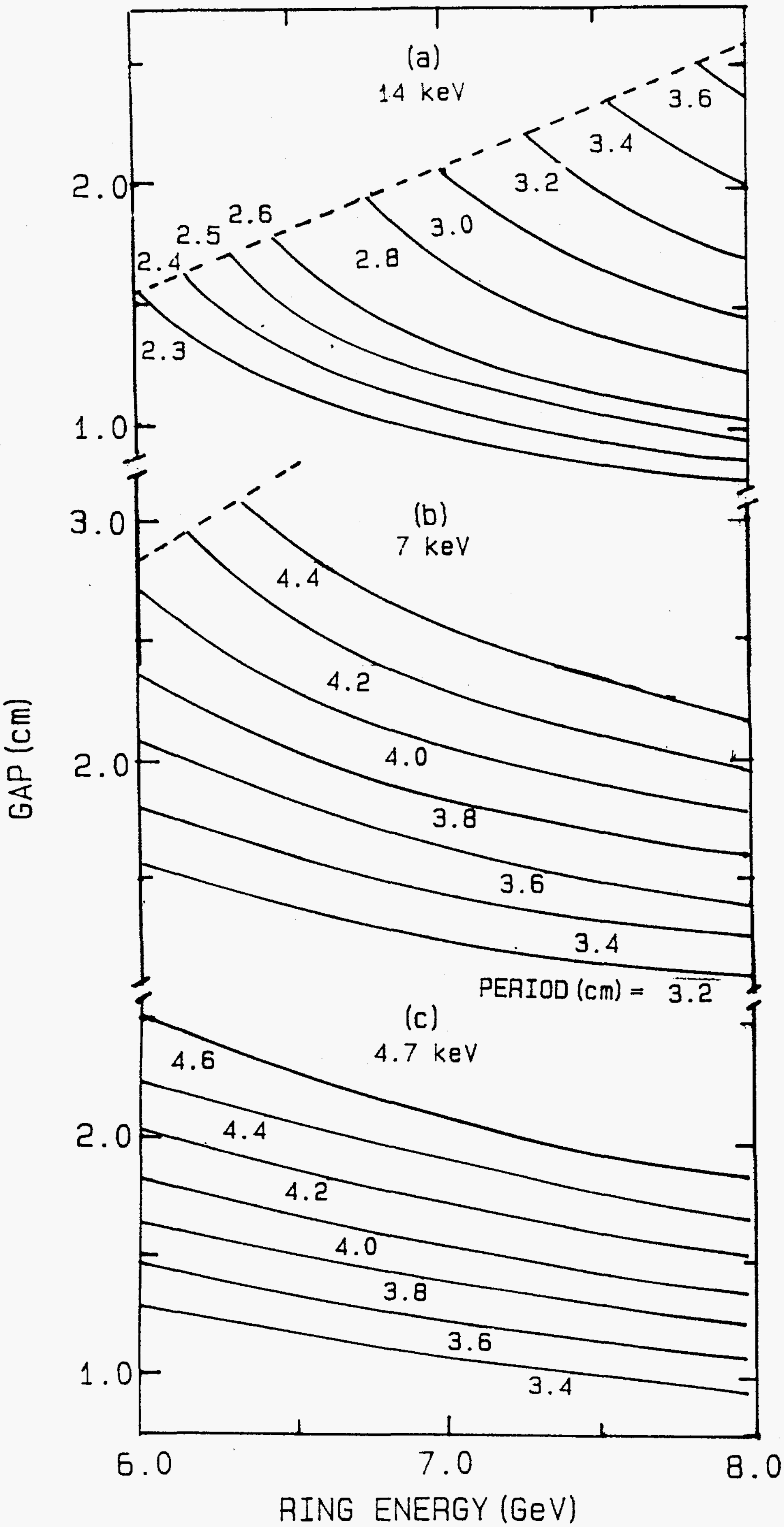
[2] "Conceptual Design Report of a 6 GeV Synchrotron X-Ray Source", Argonne National Laboratory, March 1986.

[3] G. K. Shenoy and P. J. Viccaro, "An Overview of the Characteristics of the 6-GeV Synchrotron Radiation: A Preliminary Guide to Users", Argonne National Laboratory Report, ANL 85-69 (October 1985).

[4] "Conceptual Design Report of a 6-GeV Synchrotron X-Ray Source", Supplement A, "Characteristics of Insertion Devices for the 6-GeV Synchrotron Source", Argonne National Laboratory Light Source Document, LS-52, March 1986.

[5] National Task Group on Ring Energy Section for APS is made up of G. Brown, J. Hastings, S. Krinsky, D. Moncton, G. Shenoy (chairman), P. J. Viccaro and Y. Cho.

[6] K. Halbach, J. Appl. Phys. 57, 3605 (1985) and references therein.



(a)
14 keV
3.6
3.4
3.2
3.0
2.8
2.6
2.5
2.4
2.3
2.0
1.0
(b)
7 keV
4.4
4.2
4.0
3.8
3.6
3.4
3.0
2.0
PERIOD (cm) = 3.2
(c)
4.7 keV
4.6
4.4
4.2
4.0
3.8
3.6
3.4
2.0
1.0
GAP (cm)
6.0
7.0
8.0
RING ENERGY (GeV)

Fig. 1

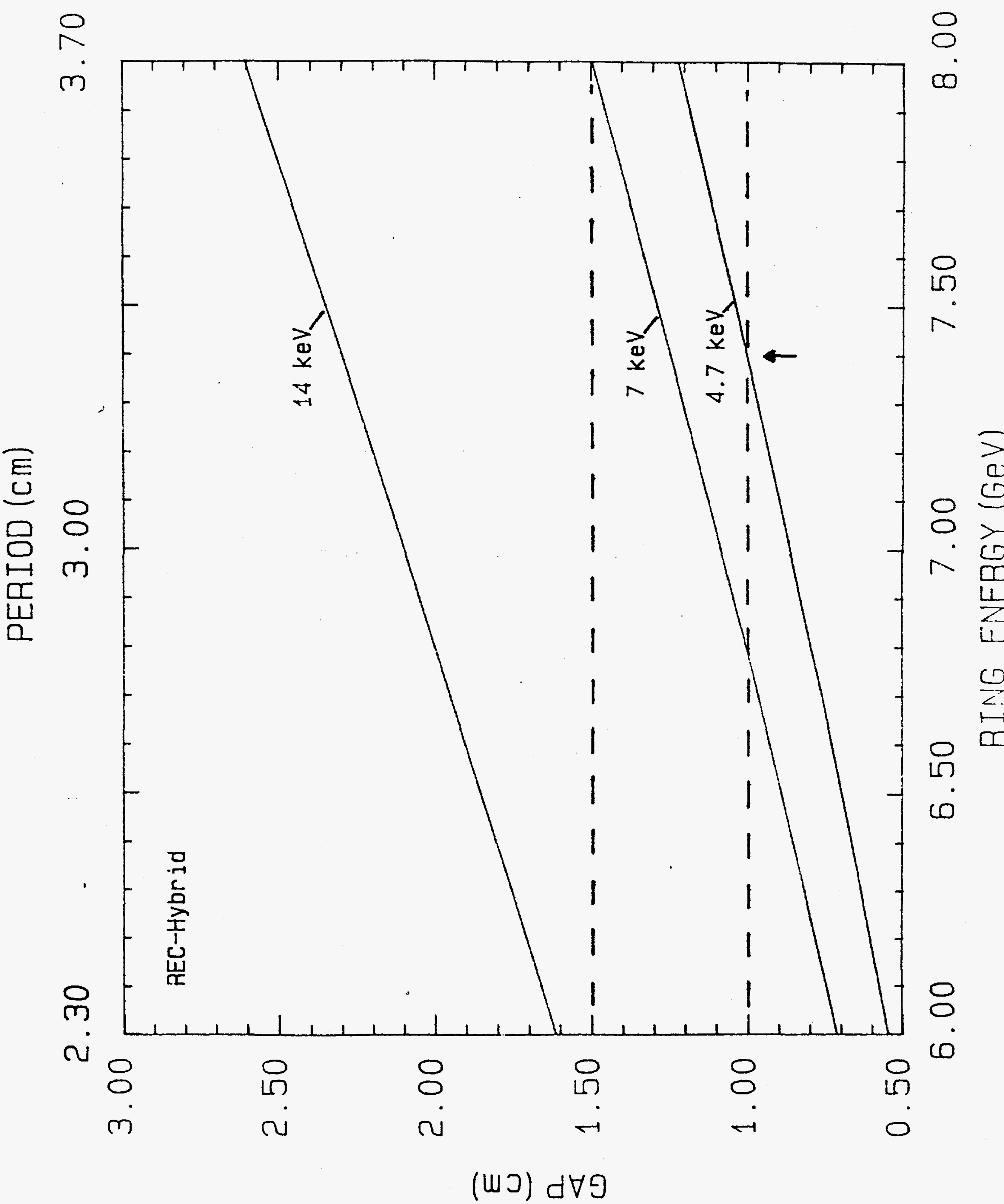
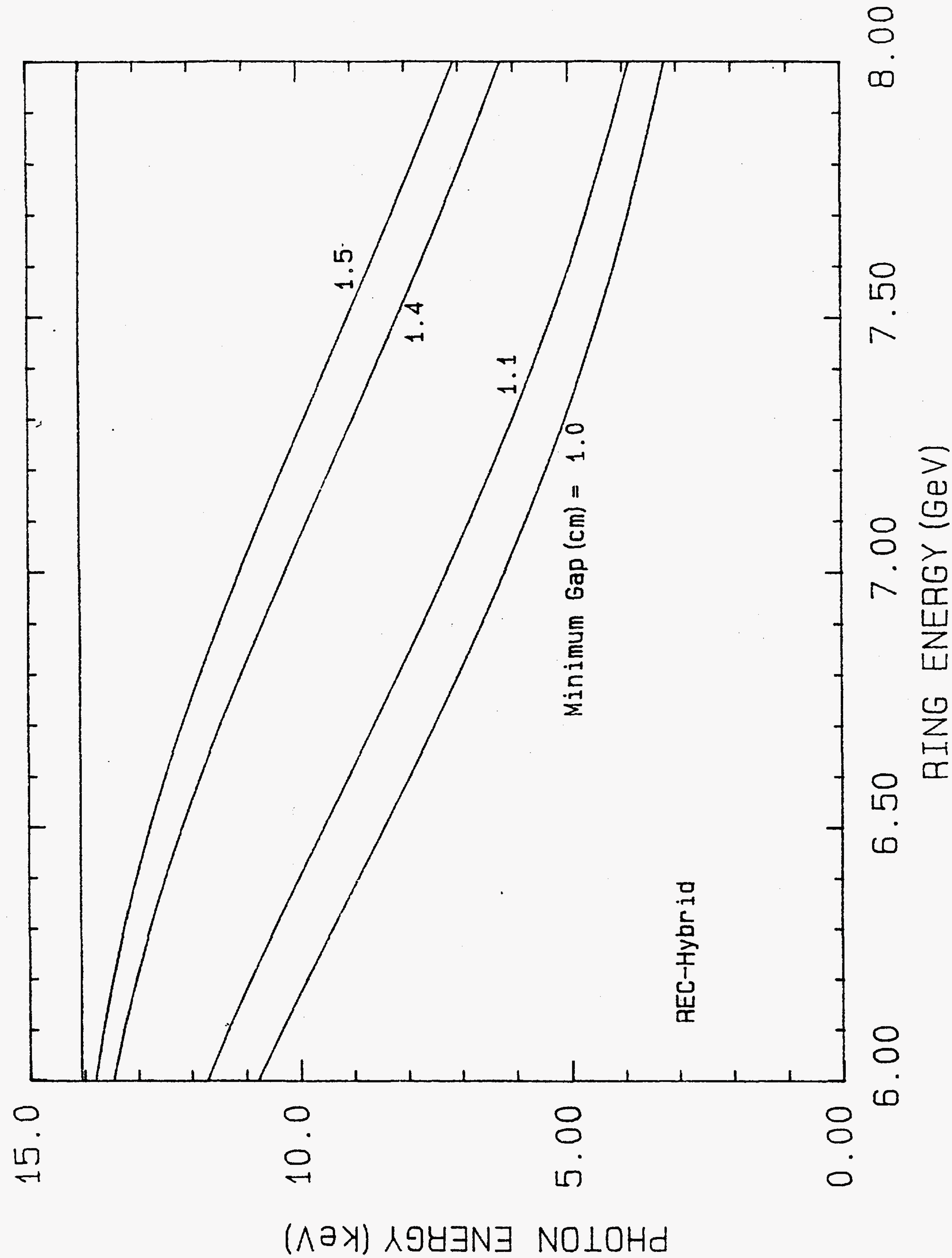
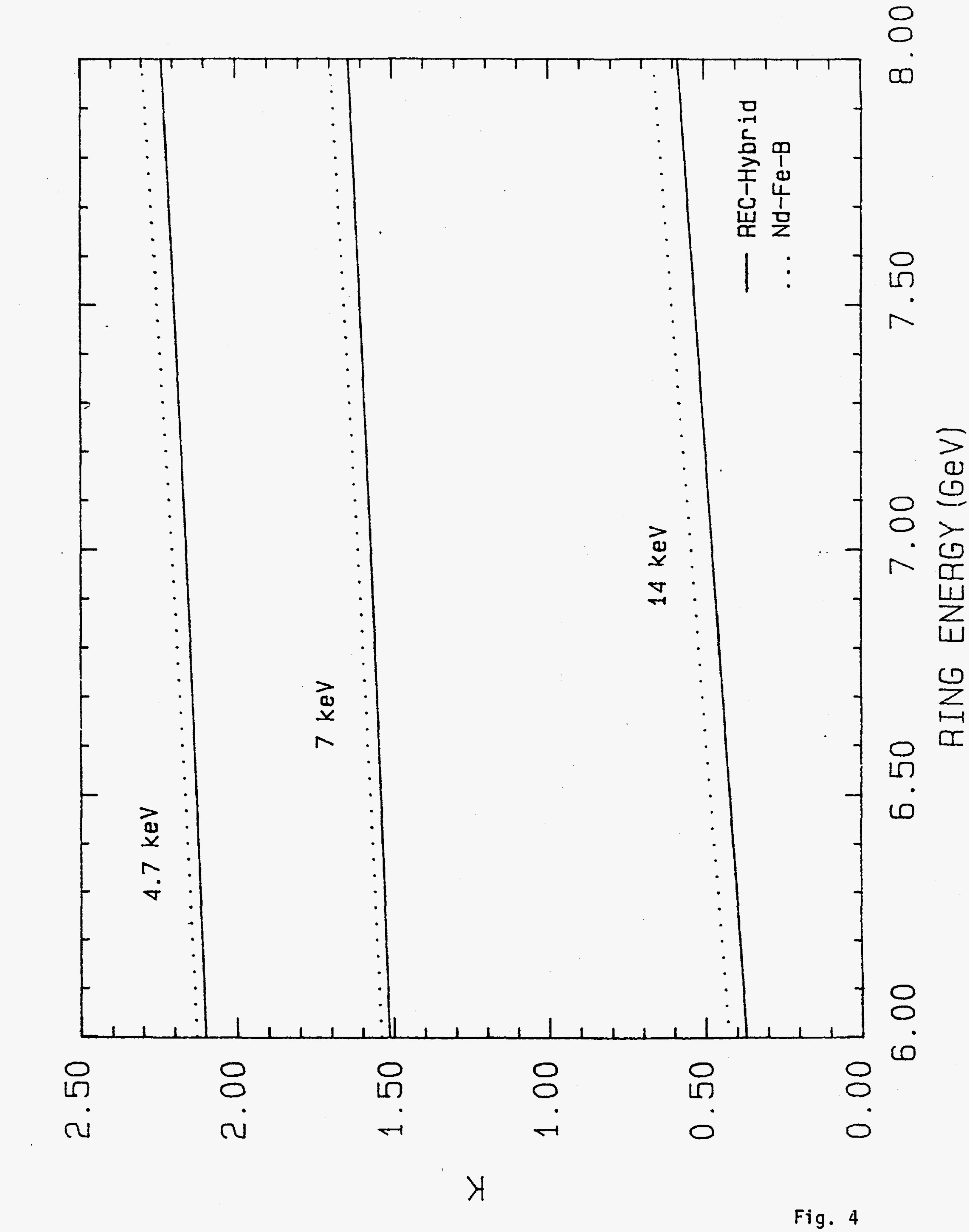

PERIOD (cm)
2.30
3.00
3.70
GAP (cm)
3.00
2.50
2.00
1.50
1.00
0.50
RING ENERGY (GeV)
6.00
6.50
7.00
7.50
8.00
REC-Hybrid
14 keV
7 keV
4.7 keV

Fig. 2



PHOTON ENERGY (KeV)
RING ENERGY (GeV)
0.00
5.00
10.0
15.0
6.00
6.50
7.00
7.50
8.00
1.5
1.4
1.1
Minimum Gap (cm) = 1.0
REC-Hybrid

Fig. 3



K
0.00
0.50
1.00
1.50
2.00
2.50
RING ENERGY (GeV)
6.00
6.50
7.00
7.50
8.00
4.7 keV
7 keV
14 keV
REC-Hybrid
Nd-Fe-B

Fig. 4

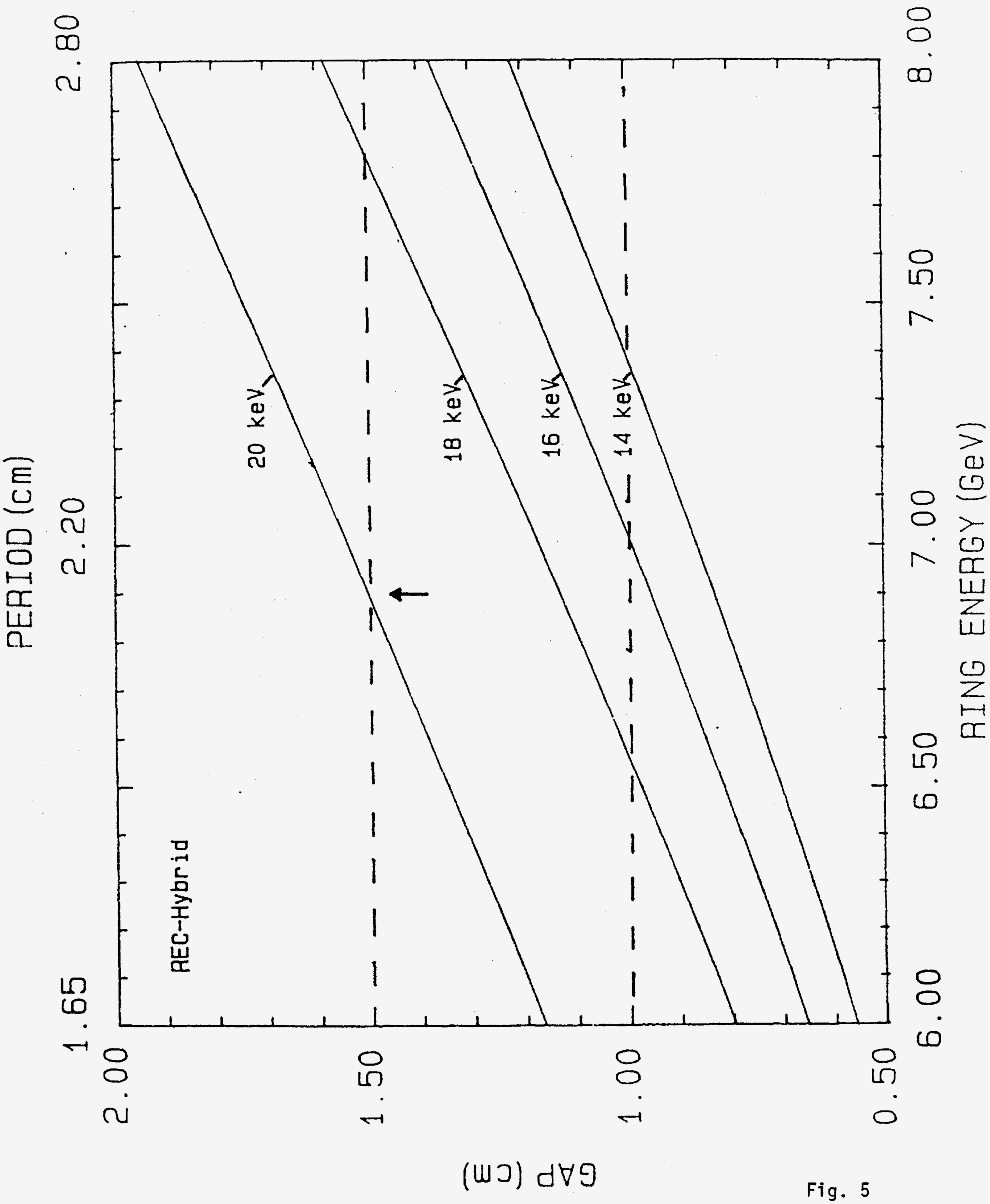
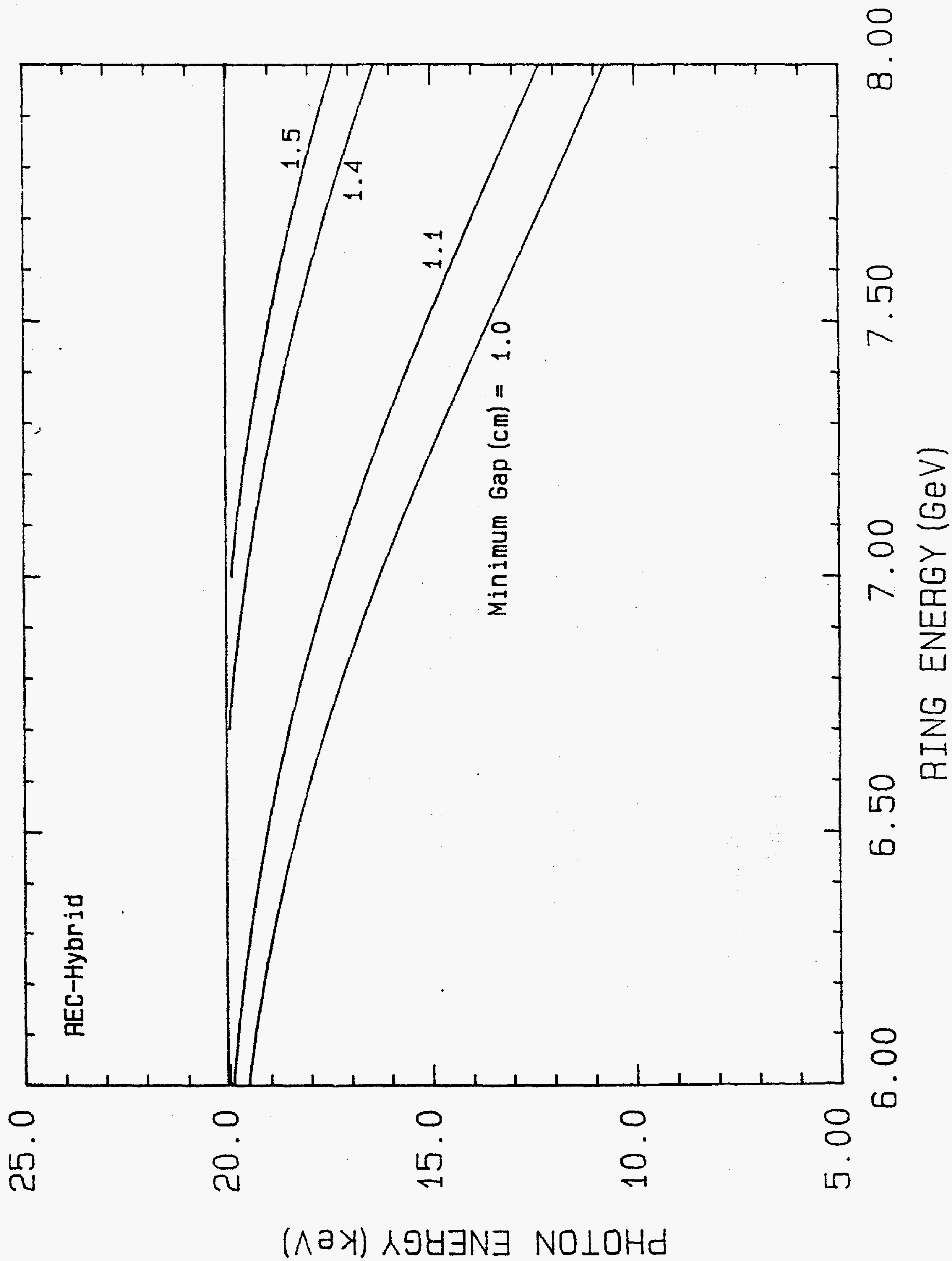
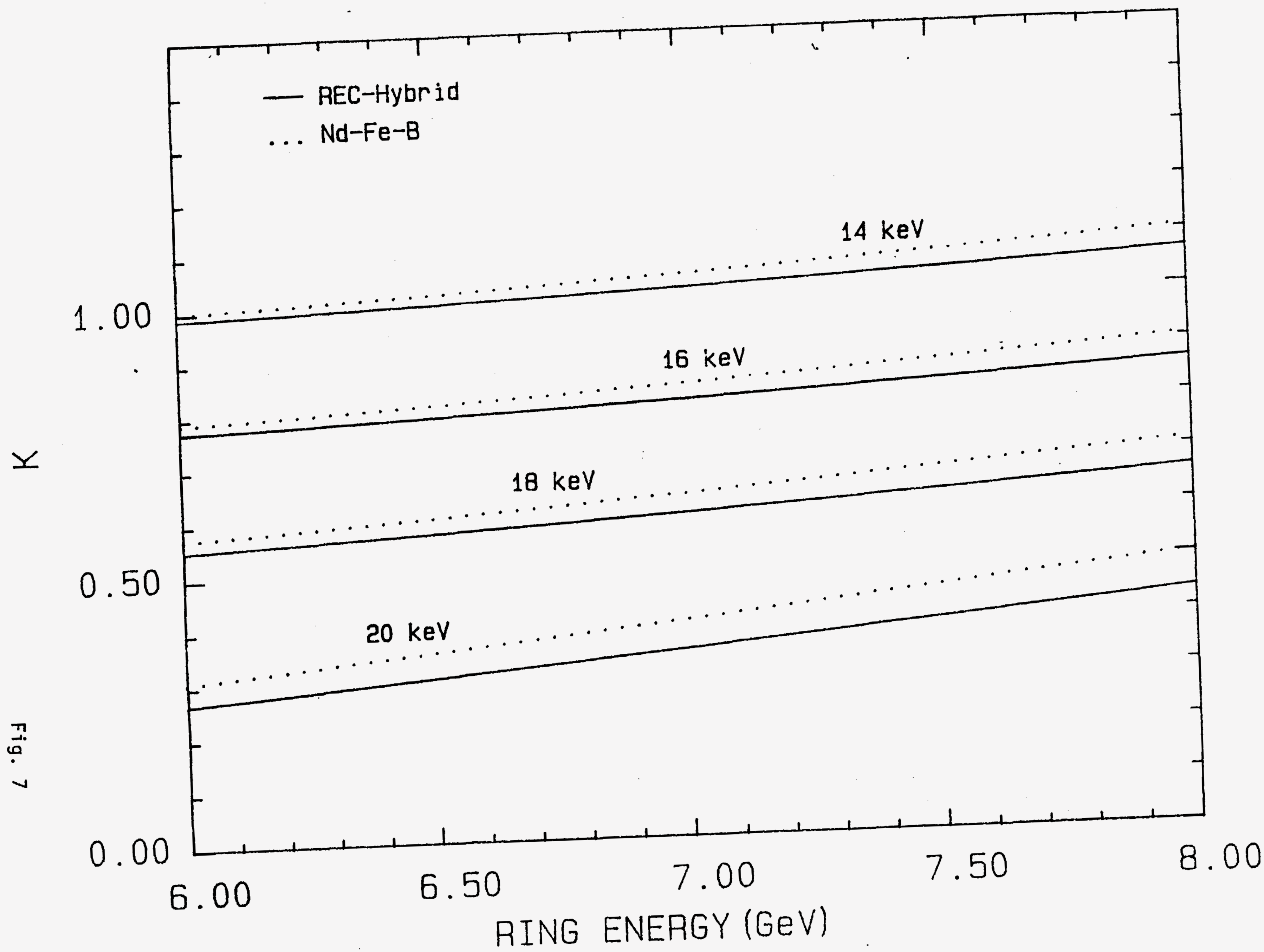

PERIOD (cm)
2.80
2.20
1.65
GAP (cm)
2.00
1.50
1.00
0.50
RING ENERGY (GeV)
6.00
6.50
7.00
7.50
8.00
20 keV
18 keV
16 keV
14 keV
REC-Hybrid

Fig. 5



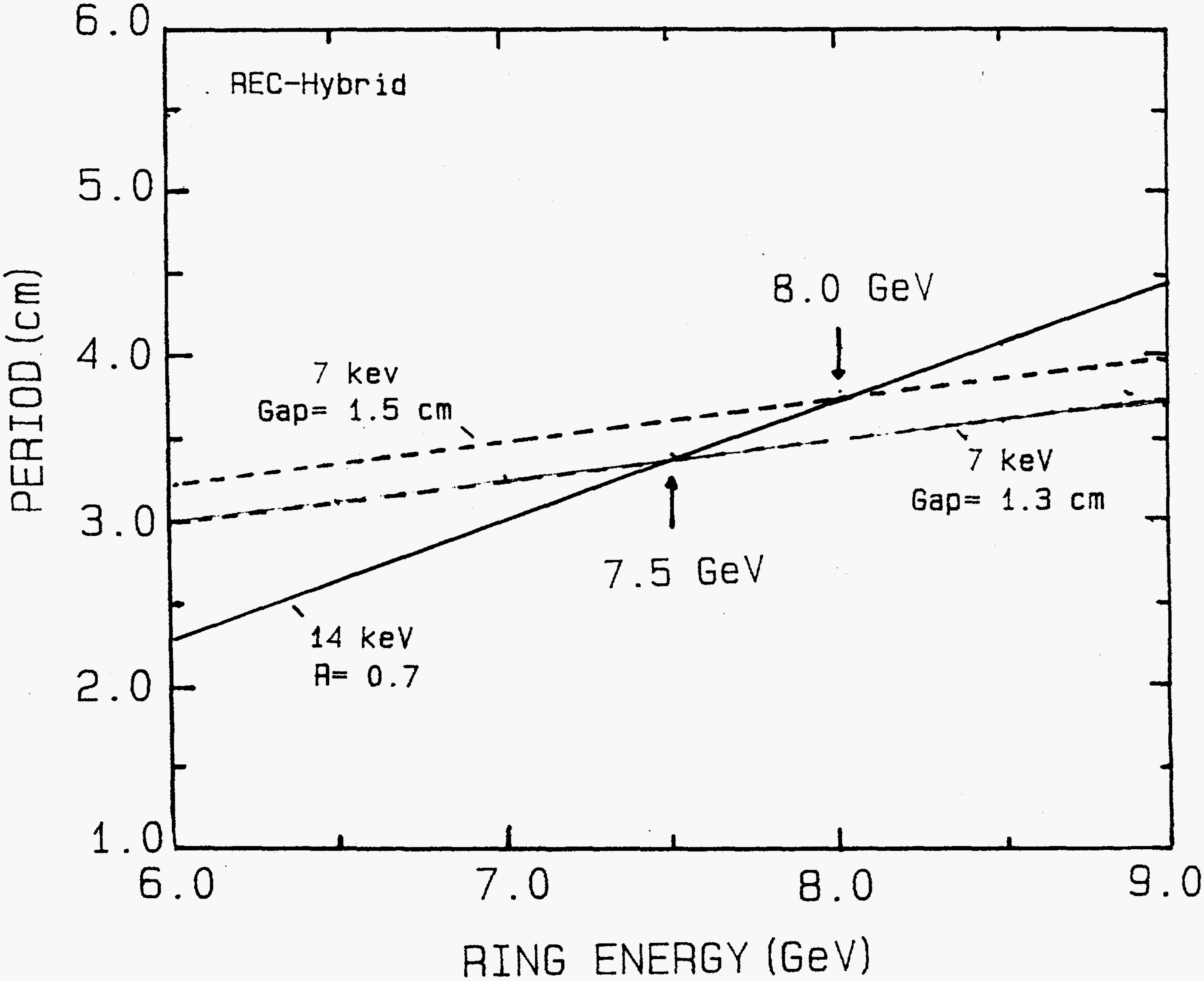
REC-Hybrid
Minimum Gap (cm) = 1.0
1.1
1.4
1.5
RING ENERGY (GeV)
PHOTON ENERGY (KeV)
6.00
6.50
7.00
7.50
8.00
5.00
10.0
15.0
20.0
25.0

Fig. 6



— REC-Hybrid
... Nd-Fe-B
14 keV
16 keV
18 keV
20 keV
K
1.00
0.50
0.00
6.00
6.50
7.00
7.50
8.00
RING ENERGY (GeV)

Fig. 7



REC-Hybrid
8.0 GeV
7.5 GeV
7 kev
Gap= 1.5 cm
7 keV
Gap= 1.3 cm
14 keV
A= 0.7
PERIOD (cm)
RING ENERGY (GeV)
6.0
5.0
4.0
3.0
2.0
1.0
6.0
7.0
8.0
9.0

Fig. 8